Satisfactory damping of power oscillations is an important issue to be addressed when dealing with the angle stability of power systems. This phenomenon is well-known and observable especially when a fault occurs. To improve the damping of oscillations in power systems, supplementary control laws can be applied to existing devices. These supplementary actions are referred to as power oscillation damping (POD) control. In this work, POD control has been applied to two FACTS devices, TCSC and UPFC. The design method utilizes the residue approach, [1]. The presented approach solves the optimal siting of the FACTS as well as selection of the proper feedback signals and the controller design problem.

1 TCSC and UPFC current injection models

In order to investigate the impact of FACTS devices on power systems effectively, appropriate models of FACTS devices are very important. The FACTS devices of the interest in this report are the TCSC and the UPFC. Usually, for the modeling of those FACTS devices, power injection models are used. However, for a dynamic analysis, due to model requirements in MATLAB/SIMULINK, current injection models are more appropriate.

1.1 TCSC current injection model

The TCSC is assumed to be connected between buses $i$ and $j$ in a transmission line as shown in Figure 1, where the TCSC is presented simplified like a continuously controllable reactance (capacitive).

$$I_{se} = \frac{\bar{V}_i - \bar{V}_j}{r_l + j(x_i - x_c)} \tag{1}$$

Figure 1: TCSC located in a transmission line
The influence of the capacitor is equivalent to a voltage source which depends on voltages $V_i$ and $V_j$. The current injection model of the TCSC is obtained by replacing the voltage across the TCSC by an equivalent current source $I_s$ in Figure 2. In Figure 1, $V_s = -jx_c I_{se}$, and from Figure 2, we have

$$I_s = \frac{V_s}{r_i + jx_i} = -\frac{jx_c I_{se}}{r_i + jx_i} \quad (2)$$

![Figure 2: Replacement of a voltage source by a current source](image)

Current injections into nodes $i$ and $j$ are

$$T_{sj} = \frac{-jx_c}{r_i + jx_i} \cdot \frac{V_i - V_j}{r_i + j(x_i - x_c)} \quad (3)$$

$$T_{si} = -T_{sj} \quad (4)$$

and the current injection model of the TCSC as (3) and (4) is shown in Figure 3.

### 1.2 UPFC current injection model

Current injection model, derived in [2] in similar way as the TCSC, can be presented as in Figure 4, where

$$T_{si} = (-rb_s V_j \sin(\theta_{ij} + \gamma) + rb_s V_i \sin \gamma + jI_q)e^{j\theta_i} + jrb_s V_i e^{j(\theta_i + \gamma)} \quad (5)$$

$$T_{sj} = -jrb_s V_i e^{j(\theta_i + \gamma)} \quad (6)$$
2 FACTS power flow controller design

The main function of the FACTS devices considered is assumed to be power flow control. The control loop is shown in Figure 5 for the FACTS devices, in general. The design method for power flow controller is based on linearization of power flow equations around an operating point see [5]. The linearization yields gain matrices shown in Figure 5. The outputs of the gain matrices corresponds to the change of the compensation degree in the case of TCSC impedance and in case for UPFC amplitude and angle of the injected voltage.

3 Modal analysis for power system

Some basics about modal analysis, which are necessary to understand the controller design methods, are introduced in this section. Power systems are essentially nonlinear systems. To contract a linear model of a nonlinear system, linearization is performed at a given operating point. The total linearized system can be represented by the following equation:

\[
\Delta \dot{x} = A\Delta x + B\Delta u
\]

\[
\Delta y = C\Delta x + D\Delta u
\]
where
\( \Delta x \) is the state vector of length equal to the numbers of states \( n \)
\( \Delta y \) is the output vector of length \( m \)
\( \Delta u \) is the input vector of length \( r \)
\( A \) is the \( n \) by \( n \) state matrix
\( B \) is the control or input matrix of size \( n \) by \( r \)
\( C \) is the output matrix of size \( m \) by \( n \)
\( D \) is the feedforward matrix of dimensions \( m \) by \( r \).

The equation
\[
\det(A - \lambda I) = 0
\]
(8)
is refereed to as the characteristic equation of matrix \( A \) and the values of \( \lambda \) which satisfy the characteristic equation are the eigenvalues of matrix \( A \). Because the matrix \( A \) is a \( n \) by \( n \) matrix, it has \( n \) solutions of eigenvalues
\[
\lambda = \lambda_1, \lambda_2, ... \lambda_n
\]
(9)
were \( \lambda_i \neq \lambda_j, i \neq j \).

For any eigenvalue \( \lambda_i \), there is the eigenvector \( \phi_i \) which satisfies Equation
\[
A \phi_i = \lambda_i \phi_i
\]
(10)
\( \phi_i \) is called the right eigenvector of the state matrix \( A \) associated with the eigenvalue \( \lambda_i \). Each right eigenvector is a column vector with the length equal to the number of the states.

Left eigenvector associated with the eigenvalue \( \lambda_i \) is the \( n \)-row vector that satisfies
\[
\psi_i A = \lambda_i \psi_i
\]
(11)

Let \( \lambda_i = \sigma_i \pm j \omega_i \) be the \( i \)-th eigenvalue of the state matrix \( A \). The real part of the eigenvalues gives the damping, and the imaginary part gives the frequency of oscillation. The relative damping ratio is given by
\[
\xi = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}
\]
(12)
The oscillatory modes having damping ratio less than 3% are said to be critical. Using the left and right eigenvectors, the set of coupled differential equations, Equation 7, can be converted into a set of uncoupled equations
\[
\Delta \dot{z} = \Lambda z + B' \Delta u
\]
\[
\Delta y = C' \Delta z + D \Delta u
\]
(13)
where
\[
z = \Phi^{-1} \Delta x
\]
(14)
\[
\Lambda = \Phi^{-1} A \Phi
\]
(15)
\[
B' = \Phi^{-1} B
\]
(16)
\[
C' = C \Phi
\]
(17)
Each element of the vector $z$ is called a mode. The original state vector $x$ is a combination of the modes determined by the right eigenvector matrix $\Phi$. Thus, the right eigenvector describes how each mode of oscillation is distributed among the system states. In other words, it indicates on which system variables the mode is more observable. The right eigenvector is called *mode shape*.

The left eigenvector, together with system’s initial state, determines the amplitude of the mode. A left eigenvector carries mode controllability information.

Numerous indices, such as participation factors, transfer function residues and mode sensitivities can be calculated from eigenvectors. Those indices are very useful in system analysis and controller design.

### 3.1 Controllability and observability

In order to modify a mode of oscillation by feedback, the chosen input must excite the mode and it must be visible in the chosen output. The measures of these two properties are controllability and observability, respectively.

The modal controllability and observability matrices are defined with Equation (16) and (17), respectively. The $i^{th}$ mode is controllable by the $j^{th}$ input if the product of $\psi_i B_j$ is not zero. The $i^{th}$ mode is observable in the $j^{th}$ output if the product of $C_j \phi_i$ is not zero.

If a mode is either uncontrollable or unobservable, the feedback between the output and the input will have no effect on the mode. A mode of the interest must be both controllable by the chosen input and observable in the chosen output for a feedback control to have any effect on the mode. Therefore, determination of suitable feedback variables is an important objective in FACTS damping controller design procedure.

### 3.2 Residues

The transfer function of the SISO (single input and single output) system is:

$$G(s) = \frac{\Delta y(s)}{\Delta u(s)} = C \Phi [sI - \Lambda^{-1}] \Psi B$$

Equation (18)

$G(s)$ can be expanded in partial fractions of the Laplace transform of $y$ in terms of $C$, $B$, matrices and the right and left eigenvectors as:

$$G(s) = \sum_{i=1}^{N} \frac{C\phi(:,i)\psi(:,i)B}{(s - \lambda_i)} = \sum_{i=1}^{N} \frac{R_i}{(s - \lambda_i)}$$

Equation (19)

were $\lambda_i \neq \lambda_j, i \neq j$.

Each term in the denominator, $R_i$, of the summation is a scalar called residue. The residue $R_i$ of a particular mode $i$ gives the measure of that mode’s sensitivity to a feedback between the output $y$ and the input $u$. It is the product of the mode’s observability and controllability.

### 4 FACTS POD controller design

To damp electromechanical oscillations in power system, supplementary control action can be applied to some FACTS devices to increase the system damping. The supplementary
control is called power oscillation damping (POD). Since the FACTS devices are located in transmission systems, local input signals are always preferred, usually the active or reactive power flow through the FACTS device or the FACTS terminal voltages. POD control is applied very often on PSS. In that case the local rotor speed is the input signal for POD controller.

Figure 6 shows the considered closed-loop system where $G(s)$ represents the power system and $H(s)$ the FACTS POD controller. The POD controller consists of an amplification block, a wash-out and low-pass filters and $m_c$ stages of lead-lag blocks as depicted in Figure 5. The transfer function, $H(s)$, of the POD controller is given by Equation (20). $K$ is a positive constant gain and $H_1(s)$ is the transfer function of the wash-out and lead-lag blocks. $T_m$ is a measurement time constants and $T_w$ is the washout time constant. $T_{\text{lead}}$ and $T_{\text{lag}}$ are the lead and lag time constant respectively.

$$H(s) = K \frac{1}{1 + sT_m} \frac{sT_w}{1 + sT_{\text{lead}}} \left[ 1 + \frac{sT_{\text{lead}}}{1 + sT_{\text{lag}}} \right]^{m_c} = KH_1(s)$$  \hspace{1cm} (20)

Changes of an eigenvalue $\lambda_i$ can be described by Equation (21). The objective of the FACTS damping controller is to improve the damping ratio of the selected oscillation mode $i$. Therefore, $\Delta \lambda_i$ must be a real negative value in order to move the real part of the eigenvalue $\lambda_i$ to the left half complex plane. Figure 7 shows the displacement of the eigenvalue after FACTS damping control action.

$$\Delta \lambda_i = R_i KH_1(\lambda_i)$$ \hspace{1cm} (21)

From Equation (21), it can be clearly seen that with the same gain of the feedback loop, a larger residue will result in a larger change of the corresponding oscillatory mode. Therefore the best feedback signal for the FACTS damping controller is the one with the largest residue for the considered mode of oscillation. The same is true for the optimal location of the POD controller, which automatically means also the best location for the FACTS device.

In Figure 7, the phase angle $\varphi_{\text{comp}}$ shows the compensation angle which is needed to move the eigenvalue direct to the left parallel with the real axis. This angle will be achieved by the lead-lag function and the parameters $T_{\text{lead}}$ and $T_{\text{lag}}$ determined by the Equation. 22,
Figure 7: Shift of eigenvalues with the POD controller.

see [1] for details,

$$\varphi_{\text{comp}} = 180^\circ - \arg(R_i)$$

$$\alpha_c = \frac{T_{\text{lead}}}{T_{\text{lag}}} = \frac{1 - \sin(\varphi_{\text{comp}}/m_c)}{1 + \sin(\varphi_{\text{comp}}/m_c)}$$

$$T_{\text{lag}} = \frac{1}{\omega_i \sqrt{\alpha_c}}, \quad T_{\text{lead}} = \alpha_c T_{\text{lag}}$$

where $\arg(R_i)$ denotes the phase angle of the residue $R_i$, $\omega_i$ is the frequency of the mode of oscillation in rad/sec, $m_c$ is the number if compensation stages (usually $m_c = 2$).

The controller gain $K$ is computed as a function of the desired eigenvalue location $\lambda_{i,\text{des}}$ according to Equation 21:

$$K = \left| \frac{\lambda_{i,\text{des}} - \lambda_i}{R_i H(\lambda_i)} \right|$$

5 Application to FACTS devices

As in case of the feedback signal, the optimal sitting of the FACTS device is also very important, since the FACTS damping behavior is influenced not only by its controller parameters $H(s)$ but also by the residues. The residue method gives very effective approach to determine the sitting of FACTS devices.

As mentioned above, since the FACTS devices are located in transmission lines, local input signals like power deviation $\Delta P$, bus voltages or bus currents, are preferably used. Here, the active power deviation $\Delta P$ is used as the input signal for the TCSC controller and active and reactive power deviations $\Delta P$ and $\Delta Q$, as the input signals for the UPFC controller.

To find the best sitting for the TCSC and UPFC, different location in the system are
tested. Residues associated with critical mode are calculated using the transfer function between the TCSC active power deviation $\Delta P$ and the TCSC inputs, characterized by the line reactance deviation, $\Delta x_c$. For the UPFC the residues are calculated between active and reactive power deviations $\Delta P$ and $\Delta Q$ individually, and the UPFC inputs, which are the changes of the UPFC injected series voltage magnitude and angle, $\Delta r$ and $\Delta \gamma$. Tables 1 and 2 show the numerical results of sitting TCSC and UPFC, respectively. The test system is shown in Figure 8. The uncontrolled system has one critical oscillatory mode characterized with eigenvalue $\lambda = -0.0641 \pm j5.2713$.

5.1 Application to TCSC

According to Table 1, the line 24-25 has the largest residue and therefore it should have the most effective location to apply the feedback control. However, it was shown during simulations that this location does not have really the best influence on the oscillatory mode. That line is very short line ($x_l = 0.0089$) so the range of the line’s compensation is not wide. The next choice for POD location is the line 36-37. Here, the most effective location means the best location of the POD controller, while the compromise has to be found for the location of both, the power flow controller and POD controller. Using the method presented above, POD controller parameters are calculated in order to shift the real part of the oscillatory mode, to the left half complex plane. Figure 9 gives direct comparison between the shift of the eigenvalues before and after POD controller is applied. The POD controller influences oscillatory mode satisfactorily and, at the same time does not influence the other modes. The influence on the other modes is negligible.
The obtained transfer function for the obtained TCSC POD controller is:

\[ H(s) = 0.1191 \left( \frac{1}{0.1s + 1} \right) \left( \frac{10s}{10s + 1} \right) \left( \frac{0.0688s + 1}{0.5032s + 1} \right)^2 \]  

(24)

The gain K is calculated in order to reach the relative damping ratio bigger than 3%. Figure 10 shows the root-locus when the gain of POD controller \( K_{FACTS} \) varies from 0 to 10. It is obvious that FACTS POD controller has little influence on the local modes whereas it has significant influence on the inter-area modes of oscillations.

In order to check controller ability to stabilize the system, the three-phase fault is applied.
Figure 9: Displacement of eigenvalues without and with the proposed POD control in the line 34-37. The fault is cleared after 100 ms by opening the faulted line. In Figure 11, a direct comparison between the power flow response of the system to the fault with and without damping control is given. The reference value for the active power flow is calculated from the steady state calculation for the faulted line out of service.

Figure 10: Root-locus of the FACTS POD controller $K_{FACTS}$ varies from 0(∗) to 10(□)
5.2 Application to UPFC

There are two control variables for the UPFC, namely $\Delta r$ and $\Delta \gamma$, and two input signals for the controller, $\Delta P$ and $\Delta Q$. Therefore, it is theoretically possible to consider four possible POD control loops. However, from Table 2, where the critical mode residues of the resulting four transfer functions are calculated, one can see that $\Delta Q$ is not a good choice for the POD controller as an input signal, since the residues of $\Delta P/\Delta r$ and $\Delta P/\Delta \gamma$ have always larger values than $\Delta Q/\Delta r$ and $\Delta Q/\Delta \gamma$. Based on this fact, $\Delta P$ is considered to be a better input signal than $\Delta Q$. Hence, there are two suitable loops remaining: the first one based on the feedback signal $\Delta r$ and the second one based on the signal $\Delta \gamma$.

The corresponding transfer functions employed here are given in (13) and (14) where the lead-lag parameters were obtained according to (10), the low-pass filter time constant corresponds to approx. 2Hz (the typical upper bound for electromechanical oscillations) and the wash-out filter time constant corresponds to approx. 0.015Hz (a lower cut off frequency to remove the DC component).

\[
\frac{\Delta P}{\Delta r} = H(s) = 0.5697 \left( \frac{1}{0.1s + 1} \right) \left( \frac{10s}{10s + 1} \right)^2 \left( \frac{0.0633s + 1}{0.5363s + 1} \right)^2
\] (25)

\[
\frac{\Delta P}{\Delta \gamma} = H(s) = 3.5342 \left( \frac{1}{0.1s + 1} \right) \left( \frac{10s}{10s + 1} \right) \left( \frac{0.0805s + 1}{0.4209s + 1} \right)^2
\] (26)

From Table 2, the line 25-35 has the largest residue for the transfer function $\Delta P/\Delta r$ and therefore it would be the most effective location to apply the feedback control on $\Delta r$ variable. However, the residue of the other transfer function, $\Delta P/\Delta \gamma$, is not large. This means, the contribution of the damping controller applied to that control variable in this chosen line will be rather small, see Figures 12 and 13. According to this observation, another line should be selected for the UPFC location. The contribution to damping of oscillations with two applied POD controllers does not differ much, compared to the results when the only one POD controller, with $\Delta r$ as the input signal, is applied. That
could be expected, due to small residue value of $\Delta P/\Delta \gamma$.

Good candidates for UPFC sitting might also be the lines 36-37, 34-36 and 34-36, see Table 2. The effect of the UPFC sitting in the line 36-37 is shown in Figures 14 and 15. The corresponding transfer functions employed in this line is given by Eq. (15) and (28). The reference values for the active and reactive power flows are calculated from the steady state calculations, for the case when the faulted line is out of service.

\[
\frac{\Delta P}{\Delta r} = H(s) = 1.0644 \left( \frac{1}{0.1s + 1} \right) \left( \frac{10s}{10s + 1} \right) \left( \frac{0.0587s + 1}{0.5818s + 1} \right)^2
\]  

(27)

\[
\frac{\Delta P}{\Delta \gamma} = H(s) = 0.2585 \left( \frac{1}{0.1s + 1} \right) \left( \frac{10s}{10s + 1} \right) \left( \frac{0.0858s + 1}{0.4256s + 1} \right)^2
\]  

(28)

Figure 12: Active power flow in controlled line 25-35 after three phase fault applied to line 24-25 cleared after 100 ms.

Figure 14 shows system without damping control. Figure 15 shows the response of the controlled system with two applied damping controllers on both control variables, and with one damping controller applied on control variables separately. One can see that one damping controller satisfies the specified requirements.

6 Conclusion

This report presents a tuning method based on the residue approach applied to POD controllers for TCSC and UPFC. The line active power is used as a input signal to TCSC POD controller. The simulation results show the effectiveness of the proposed approach in damping of power system oscillations.

The residues can be used very efficiently to determine which local signal should be utilized as the input to POD controller. Besides line power flow, other local signals (the voltages or the currents) can also be considered as suitable feedback signal candidates. The influence
of the UPFC POD controller for different input signals were particularly investigated. The nonlinear simulations results show that only one UPFC POD controller applied to proper feedback signal, selected through the residue analysis, can significantly improve the dynamic performance of the power system.
Figure 15: Active and reactive power flow with damping control in controlled line 36-37 after three phase fault applied to line 34-37 cleared after 100 ms.

References


