An Entropy Method for Diversified Fuzzy Portfolio Selection

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Abstract

This paper proposes an entropy method for diversified fuzzy portfolio selection. Proportion entropy is introduced and credibilistic mean-variance and mean-semivariance diversification models for fuzzy portfolio selection are proposed. The crisp forms of the proposed models are also provided when the security returns are all triangular fuzzy variables. As an illustration, an application example of mean-variance diversification model is given using real data from Shanghai Stock Exchange. The computation results show that the proposed model results in a more diversified investment than the credibilistic mean-variance model.

Keywords: entropy, fuzzy portfolio selection, mean-variance model, mean-semivariance model.

1. Introduction

Since the introduction of entropy by Shannon [1], entropy has been applied in financial investment. In the area of portfolio selection, the applications of entropy were researched mainly in two directions. In the first direction, entropy was used as the uncertainty measure of the portfolio return. In this direction, entropy was applied mainly in two ways. In one way, scholars such as Cozzolino and Zahner [2], Popkov [3], Smimou et al. [4], tried to use the principle of maximum entropy to derive the most unbiased probability distributions of future security returns for investors having limited information in different situations. The idea proposed by them, as what Cozzolino and Zahner said, is that: “... the probability distribution desired has maximum uncertainty (minimum information content) subject to representing some explicitly stated known information ...”. In another way, researchers tried to use entropy of a portfolio return as the measure of risk in the sense that uncertainty causes loss and so investors dislike uncertainty. In this context, Philippatos and Wilson [5] were the first to suggest that entropy be an alternative measure of risk to replace variance. Philippatos and Gressis [6] further provided conditions of equivalence among the mean-variance criterion, second-degree stochastic dominance criterion and mean-entropy criterion for portfolios whose returns are uniformly, normally, or lognormally distributed. Later, Nawrocki and Harding [7] proposed two alternative calculations of entropy as the measure of risk. Simonelli [8] used Shannon's entropy as an index of indeterminacy to reflect the increase of the risk from a portfolio. Rödder et al. [9], and Usta and Kantar [10] proposed mean-variance-entropy models. Huang [11] employed credibility-based fuzzy entropy [12] to measure the risk of fuzzy portfolios and proposed two fuzzy mean-entropy models. Xu et al. [13] proposed $\lambda$ mean-entropy model to deal with portfolio selection problem with both random uncertainty and fuzzy uncertainty.

The second direction of applications of entropy is connected with the investment diversification. There is a famous saying that one should not put all the eggs into one basket. However, it is found that optimal portfolio produced by traditional mean-variance model is often extremely concentrated on a few securities [14]. Since variance is only an average level of deviation from the expected return, it is still likely that the selected security returns deviate greatly from the mean even when the variance constraint is satisfied. If the selected portfolio is quite concentrative, once one selected security return deviated greatly from the mean, the investors would suffer greatly. However, if the portfolio is diversified enough, it is rare that several or all the selected security returns would simultaneously deviate greatly from the mean. Therefore, Kapur and Kesavan [15], Kapur [16], Fang, Rajasekera and Tsao [17], Jana et al. [14] used investment proportions to replace probabilities and used entropy as the divergence measure of asset portfolio to improve the traditional mean-variance models in different situations.

Since the security market is so complex that in many situations the historical data of the security performances can hardly reflect their future returns. The evaluation and prediction of the security returns contains much subjective guessing. With the introduction and development of fuzzy set theory and credibility theory, scholars began to use them to help solve fuzzy portfolio selection problems. Numerous portfolio selection models containing fuzzy parameters are proposed. For example, Bilbao-Terol et al. [18], Gupta, et al. [19], Zhang et al. [20]
proposed different possibilistic mean-variance models. Based on credibility measure, Huang developed fuzzy mean-variance models [21] and further fuzzy mean-semivariance models [22]. Qin, Li and Ji proposed fuzzy mean-variance-cross entropy models [23]. Li et al. [24] proposed fuzzy mean-variance-skewness models, and Bhattacharyya et al. [25] proposed fuzzy mean-variance-skewness portfolio selection models by interval analysis. A review of the fuzzy portfolio selection can be found in book [26]. However, it is also found that optimal portfolio produced by fuzzy mean-variance [21] and mean-semivariance [22] models may also be concentrative sometimes. Motivated by the applications of entropy in random portfolio selection in research works [14, 15, 16, 17], in this paper, we will use the entropy method to fuzzy environment to propose the fuzzy diversification models. It is interesting to point out that though we are selecting fuzzy portfolios, we are using the concept of random entropy and its properties to measure the divergence degree of asset portfolio and to help select a diversified portfolio within the framework of mean-variance and mean-semivariance selection idea.

The rest of the paper is organized as follows. Section 2 will briefly review the important knowledge about Shannon entropy and discuss its usefulness in portfolio selection. Section 3 will propose fuzzy diversification models using entropy to help investors hold a diversified securities within the framework of mean-variance and mean-semivariance selection idea. Section 4 will provide the crisp equivalents of the optimization models when security returns are all triangular fuzzy variables, and Section 5 will present one example using real data from Shanghai Stock Exchange. Finally, Section 6 will give the conclusion remarks.

2. Entropy and Diversification

Entropy is a measure of uncertainty proposed by Shannon. Let \( \eta \) be a discrete random variable taking values \( y_i \) at probabilities \( p_i, i = 1, 2, \ldots, n \), respectively. Then its entropy is defined by [1]

\[
H[\eta] = -\sum_{i=1}^{n} p_i \ln p_i. \tag{1}
\]

The entropy value does not depend on the actual values that the random variable takes, but depends only on the number of values and their probabilities of occurrence. It has been proven that the entropy has following properties:

1. The entropy value will reach its minimum of 0 if and only if there exists an index \( k \) such that \( p_k = 1 \).

2. The entropy value will reach its maximum of \( \ln n \) if and only if \( p_i \equiv 1/n \) for all \( i = 1, 2, \ldots, n \).

The two properties tell us that the entropy measure reflects how equal the probabilities \( p_1, p_2, \ldots, p_n \) are among themselves. The greater the value the entropy, the closer the random variable is to the equi-probable random variable, or the vice versa. In portfolio selection problem, the more uniformly the capital is allocated to all the securities, the more diverse the investment is. If all the capital is allocated to just one security, the investment is extremely concentrative. Notice that the investment proportions, denoted by \( x_i \), are required to be \( x_i \geq 0 \) for \( i = 1, 2, \ldots, n \), and \( \sum_{i=1}^{n} x_i = 1 \), which is just the requirement for \( p_i \). Thus, we replace the probabilities in Equation (1) by investment proportions and make use of the properties of entropy to reflect how much the portfolio is diversified. For clarity, we call this entropy the proportion entropy.

Let \( x_i \) denote the investment proportions in the \( i \)-th securities. Then

\[
H = -\sum_{i=1}^{n} x_i \ln x_i. \tag{2}
\]

is called the proportion entropy. From the properties of Shannon entropy we just reviewed, it is easy to see that

1. The proportion entropy value will reach its minimum of 0 if and only if there exists an index \( k \) such that \( x_k = 1 \), which implies that if the proportion entropy has the minimum value of 0, the capital is concentrated to just one security.

2. The proportion entropy value will reach its maximum of \( \ln n \) if and only if \( x_i \equiv 1/n \) for all \( i = 1, 2, \ldots, n \), which implies that if the proportion entropy has the maximum value of \( \ln n \), the capital is uniformly allocated to all the securities. The greater value the proportion entropy has, the more diversely the capital is allocated to the alternative securities.

3. Fuzzy Diversification Models

Mean-variance models were first proposed by Markowitz [27]. He regarded the expected return of the portfolio as the investment return and variance the investment risk. In paper [21], Huang extended the idea to fuzzy portfolio selection and proposed the credibilistic mean-variance model as follows:

\[
\max E[x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n] \\
\text{subject to:} \\
V[x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n] \leq \alpha \\
x_1 + x_2 + \cdots + x_n = 1 \\
x_i \geq 0, \quad i = 1, 2, \ldots, n,
\]

where \( x_i \) denotes the investment proportion in the \( i \)-th
security, $ξ_i$ the fuzzy return of the $i$-th security, $E$ and $V$ the expected value operator and variance value operator of fuzzy variables defined by Liu and Liu [28] based on credibility measure, and $α$ the investors’ maximum tolerable variance level. For easy understanding of the paper, we provide here the definitions of credibility measure and the expected and variance value operators as follows:

Let $ξ$ be a fuzzy variable with membership function $μ$. The credibility measure is defined as [28]

$$Cr[ξ ∈ A] = \frac{1}{2} \left( \sup_{x ∈ A} μ(x) + 1 − \sup_{x \notin A} μ(x) \right).$$

(4)

for any set $A$ of real numbers. It is easy to see that credibility is self-dual.

The expected value of $ξ$ is defined as [28]

$$E[ξ] = \int_{−∞}^{+∞} Cr[ξ ≥ r]dr − \int_{−∞}^{0} Cr[ξ ≤ r]dr$$

(5)

provided that at least one of the two integrals is finite. If the fuzzy variable $ξ$ has a finite expected value, then its variance is defined as [28]

$$V[ξ] = E[(ξ − E[ξ])^2].$$

(6)

It has been found that an optimal portfolio produced by the credibilistic mean-variance model (3) may be quite concentrative sometimes. As it can be seen from the equations (5) and (6), variance is only an average deviation information. The event of the selected security returns being quite low (deviating greatly from the mean) may still happen even when the variance constraint is satisfied. However, if the optimal portfolio is diversified enough, several or all the selected security returns being low simultaneously will be by far less likely to occur. Thus, for conservative investors, we propose the diversified mean-variance model as follows:

$$\begin{align*}
\max & \quad E[x_1ξ_1 + x_2ξ_2 + \cdots + x_nξ_n] \\
\text{subject to:} & \quad V[x_1ξ_1 + x_2ξ_2 + \cdots + x_nξ_n] \leq \bar{α} \\
& \quad −x_i \ln x_i − x_i \ln x_i − \cdots − x_i \ln x_i \geq \bar{β} \\
& \quad x_i + x_2 + \cdots + x_n = 1 \\
& \quad x_i ≥ 0, \quad i = 1, 2, \cdots, n,
\end{align*}$$

(7)

where $\bar{α}$ is the maximum variance level the investor can tolerate, and $\bar{β}$ the preset entropy level. It is seen that the proportion entropy

$−x_i \ln x_i − x_i \ln x_i − \cdots − x_i \ln x_i$ has nothing to do with the fuzzy security returns. Instead, it measures the diversification degree of the capital allocation, i.e., whether the capital is allocated to only a few securities or to a variety of securities. Therefore, the proportion entropy cannot be used as a risk measure itself, and must be combined with a measure of risk such that it can serve as a complementary means to reduce risk. It has been previously pointed out that the mean-variance selection framework without entropy constraint may lead to quite concentrative result [21]. However, when proportion entropy constraint is added, the solution must satisfy the preset diversification requirement, which avoids the concentrative allocation. For example, when the preset entropy value is the minimum value of 0, it has the same effect as that without entropy constraint, and the capital may be allocated to just one security. However, when the entropy value is set higher, the allocated money must be diversified enough to meet the entropy requirement. When the preset entropy value reaches the maximum value of $\ln n$, the capital will be evenly distributed to all the $n$ numbers of securities. Thus, the investors can choose the entropy value $β$ from the interval $(0, \ln n)$ according to their own requirement of diversification. We can see that compared with the traditional mean-variance idea, the new model (7) will ensure to produce a more diversified portfolio.

Though it is usually adopted that the security returns are symmetrical, there do exist empirical evidences [29] indicating that many security returns are not symmetrically distributed. In the case where the fuzzy security returns are asymmetrical, we can use semivariance to replace the variance. Then the model (7) becomes

$$\begin{align*}
\max & \quad E[x_1ξ_1 + x_2ξ_2 + \cdots + x_nξ_n] \\
\text{subject to:} & \quad SV[x_1ξ_1 + x_2ξ_2 + \cdots + x_nξ_n] \leq \bar{α} \\
& \quad −x_i \ln x_i − x_i \ln x_i − \cdots − x_i \ln x_i \geq \bar{β} \\
& \quad x_i + x_2 + \cdots + x_n = 1 \\
& \quad x_i ≥ 0, \quad i = 1, 2, \cdots, n,
\end{align*}$$

(8)

where $\bar{α}$ is the maximum semivariance level the investor can tolerate, and $SV$ the semivariance operator of fuzzy variables defined as [22]

$$SV[ξ] = E[(ξ − E[ξ])^2].$$

(9)

where $ξ$ is a fuzzy variable and $(ξ − E[ξ])^2 = \min{ξ − E[ξ], 0}$.

4. Crisp Forms of the Models

Assume each security return is the symmetrical triangular fuzzy variable denoted by $ξ_i = (a_i, b_i, c_i)$ with $c_i − b_i = b_i − a_i, i = 1, 2, \cdots, n$, whose membership function is given by

$$μ_ξ(r_i) = \begin{cases} r_i − a_i / b_i − a_i, & \text{if } a_i ≤ r_i ≤ b_i \\ r_i − c_i / b_i − c_i, & \text{if } b_i ≤ r_i ≤ c_i \\ 0, & \text{otherwise.} \end{cases}$$

According to the equations (4), (5) and (6), it can be
obtained that for a symmetrical triangular fuzzy portfolio return \( \eta = (a, b, c) \), its expected value is \( E[\eta] = b \) and variance value is \( V[\eta] = \frac{(c-a)^2}{24} \). It also has been given [30] that a weighted sum of triangular fuzzy variables is also a triangular fuzzy variable. That is, if fuzzy variables \( \eta_1 = (a_1, b_1, c_1) \) and \( \eta_2 = (a_2, b_2, c_2) \) are triangular fuzzy variables, then for any real numbers \( \lambda_1 \geq 0 \) and \( \lambda_2 \geq 0 \), the fuzzy variable \( \lambda_1 \eta_1 + \lambda_2 \eta_2 = (\lambda_1 a_1 + \lambda_2 a_2, \lambda_1 b_1 + \lambda_2 b_2, \lambda_1 c_1 + \lambda_2 c_2) \) is also a triangular fuzzy variable. Thus the model (7) can be converted into the following model:

\[
\begin{align*}
\text{max} & \sum_{i=1}^{n} x_i b_i \\
\text{subject to:} & \sum_{i=1}^{n} x_i c_i - \sum_{i=1}^{n} x_i a_i \leq \sqrt{24} a \\
& \sum_{i=1}^{n} x_i \ln x_i \geq \beta \\
& \sum_{i=1}^{n} x_i = 1 \\
& x_i \geq 0, \quad i = 1, 2, \ldots, n.
\end{align*}
\] (10)

According to the equations (4), (5) and (9), it can be proven that for an asymmetrical triangular fuzzy portfolio return \( \eta = (a, b, c) \) its expected value and the semivariance value are \( E[\eta] = e = \frac{a+2b+c}{4} \) and \( V[\eta] = \frac{(e-a)^2}{6(b-a)} \) when \( b-a > c-b \), which is the case that the investors welcome. Thus, the model (8) can be converted into the following model:

\[
\begin{align*}
\text{max} & \sum_{i=1}^{n} x_i a_i + \sum_{i=1}^{n} x_i b_i + \sum_{i=1}^{n} x_i c_i \\
\text{subject to:} & \sum_{i=1}^{n} x_i b_i + \sum_{i=1}^{n} x_i c_i - \sum_{i=1}^{n} x_i a_i \leq 384 a \\
& \sum_{i=1}^{n} x_i c_i - \sum_{i=1}^{n} x_i b_i < \sum_{i=1}^{n} x_i b_i - \sum_{i=1}^{n} x_i a_i \\
& \sum_{i=1}^{n} x_i \ln x_i \geq \beta \\
& \sum_{i=1}^{n} x_i = 1 \\
& x_i \geq 0, \quad i = 1, 2, \ldots, n.
\end{align*}
\] (11)

5. One Example

To help understand the optimization idea, we give one example. Assume that the investor chooses 8 securities from different industries from Shanghai Stock Exchange for investment. The investment duration is one month. The security returns, denoted by \( \xi_i \) are defined as \( \xi_i = (p_{i+1} + d_i - p_i) / p_i \), \( i = 1, 2, \ldots, 8 \), respectively, where \( p_{i+1} \) are the estimated closing prices of the securities \( i \) in the next month, \( p_i \) the closing prices of the securities \( i \) at present, and \( d_i \) the estimated dividends of the securities \( i \) during the month. Historical data of the 8 securities from January 2005 to December 2007 are collected. The monthly returns of each security for 36 periods are obtained. Based on the experts’ knowledge and the data of midrange, minimum return and maximum return of 36 data of the \( i \)-th security, the security returns are regarded to be symmetrical triangular fuzzy variables, i.e., \( \xi_i = (a_i, b_i, c_i) \) with \( c_i - b_i = b_i - a_i, i = 1, 2, \ldots, 8 \). The data of \( a_i, b_i, c_i, i = 1, 2, \ldots, 8 \), are given in Table 1.

<table>
<thead>
<tr>
<th>Security</th>
<th>Security code</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>600570</td>
<td>-0.2818</td>
<td>0.2446</td>
<td>0.7710</td>
</tr>
<tr>
<td>2</td>
<td>600821</td>
<td>-0.4041</td>
<td>0.0621</td>
<td>0.5283</td>
</tr>
<tr>
<td>3</td>
<td>600019</td>
<td>-0.2156</td>
<td>0.0933</td>
<td>0.4021</td>
</tr>
<tr>
<td>4</td>
<td>600050</td>
<td>-0.1741</td>
<td>0.1557</td>
<td>0.4855</td>
</tr>
<tr>
<td>5</td>
<td>600030</td>
<td>-0.2366</td>
<td>0.1588</td>
<td>0.5511</td>
</tr>
<tr>
<td>6</td>
<td>600063</td>
<td>-0.2678</td>
<td>0.2186</td>
<td>0.7051</td>
</tr>
<tr>
<td>7</td>
<td>600791</td>
<td>-0.3712</td>
<td>0.0445</td>
<td>0.4603</td>
</tr>
<tr>
<td>8</td>
<td>600591</td>
<td>-0.2432</td>
<td>0.1855</td>
<td>0.6141</td>
</tr>
</tbody>
</table>

Example 1: Suppose the investor sets the variance level at 0.04 and entropy level at 1.0. Since the membership functions of the eight alternative security returns are all symmetrical, mean-variance diversification model is adopted. If the investor wants to pursue the maximum expected return under the variance and the entropy constraints, according to the model (10) and the data in Table 1, we have the model as follows:

\[
\begin{align*}
\text{max} & \\sum_{i=1}^{8} 0.2446x_i + 0.0621x_i + 0.0933x_i + 0.1557x_i \\
& + 0.1588x_i + 0.2186x_i + 0.0445x_i + 0.1855x_i \\
\text{subject to:} & (0.7710 + 0.2818)x_i + (0.5283 + 0.4041)x_i + (0.4021 + 0.2156)x_i \\
& + (0.4855 + 0.1742)x_i + (0.5511 + 0.2366)x_i + (0.7050 + 0.2678)x_i \\
& + (0.4603 + 0.3712)x_i + (0.6141 + 0.2432)x_i \leq 0.04 \\
& -x_i \ln x_i - x_i \ln x_i - \cdots - x_i \ln x_i \geq 1.0 \\
& x_i + x_i + \cdots + x_i = 1 \\
& x_i \geq 0, i = 1, 2, \ldots, 8.
\end{align*}
\] (12)

By making use of the command “Solver” in Microsoft

\[\begin{align*}
\text{max} & \\sum_{i=1}^{8} 0.2446x_i + 0.0621x_i + 0.0933x_i + 0.1557x_i \\
& + 0.1588x_i + 0.2186x_i + 0.0445x_i + 0.1855x_i \\
\text{subject to:} & (0.7710 + 0.2818)x_i + (0.5283 + 0.4041)x_i + (0.4021 + 0.2156)x_i \\
& + (0.4855 + 0.1742)x_i + (0.5511 + 0.2366)x_i + (0.7050 + 0.2678)x_i \\
& + (0.4603 + 0.3712)x_i + (0.6141 + 0.2432)x_i \leq 0.04 \\
& -x_i \ln x_i - x_i \ln x_i - \cdots - x_i \ln x_i \geq 1.0 \\
& x_i + x_i + \cdots + x_i = 1 \\
& x_i \geq 0, i = 1, 2, \ldots, 8.
\end{align*}\]
Excel, the optimal portfolio is obtained. In order to obtain the maximum expected return at the preset variance value 0.04 and entropy value 1.0, the investor should allocate his or her money according to Table 2. The maximum expected return is 0.2257. We can see that the capital is allocated comparatively more to the securities 1, 4 and 6 whose expected returns are high, and in the meantime the portfolio is comparatively diversified since the portfolio contains six securities.

Table 2. Allocation of money to 8 securities (%).

<table>
<thead>
<tr>
<th>Security</th>
<th>Security code</th>
<th>Money allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>600570</td>
<td>68.87</td>
</tr>
<tr>
<td>2</td>
<td>600821</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>600019</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>600050</td>
<td>11.99</td>
</tr>
<tr>
<td>5</td>
<td>600030</td>
<td>3.81</td>
</tr>
<tr>
<td>6</td>
<td>600063</td>
<td>12.56</td>
</tr>
<tr>
<td>7</td>
<td>600791</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>600591</td>
<td>2.66</td>
</tr>
</tbody>
</table>

Table 3. Optimal money allocation with different preset entropy values (%)..

<table>
<thead>
<tr>
<th>β (β)</th>
<th>0.6000</th>
<th>1.0000</th>
<th>1.6000</th>
<th>1.8000</th>
<th>2.0000</th>
<th>2.0794</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obj</td>
<td>0.2281</td>
<td>0.2257</td>
<td>0.2070</td>
<td>0.1938</td>
<td>0.1720</td>
<td>0.1460</td>
</tr>
<tr>
<td>600570</td>
<td>79.17</td>
<td>68.87</td>
<td>39.88</td>
<td>31.31</td>
<td>21.12</td>
<td>12.67</td>
</tr>
<tr>
<td>600821</td>
<td>0.00</td>
<td>0.00</td>
<td>1.47</td>
<td>3.16</td>
<td>6.90</td>
<td>12.36</td>
</tr>
<tr>
<td>600019</td>
<td>0.00</td>
<td>0.10</td>
<td>2.58</td>
<td>4.67</td>
<td>8.35</td>
<td>12.41</td>
</tr>
<tr>
<td>600050</td>
<td>17.97</td>
<td>11.99</td>
<td>7.98</td>
<td>10.23</td>
<td>12.24</td>
<td>12.52</td>
</tr>
<tr>
<td>600030</td>
<td>0.00</td>
<td>3.81</td>
<td>8.44</td>
<td>10.64</td>
<td>12.47</td>
<td>12.52</td>
</tr>
<tr>
<td>600063</td>
<td>2.74</td>
<td>12.56</td>
<td>24.91</td>
<td>22.58</td>
<td>18.02</td>
<td>12.63</td>
</tr>
<tr>
<td>600791</td>
<td>0.00</td>
<td>0.00</td>
<td>1.07</td>
<td>2.53</td>
<td>6.20</td>
<td>12.33</td>
</tr>
<tr>
<td>600591</td>
<td>0.12</td>
<td>2.66</td>
<td>13.68</td>
<td>14.89</td>
<td>14.69</td>
<td>12.57</td>
</tr>
</tbody>
</table>

Table 4. Optimal money allocation without entropy constraint (%).

<table>
<thead>
<tr>
<th>Objective value</th>
<th>600570 (Security 1)</th>
<th>600050 (Security 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2281</td>
<td>81.41</td>
<td>18.59</td>
</tr>
</tbody>
</table>

We further change the values of the preset entropy value β, and run the Microsoft Excel again. The preset entropy value, the maximum expected return and the optimal allocation of the money to the 8 securities are given in Table 3. In the meantime, we solve the mean-variance model without the entropy constraint. The maximum expected return and the optimal allocation of the money to the 8 securities are given in Table 4. It can be seen that without entropy constraint, the solution of the mean-variance model is concentrated on two securities 1 and 4. With the entropy constraint, the solution becomes diversified. In addition, when the preset entropy value becomes bigger, the allocation of the capital becomes more diversified. When the preset entropy value is 0.6, money is only allocated to four securities with code 600570, 600050, 600063 and 600591; when the preset entropy value is 1.0, money is allocated to six securities; when the preset entropy value is 2.0794, money is almost uniformly allocated to all the eight securities. It is also seen that accompanied with more diversified investment, the expected return becomes smaller (from 0.2281 to 0.1460), which implies that the strategy of mean-variance diversification model is not to pursue simply the uniform allocation of the money. Instead, it helps to find a required diversified portfolio in the framework of mean-variance selection philosophy.

6. Conclusions

This paper has used a proportion entropy to propose the credibilistic mean-variance-entropy and mean-semivariance-entropy models for fuzzy portfolio selection. Crisp forms of the optimization models have also been provided when the security returns are all triangular fuzzy variables. Compared with traditional mean-variance and mean-semivariance idea which may produce concentrative portfolio, the proposed method requires that the selected portfolio must satisfy the preset entropy requirement, and thus ensures a more diversified solution. The paper shows that when the preset entropy value is the minimum value of 0, the new models have the same effect as the traditional mean variance or mean semivariance method, but when the entropy value is set higher, the allocated money must be diversified enough so that the entropy requirement can be met. The capital will be evenly distributed to all the n numbers of securities when the preset entropy value reaches the maximum value of ln n. Therefore, by choosing the entropy value β from the interval (0, ln n), the investors can obtain an optimal portfolio that meet their requirement for diversification. The results of the example provided in the paper illustrate the advantage of the new method.

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References

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