Analysis Of Interference Reduction In Mc-Cdma System Using Binary Spreading Sequences

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Abstract: In this paper, we explore the effects of various spreading codes chiefly GMW, WG and 3-term sequences on interference levels in the multicarrier code-division multiple access (MC-CDMA) system. We evaluate the performances of these codes in terms of interference cancellation in different subcarrier systems. In the system, the data from the user is spreaded using a special sequence (code). The spreaded data is transmitted using the orthogonal frequency division multiplexing (OFDM) notion.i.e., the data is transmitted simultaneously via numerous subcarriers. The orthogonality among the subcarriers further reduces the interference level. Simulation results show that 3-term sequence performs better than other spreading sequences. Based on the simulations, we propose that 3-term sequence can be used as the spreading code in higher order subcarrier systems for mitigating interference and achieving a better bit error rate (BER) performance.

Key words: 3-term sequence, BER, GMW, MC-CDMA, OFDM, WG sequence.

INTRODUCTION

The technologies in communication systems of the future will have an extensive range of services and data rates by utilizing a number of techniques that can attain the highest achievable spectrum efficiency. Multicarrier code division multiple access (MC-CDMA), which is an arrangement of code-division multiple access (CDMA) and orthogonal frequency-division multiplexing (OFDM), has been drawing a great deal of interest (Hara S. and R. Prasad, 1997). The advantages of MC-CDMA include maximum utilization of spectrum, easy adjustment to strict channel conditions without using complex detection schemes, and high resistance to intersymbol interference (ISI) and fading caused by multipath propagation (Fazel K. and S. Kaiser, 2008). During the years, researchers have developed several versions of MC-CDMA, like multicarrier direct sequence CDMA (MC-DS-CDMA) proposed by Vandendorpe (L. Vandendorpe, 1993) and multicarrier direct sequence CDMA (MC-DS-CDMA) proposed by DaSilva and Sousa (Victor Dasilva and Elvino S Sousa, 1993). To avoid transmitter and receiver complexities and achieve maximum spectrum utilization, fast Fourier transform (FFT) is used on these signals.

Since MC-CDMA is an OFDM-based technique, it is critical to sudden time variations of the channel. The frequency offset and timing jitter cause the subcarriers to lose their orthogonality, a significant issue for MC-CDMA (Shah Set al., 2011). The loss of orthogonality between the subcarriers of a user or unwanted correlation between the spreading codes of numerous users can lead to an increase in multi-access interference (MAI).

To the best of our knowledge, in the literature, no work related to interference cancellation in MC-CDMA has considered the effect of different spreading sequences, especially GMW, WG and 3-term sequence, on interference levels and bit error rate (BER) performance. Here, we have considered the above said three codes and analysed them along with three existing and well-known spreading codes, Gold, Walsh and Kasami. In this paper, we propose, for the first time, usage of 3-term sequence as the spreading sequence for higher order subcarrier systems. We also propose the usage of the GMW sequence as an alternative to Gold and Kasami sequences for a 64 subcarrier system.

This paper is ordered as follows. First section describes the MC-CDMA system model and the parameters & expressions involved. Second section provides the literature for the different spreading sequences used in the paper. The simulation results of the comparison of the spreading codes are presented in Third section. The paper is concluded in Fourth section.

System Model:
In this work for the MC-CDMA system, multiple active users have been considered. For simplicity, pulse shaping is neglected. The MC-CDMA system model is presented in Fig. 1.
We consider an uncoded BPSK (binary phase-shift keying) MC-CDMA system with $U$ users and $N$ subcarriers. We denote the $j$th block data for user $u$ and each block has $M$ symbols as $d_u^j(i) = [d_1^u(i), d_2^u(i), ..., d_M^u(i)]^T$, (i.e.) $d_m^u(i) \in \{\pm 1\}$, $u = 1, ..., U$ and $m=1,2,....M$, where $c_u$ is the spreading code for user $u$.

The data bits are multiplied with the spreading code chips which results in a spreaded data matrix given by

$$s_u[k] = c_u[k] d_u \quad 0 \leq k \leq N-1 \quad (1)$$

where $c_u[k]$ is the $k$th element of the orthogonal code.

Since we have $N$ subcarriers, we use $N$ - point fast Fourier Transform (FFT) and inverse FFT (IFFT). It is to split the multipath channel, in the frequency domain, into $N$ narrowband channels (Cai et al., 2011). This notion is the Orthogonal Frequency Division Multiplexing (OFDM) concept.

After spreading, it is passed through the IFFT. Then, the output undergoes parallel-to-serial (P/S) conversion and gets transmitted. At the receiver side, the receiver does the S/P conversion and then passes each data block through the FFT.

The $k$th element of the FFT output can be expressed as

$$\hat{s}_u[k] = \sum_{v=0}^{U-1} s_v[k] CM_v[k] + \mu [k] \quad 0 \leq k \leq N-1 \quad (2)$$

where $CM_u[k]$ is the $k$th component of $N$ - point FFT of user $v$’s channel impulse response and $\mu$ is Additive White Gaussian Noise (AWGN) - zero mean and $N_0/2$ Power spectral density (PSD).

To detect the data transmitted by the $u$th user, $\hat{s}_u$ is multiplied by $c_u^*[k]$ and then by $CM_u^*[k]$ where $^*$ stands for complex conjugate. After the multiplication operation with frequency gains, summation of $N$ chips is done to form reconstructed data bit $\hat{d}_u$.

$$\hat{d}_u = \sum_{k=0}^{N-1} CM_u[k] c_u^*[k] \hat{s}_u[k]$$

$$= \sum_{k=0}^{N-1} CM_u[k] c_u^*[k] \left( \sum_{v=0}^{U-1} s_v[k] CM_v[k] + \mu [k] \right)$$

$$= d_u \sum_{k=0}^{N-1} CM_u[k] \hat{d}_u^k + \sum_{v=0, v \neq u}^{U-1} \sum_{k=0}^{N-1} CM_v[k] c_u^*[k] CM_v[k] c_v[k] + \sum_{k=0}^{N-1} CM_v^*[k] c_v^*[k] \mu [k] \quad (3)$$

The first term in (3) represents the multipath effect, while the second term represents MAI from user $v$ to user $u$. The orthogonality of the subcarriers and the properties of the spreading codes reduce the MAI to acceptable levels so as to increase the BER.

**Different Spreading Codes:**

The most common usage of orthogonal codes is for uplink and downlink operations of cellular CDMA systems. More than half of the spreading codes used in the current communication systems are fixed power codes and have restricted power levels. Different types of spreading codes will cause different performance results for linear detectors. This has to do with the correlation property of the codes.
A. 32 bit code:
For the generation of the 32 bit code, we initialize with the notion that the 32 bit code set has two characteristics: zero mean and linear phase. So, using the two properties, we arrive at the fact that there are around 38,000 potential codes (Garg, S. and N. Srivastava, 2011). To get the optimal orthogonal codes from these many potential codes, the orthogonal code sets are searched iteratively from the binary sample space according to the following algorithm:

1. Select an integer in the sample space and accordingly choose the first basis function of the orthogonal set. Convert the integer in the sample notation and then to bipolar notation.
2. Convert the integers in the sample set, sequentially, into binary codes and check the orthogonality of the formed codes by matching it with the first basis function to choose the next basis function.
3. Run this process for m-1 iterations to obtain m-1 orthogonal codes, while evaluating the orthogonality between the codes at the same time. Add the DC code to form an m-dimensional binary set where m is the generated code’s length.
4. Different orthogonal code pairs can be obtained by initializing the process with a different integer number basis function combination.

The code is well suited for both, synchronous and asynchronous AWGN environments. Also, it performs well in a synchronous Rayleigh environment (comparable to Walsh codes) while poorly in an asynchronous environment.

B. Walsh-Hadamard Code:
The code gets its name from an American mathematician Joseph Leonard Walsh and a French mathematician Jacques Hadamard. Walsh codes are entirely orthogonal codes, which leads to zero cross correlation between any 2 Walsh-Hadamard codes in a synchronous system. However, they perform unsuccessfully in an asynchronous environment due to low cross correlation between the codes in the asynchronous conditions.

The Walsh-Hadamard code is generated iteratively from kernels. Also, lower order Walsh code generate higher order Walsh codes iteratively. Walsh codes, The Walsh-Hadamard code converts length x messages to ‘length 2^x’ codewords. The general Hadamard matrix is shown below.

\[ H_{2M} = \begin{bmatrix} H_M & H_M \\ H_M & -H_M \end{bmatrix} \]

where M is a power of 2

Generation of Kernels is as follows:

\[ H_1 = [1] \]
\[ H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]
and so on.

C. Gold code:
The code gets its name from its creator Robart Gold. These are constructed by EXOR-ing two PN sequences of the same length with each other. Fig. 2 shows an example of the generation of the gold code using PN sequences. Gold codes are popular because of the low cross correlation values between the codes and are considered idealistic for asynchronous systems. Gold sequences exist for lengths 2^x – 1, where x ≠ multiple of 4 i.e. Gold codes are undefined for lengths of 15, 255, etc. These codes are easy to create, but the lengths of the available codes are restricted for multi-user systems. Gold code is used in MC-CDMA systems as chipping sequences that enable multiple users to have the same frequency, resulting in lower interference levels and higher spectral efficiency. A set of x Gold sequences can be generated from a chosen pair of m-sequences having the same length 2^x – 1, such that their cross-correlation value is ≤2^{(x-1)/2}. It is done by modulo-2 addition of the x cyclically shifted versions of the second chosen m-sequence and the first chosen m-sequence. The three valued autocorrelation and cross-correlation function of the Gold code has values {-1, -f(c), f(c) - 2}, where

\[ f(c) = \begin{cases} 2^{c_1} + 1 & \text{for odd } c \\ 2^{c_2} + 1 & \text{for even } c \end{cases} \]

Fig. 2: Generation of Gold sequence using PN sequences
D. Kasami sequence:
Kasami sequences are PN sequences of length \( L = 2^x - 1 \), defined only for even values of \( x \). Kasami sequences are of two types: (i) small set of Kasami sequences (ii) large set of Kasami sequences.
Small set of Kasami sequences possesses the property of matching the Welch’s lower bound for correlation functions and is thus highly favourable for usage as a spreading code. A small set of Kasami sequences is a set of \( 2^x \) binary sequences (Kumar et al., 2007).
The small set of Kasami sequences are more favourable sequences and have superior correlation properties than that of Gold sequences. But the set has a lesser number of sequences i.e. for a shiftregister of length \( x \), the number of sequences possible for the small set is only \( 2^x \) sequences, whereas Gold code set has \( 2^x + 2 \) sequences. By allowing some relaxations on the range of the correlation values, the number of Kasami sequences in the set can be increased. The set of sequences generated as a result of these relaxations is called, large set of Kasami sequences.

Construction of Kasami (small) signal sets:
Trace function \( T_{\text{Tr}}^1 (x) \) used in Kasami (small) set construction can be replaced by any orthogonal function from \( F_2^m \) to \( F_2 \). So, let \( h(x) : F_2^m \rightarrow F_2 \) be an orthogonal function & let \( t_\gamma = \{ t_\gamma, i \}_{i \geq 0} \) whose elements are given by

\[
t_{\gamma, i} = f_\gamma (a_i), \ i = 0, 1, ... \quad \text{where} \]

\[
f_\gamma (z) = h (T_{\text{Tr}}^1 (z^{2^\gamma} + z^d), \gamma \in F_2^m, z \in F_2^n
\]

So, \( f_\gamma (z) \) is the trace representation of \( t_\gamma \). A signal set \( T(h) \) consists of \( t_\gamma \forall \gamma \in F_2^m \) i.e.

\[
T(h) = \{ t_\gamma \text{ such that } \gamma \in F_2^m \} \quad \text{(6)}
\]

\( T(h) \) is called a generalized Kasami (small) signal set.

E. GMW sequence:
The Gordon, Mills, and Welch (GMW) design produces sequences that are periodical and have autocorrelation properties same as those of m-sequences (No. J, 1996). There are 4 types of such sequences: GMW sequences, Cascaded GMW and Generalized GMW (Type 3 and Type 4).
There are two methods to generate the four types of GMW sequences. First one is to apply the finite field configuration and the other is to use an interleaved structure with precomputation.

We’ve used the first approach to generate the GMW sequence.

Finite Field Configuration:
For an m-sequences having degree \( d \), we assume that \( s \leftrightarrow f(z) = T_{d} (z^m) \), where \( \text{gcd} (m, 2^d - 1) = 1 \). If \( d \) is a composite number, the resulting subfield decomposition of \( f(z) \) is:

\[
\begin{array}{c}
F_d \\
\downarrow \\
T_{d} (z^m) \\
\downarrow \\
F_p \\
\downarrow \\
T_{1} (z) \\
F_1
\end{array}
\]

where \( T_{d} (z^m) \) is a trace representation of an m-sequence over \( F_2^p \) of degree \( l = d/p \) and \( T_{1} (z) \) is a trace representation of an m-sequence over \( F_2 \) of degree \( p \). Let \( \beta \) be a primitive element in \( F_2^{d} \). Let \( s = \{ s_i \} \) be sequence of period \( 2^d - 1 \) and \( a = \{ a_i \} \) be an m-sequence over \( F_2^p \) of degree \( l = d/p \). Elements of \( s \) are given by

\[
s_i = g (a_i) \quad i = 0, 1, ...
\]

If \( g (z) = T_{1} (z) \) i.e. if \( s \) is a GMW sequence, then \( s_i \) can be produced by first raising the elements of \( \{ a_i \} \) to power \( r \) and then applying a trace function from \( F_2^p \) to \( F_2 \). So, we get

\[
s_i = T_{r} (a_i^r) \quad i = 0, 1, ...
\]

Algorithm for generating GMW sequences using Finite Fields

Input:
- \( d \), a positive integer, and \( 1 < p | d \).
- \( w (z) = z^p + w_{p-1}z_{p-1} + ... + w_1z + w_0, w_i \in F_2 \), a primitive polynomial over \( F_2 \) for generating \( F_2^p \).
- \( t (z) = z + t_{l}z_{l} + ... + t_1z + t_0, t_i \in F_2^p \) is a primitive polynomial over \( F_2^p \) of degree \( l = d/p \).
• A = (a₀, a₁, ..., a_{l-1}), b_i \in F_2^p, a nonzero vector.
• r: 1 < r < 2^{p-1}, coprime to 2^{p-1}.

Output: \( s = s_0, s_1, ..., \) a binary GMW sequence of period 2^{d-1}.

Procedure (d, p, w(z), t(z), A, r, s)
1. Generate a finite field \( F_2^p \) using \( w(z) \).
2. Use \((a_0, a_1, ..., a_{l-1})\) as the initial state of an LFSR with characteristic polynomial \( t(z) \) to generate a sequence \( a = \{a_i\} \):
   for \( i = 0, 1, ..., l-1 \), compute: \( d_i = a_i^r, s_i = Tr_1^p (d_i); \)
   for \( j = l, l+1, ..., 2^d-2 \), compute:
   \[ a_i = \sum_{j=0}^{l-1} t_j a_{j+i} \]
   \[ d_i = a_i^r \]
   \[ s_i = Tr_1^p (d_i) \]
   all computations are done in \( F_2^p \).
3. Return \( s = s_0, s_1, ..., s_{2^d-2} \).

F. Welch-Gong (WG) sequence:
Let \( d \) be a natural number (where \( d \) is not a multiple of 3), \( \beta \) be a primitive element of \( F_2^d \) and \( t(z) = z + z^{r_1} + z^{r_2} + z^{r_3} \), \( z \in F_2^d \) where the \( r_i \)'s are:

When \( d = 3m-1 \):
\[ r_1 = 2^{m} + 1 \]
\[ r_2 = 2^{2m-1} + 2^{m-1} + 1 \]
\[ r_3 = 2^m + 2^{m-1} + 1 \]
\[ r_4 = 2^{2m-1} + 2^m - 1 \]

When \( d = 3m-2 \):
\[ r_1 = 2^{m-1} + 1 \]
\[ r_2 = 2^{2m-2} + 2^{m-1} + 1 \]
\[ r_3 = 2^{2m-2} - 2^{m-1} + 1 \]
\[ r_4 = 2^{2m-1} - 2^{m-1} + 1 \]

The function defined as
\[ f(z) = Tr(t(z+1) + 1), z \in F_2^d \] (8)

is called the Welch-Gong transformation of \( Tr(t(z)) \).
Here \( f(z) \) is a function from \( F_2^d \) to \( F_2 \). Let \( p = \{p_i\} \) and \( q = \{q_i\} \) & their elements are given by
\[ p_i = Tr(t(\beta^i)), i = 0, 1, ..., \]
\[ q_i = f(\beta^i) = Tr(t(\beta^{i+1}) + 1) \] \( i = 0, 1, ... \)
\( q \) is a WG sequence and is called a WG transformation sequence of \( p \).
The period of the WG sequence is \( 2^d-1 \) and it has a 2-level autocorrelation(Gong, G. and A. M. Youseef, 2002). We have generated the WG sequence by using trinomial decimation approach.

Trinomial Decimation (For small \( d \)):
Let \( \beta \) be primitive element of \( F_2^d \) and \( q \) the WG sequence from \( p \). The elements of \( q \) are then obtained by applying an irregular decimation on \( p \):
\[ q_0 = p_0 \quad \text{and} \quad q_i = \begin{cases} p_{r(i)} & \text{if even} \ n \\ p_{r(i)} + 1 & \text{if odd} \ n \end{cases} \] (9)

or in other words, \( q_i = p_{r(i)} + d \mod 2 \) for \( i > 0 \), where \( r(i) \) is computed from
\[ \beta^{r(i)} = \beta^i + 1 \] (10)

Algorithm for WG Sequence Generator (small \( d \)):
Input :
• d – natural number (not a multiple of 3)
• h (z), primitive polynomial over F_2 of degree d
• β, a root of h(z) in F_2^d
• 5 term sequence p with p_i = Tr(t(β^i)).

Output : q = {qi}, a WG sequence of period 2^d-1

Procedure (d, p, q):
1. Generate the trinomial table of F_2^d with primitive polynomial h (z): listing τ(i) such that β^τ(i) = β^i + 1.
2. Compute q_0 = p_0, q_i = p_τ(i) + d (mod 2), i = 1, … , 2^d - 2.
3. Return {qi}.

G. 3-term sequence:
For odd d ≥ 5 and d = 2c + 1, with period p = 2^d -1, the binary sequence is

b_i = Tr(β^i) + Tr(β_1^i) + Tr(β_2^i), i = 0,1, …

where β is a primitive element of F_2^d, where
v_1 = 2^c + 1, v_2 = 2^c + 2^c-1 + 1

The period of the 3-term sequence is 2^d -1 and it has a 2-level autocorrelation (Gao et al., 2010).

Algorithm for 3-term Sequence Generator:
Input: d, f_i, i = 0, 1, 2; P_0, Q_0, R_0
Output: b = b_0, b_1, … , a binary 3-term sequence of period 2^d - 1

Procedure (d, f_0, f_1, f_2, P_0, Q_0, R_0, b):
Compute:
p_d+i = ∑_{j=0}^{d-1} c_j P_{j+i}, q_d+i = ∑_{j=0}^{d-1} d_j Q_{j+i}, and r_d+i = ∑_{j=0}^{d-1} e_j R_{j+i}, i ≥ 0
b_i = p_i + q_i + r_i, i ≥ 0
Return b

Simulations:
In MC-CDMA system, the analysis of interference reduction using various spreading codes has been done. We performed simulations to measure the average bit-error-rate performance of the system for different signal to noise ratios (SNRs). We compared the impact of GMW, WG and 3-term sequences on interference (MAI) with that of the spreading codes currently used. The number of users is 4 and the number of data bits per user is 32.

Different spreading codes of varying lengths are used in the simulations. The channel is Rayleigh fading channel. The modulation scheme chosen for the system is BPSK. The noise assumed is Additive White Gaussian Noise (AWGN). Three types of subcarrier systems are considered viz. 32, 64 and 128.

Fig. 3 shows the average BER performance of a MC-CDMA system (maximum 32 subcarriers). The spreading codes used for this system are: 32-bit code, Gold code and Walsh-Hadamard code. The simulation results show that the interference is least when the Walsh code is used. So, Walsh code is optimal for usage as spreading code in such a system to reduce MAI levels.
The average BER performance of a MC-CDMA system (maximum 64 subcarriers) is shown in Fig. 4. The spreading codes used in this system are: Gold code, Kasami sequence, GMW sequence and Walsh-Hadamard code. We find that the GMW sequence performs better in cancelling the interference than the Gold code and the Kasami sequence. So, in the systems currently being used in the world, GMW can replace Gold and Kasami as the spreading code. Again, it is evident from the simulation results that Walsh-Hadamard code is best in tackling interference and improving BER for this system.

The average BER performance of a MC-CDMA system (maximum 128 subcarriers) is shown in Fig. 5. The spreading codes used in this system are: Gold code, Welch-Gong (WG) sequence, Walsh-Hadamard code and 3-term sequence. From the results, we find that the WG sequence performs better than the Gold code and can replace the code in the communication systems of the current scenario. It is seen in the simulation results that 3-term sequence outperforms Walsh-Hadamard code in reducing the interference to a minimum for the complete range of SNR (0-30 dB).
Conclusion:

In this paper, the interference reduction in MC-CDMA system has been analysed using various spreading codes. Here we have considered three different subcarriers such as 32, 64 and 128. In 32 subcarriers we have used Gold codes, Kasmi, GMW and Walsh Hadamard code. In these codes, GMW can act as a spreading code instead of Gold code and Kasmi. But compare to all the Walsh Hadamard code performance is better than all other codes. So in 32 and 64 subcarriers Walsh Hadamard code performance is better. In 128 subcarrier we have used Gold code, WG, Walsh Hadamard code and 3-term. In these codes WG sequence perform better than Gold codes, so WG sequence can act as spreading codes instead of Gold code. In 128 subcarriers system the 3-term sequences outperforms all the codes including Walsh Hadamard code. We suggest that the 3-term sequence can be used as spreading codes in MC-CDMA system for the higher order subcarriers.

REFERENCES


