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Neuro-Fuzzy based Approach for Inverse Kinematics Solution of Industrial Robot Manipulators

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Abstract: Obtaining the joint variables that result in a desired position of the robot end-effector called as inverse kinematics is one of the most important problems in robot kinematics and control. As the complexity of robot increases, obtaining the inverse kinematics solution requires the solution of non linear equations having transcendental functions are difficult and computationally expensive. In this paper, using the ability of ANFIS (Adaptive Neuro-Fuzzy Inference System) to learn from training data, it is possible to create ANFIS, an implementation of a representative fuzzy inference system using a BP neural network-like structure, with limited mathematical representation of the system. Computer simulations conducted on 2 DOF and 3DOF robot manipulator shows the effectiveness of the approach.
Keywords: Neuro-Fuzzy, ANFIS, Robot manipulator, Inverse kinematics

1 Introduction

A robot manipulator is composed of a serial chain of rigid links connected to each other by revolute or prismatic joints. A revolute joint rotates about a motion axis and a prismatic joint slide along a motion axis. Each robot joint location is usually defined relative to neighboring joint. The relation between successive joints is described by 4×4 homogeneous transformation matrices that have orientation and position data of robots. The number of those transformation matrices determines the degrees of freedom of robots. The product of these transformation matrices produces final orientation and position data of a n degrees of freedom robot manipulator. Robot control actions are executed in the joint coordinates while robot motions are specified in the Cartesian coordinates. Conversion of the position and orientation of a robot manipulator end-effector from Cartesian space to joint space, called as inverse kinematics problem, which is of fundamental importance in calculating desired joint angles for robot manipulator design and control.

For a manipulator with n degree of freedom, at any instant of time joint variables is denoted by $\theta_i = \theta(t)$, $i = 1,2,3,...,n$ and position variables $x_j = x(t)$, $j = 1,2,3,...,m$. The relations between the end-effector position $x(t)$ and joint angle $\theta(t)$ can be represented by forward kinematic equation,

$$x(t) = f(\theta(t)) \tag{1}$$

where f is a nonlinear, continuous and differentiable function. On the other hand, with the given desired end effector position, the problem of finding the values of the joint variables is inverse kinematics, which can be solved by,

$$\theta(t) = f'(x(t)) \tag{2}$$

Solution of (2) is not unique due to nonlinear, uncertain and time varying nature of the governing equations. Figure 1 shows the schematic representation of forward and inverse kinematics. The different techniques used for solving inverse kinematics can be classified as algebraic [1], geometric [2] and iterative [3]. The algebraic methods do not guarantee closed form solutions. In case of geometric methods, closed form solutions for the first three joints of the manipulator must exist geometrically. The iterative methods converge to only a single solution depending on the starting point and will not work near singularities.

If the joints of the manipulator are more complex, the inverse kinematics solution by using these traditional methods is a time consuming. In other words, for a more generalized m degrees of freedom

manipulator, traditional methods will become prohibitive due to the high complexity of mathematical structure of the formulation. To compound the problem further, robots have to work in the real world that cannot be modeled concisely using mathematical expressions. In recent years, there have been increasing

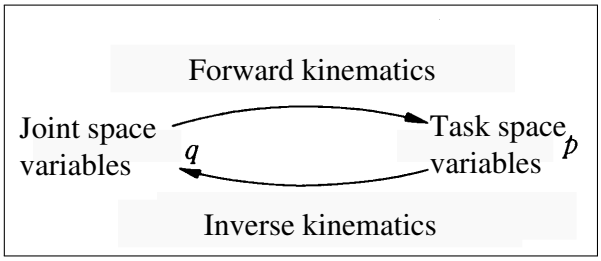

Figure 1: Schematic representation of forward and inverse kinematics

research interest of artificial neural networks and many efforts have been made on applications of neural networks to various control problems. The most significant features of neural networks are the extreme flexibility due to the learning ability and the capability of nonlinear functions approximations. This fact leads us to expect neural networks to be a excellent tool for solving the inverse kinematics problem in robot manipulators with overcoming the difficulties of algebraic, geometric and iterative methods.

Fuzzy Inference Systems are the most popular constituent of the soft computing area since they are able to represent human expertise in the form of IF antecedent THEN consequent statements. In this domain, the system behavior is modeled through the use of linguistic descriptions. Although the earliest work by Prof. Zadeh on fuzzy systems has not been paid as much attention as it deserved in early 1960s, since then the methodology has become a well-developed framework. The typical architectures of fuzzy inference systems are those introduced by Wang [4][5], Takagi and Sugeno [6] and Jang [7]. In [4], a fuzzy system having Gaussian membership functions, product inference rule and weighted average defuzzifier is constructed and has become the standard method in most applications. Takagi and Sugeno change the defuzzification procedure where dynamic systems are introduced as defuzzification subsystems. The potential advantage of the method is that under certain constraints, the stability of the system can be studied.

Utilization of Neural networks (NN) and Fuzzy logic for solving the inverse kinematics is much reported [8]-[13]. Li-Xin Wei et al [14]., and Rasit Koker et al [15]., proposed neural network based inverse kinematics solution of a robotic manipulator. There exist numerous possibilities for the fusion of neural networks and fuzzy logic technique so that both of them can overcome their individual drawbacks as well as benefit from each other's merits. Jang et al [16]., propose an Adaptive Neuro Fuzzy Inference System, in which a polynomial is used as the defuzzifier. This structure is commonly referred to as ANF1S. In this paper, neuro-fuzzy systems which provide fuzzy systems with automatic tuning using Neural network (ANFIS) is used to solve the inverse kinematics problem. The paper is organized as follows, in section 2, the structure of ANFIS used is presented. Section 3 describes simulation results and discussion. Conclusion and acknowledgment are followed in section 4 and 5 respectively.

2 ANFIS Architecture

This section introduces the basics of ANFIS network architecture and its hybrid learning rule. Inspired by the idea of basing the fuzzy logic inference procedure on a feedforward network structure, Jang [16] proposed a fuzzy neural network model - the Adaptive Neural Fuzzy Inference System or semantically equivalently, Adaptive Network-based Fuzzy Inference System (ANFIS), whose architecture is shown in Figure 2. He reported that the ANFIS architecture can be employed to model nonlinear functions, identify nonlinear components on-line in a control system, and predict a chaotic time series. It

is a hybrid neuro-fuzzy technique that brings learning capabilities of neural networks to fuzzy inference systems. The learning algorithm tunes the membership functions of a Sugeno-type Fuzzy Inference System using the training input-output data. A detailed coverage of ANFIS can be found in [7],[16]-[17]. The ANFIS is, from the topology point of view, an implementation of a representative fuzzy inference

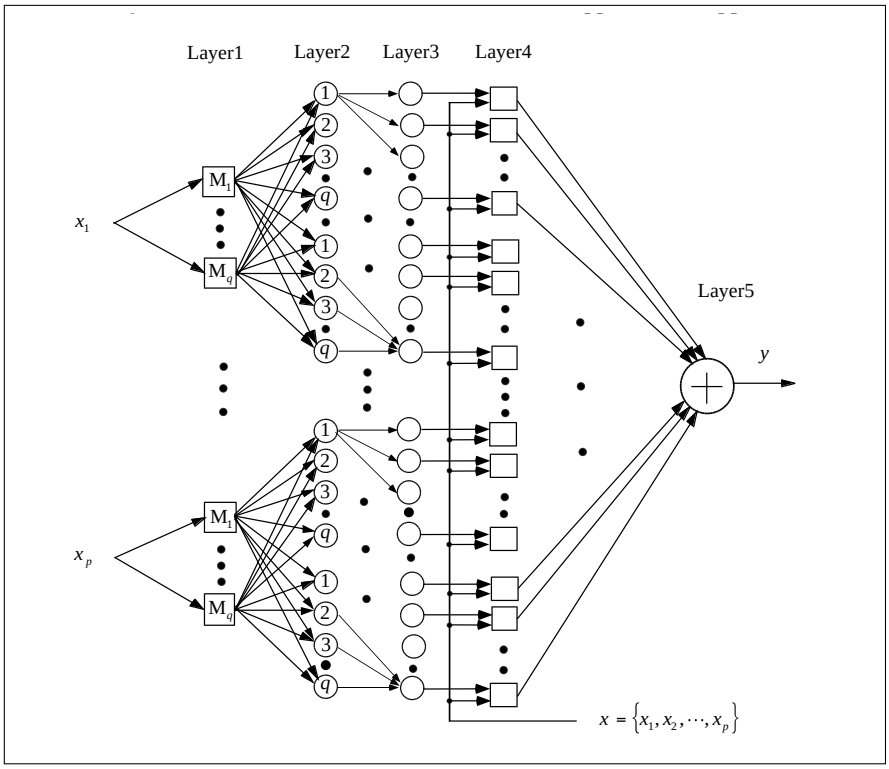

Figure 2: Structure of ANFIS

system using a BP neural network-like structure. It consists of five layers. The role of each layer is briefly presented as follows: let O_i^l denote the output of node i in layer l, and x_i is the ith input of the ANFIS, $i = 1, 2, ..., p$. In layer 1, there is a node function M associated with every node:

$$O_i^1 = M_i(x_i) \tag{3}$$

The role of the node functions $M_1, M_2, ...M_q$ here is equal to that of the membership functions $\mu(x)$ used in the regular fuzzy systems, and q is the number of nodes for each input. Gaussian shape functions are the typical choices. The adjustable parameters that determine the positions and shapes of these node functions are referred to as the premise parameters. The output of every node in layer 2 is the product of all the incoming signals:

$$O_i^2 = M_l(x_l) AND M_j(x_j) \tag{4}$$

Each node output represents the firing strength of the reasoning rule. In layer 3, each of these firing strengths of the rules is compared with the sum of all the firing strengths. Therefore, the normalized firing strengths are computed in this layer as:

$$O_i^3 = \frac{O_i^2}{\sum_i O_i^2} \tag{5}$$

Layer 4 implements the Sugeno-type inference system, i.e., a linear combination of the input variables of ANFIS, $x_1, x_2, ...x_p$ plus a constant term, $c_1, c_2, ...c_p$, form the output of each $IF - THEN$ rule. The

output of the node is a weighted sum of these intermediate outputs:

$$O_i^4 = O_i^3 \sum_{j=1}^{p} (P_j x_j + c_j) \tag{6}$$

where parameters $P_1, P_2, ..., P_p$ and $c_1, c_2, ..., c_p$, in this layer are referred to as the consequent parameters.The node in layer 5 produces the sum of its inputs, i.e., defuzziŕcation process of fuzzy system (using weighted average method) is obtained:

$$O_i^5 = \sum_i O_i^4 \tag{7}$$

The flowchart of ANFIS procedure is shown in Figure 3. ANFIS distinguishes itself from normal fuzzy logic systems by the adaptive parameters, i.e., both the premise and consequent parameters are adjustable. The most remarkable feature of the ANFIS is its hybrid learning algorithm. The adaptation

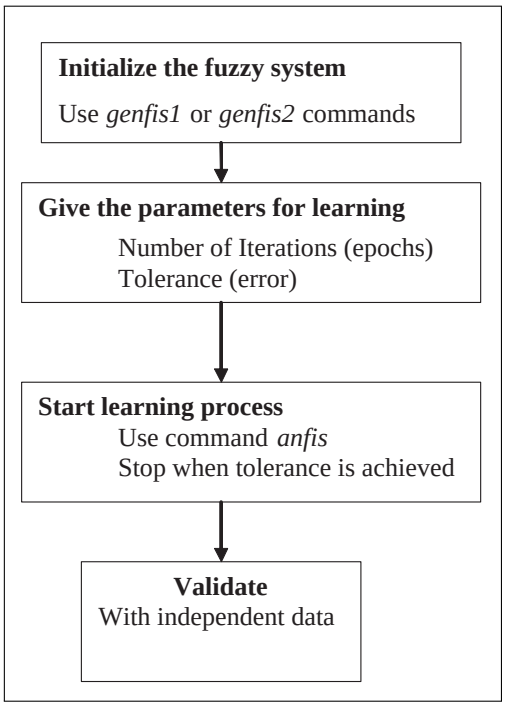

Figure 3: ANFIS procedure

process of the parameters of the ANFIS is divided into two steps. For the first step of the consequent parameters training, the Least Squares method (LS) is used, because the output of the ANFIS is a linear combination of the consequent parameters. The premise parameters are fixed at this step. After the consequent parameters have been adjusted, the approximation error is back-propagated through every layer to update the premise parameters as the second step. This part of the adaptation procedure is based on the gradient descent principle, which is the same as in the training of the BP neural network. The consequence parameters identified by the LS method are optimal in the sense of least squares under the condition that the premise parameters are fixed. Therefore, this hybrid learning algorithm is more effective than the pure gradient decent approach, because it reduces the search space dimensions of the original back propagation method. The pure BP learning process could easily be trapped into local minima. When compared with employing either one of the above two methods individually, the ANFIS converges with a smaller number of iteration steps with this hybrid learning algorithm.

This paper considers the ANFIS structure with first order Sugeno model containing 49 rules. Gaussian membership functions with product inference rule are used at the fuzzification level. Hybrid learning

algorithm that combines least square method with gradient descent method is used to adjust the parameter of membership function.

3 Simulation and Results

Figure 4 and 5 shows the two degree of freedom (DOF) and three DOF planar manipulator arm which is simulated in this work.

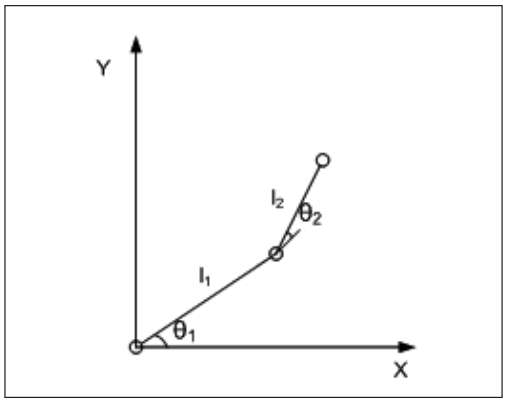

Figure 4: Two DOF manipulator

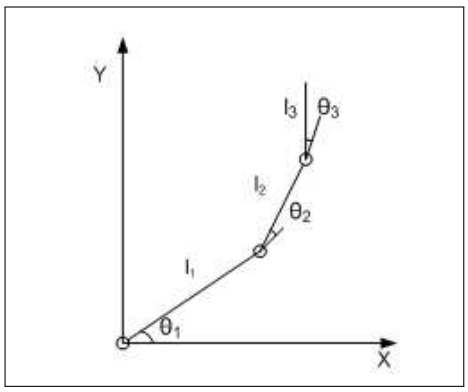

Figure 5: Three DOF manipulator

3.1 Two Degree of Freedom planar manipulator

For a 2 DOF planar manipulator having l_1 and l_2 as their link lengths and θ_1, θ_2 as joint angles with x, y as task coordinates the forward kinematic equations are,

$$x = l_1 cos(\theta_1) + l_2 cos(\theta_1 + \theta_2) \tag{8}$$

$$y = l_1 sin(\theta_1) + l_2 sin(\theta_1 + \theta_2) \tag{9}$$

and the inverse kinematics equations are,

$$\theta_1 = atan2(y, x) - atan2(k_2, k_1) \tag{10}$$

$$\theta_2 = atan2(sin\theta_2, cos\theta_2) \tag{11}$$

where, $k_1 = l_1 + l_2 cos\theta_2$, $k_2 = l_2 sin\theta_2$, $cos\theta_2 = \frac{(x^2+y^2-l_1^2-l_2^2)}{2l_1l_2}$ and $sin\theta_2 = \sqrt{\pm(1-cos^2\theta_2)}$.

Considering length of first arm l_1 = 10 and length of second arm l_2 = 7 along with joint angle constraints $0 < \theta_1 < \frac{\pi}{2}, 0 < \theta_2 < \pi$, the x and y coordinates of the arm are calculated for two joints using forward kinematics. Figure 6 shows the workspace for two link planar arm. The codes are written in MATLAB 7 Release 14.

The coordinates and the angles are used as training data to train ANFIS network with Gaussian membership function with hybrid learning algorithm. Figure 7 and Figure 8 shows the training data of two ANFIS networks for two joint angles. Figure 9 shows the difference in theta deduced analytically and the data predicted with ANFIS.

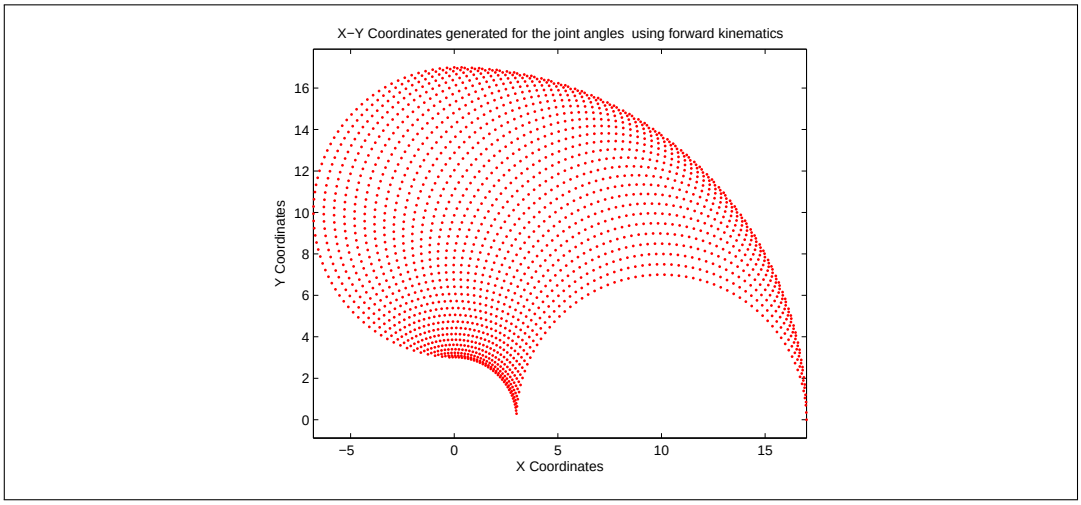

Figure 6: Workspace for two link planar arm

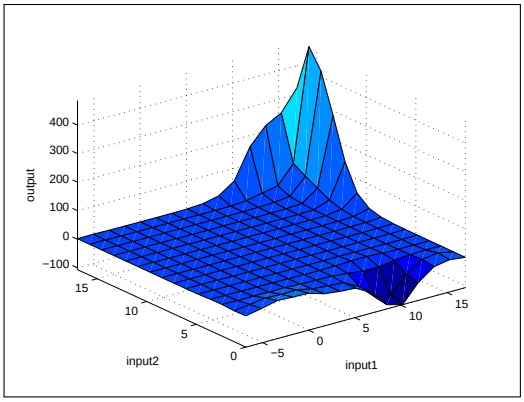

Figure 7: Training data of θ_1 for 2DOF manipulator

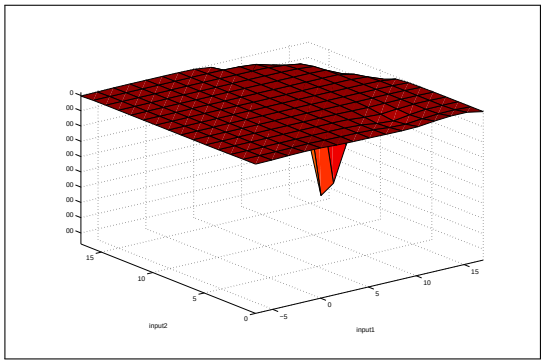

Figure 8: Training data of θ_2 for 2DOF manipulator

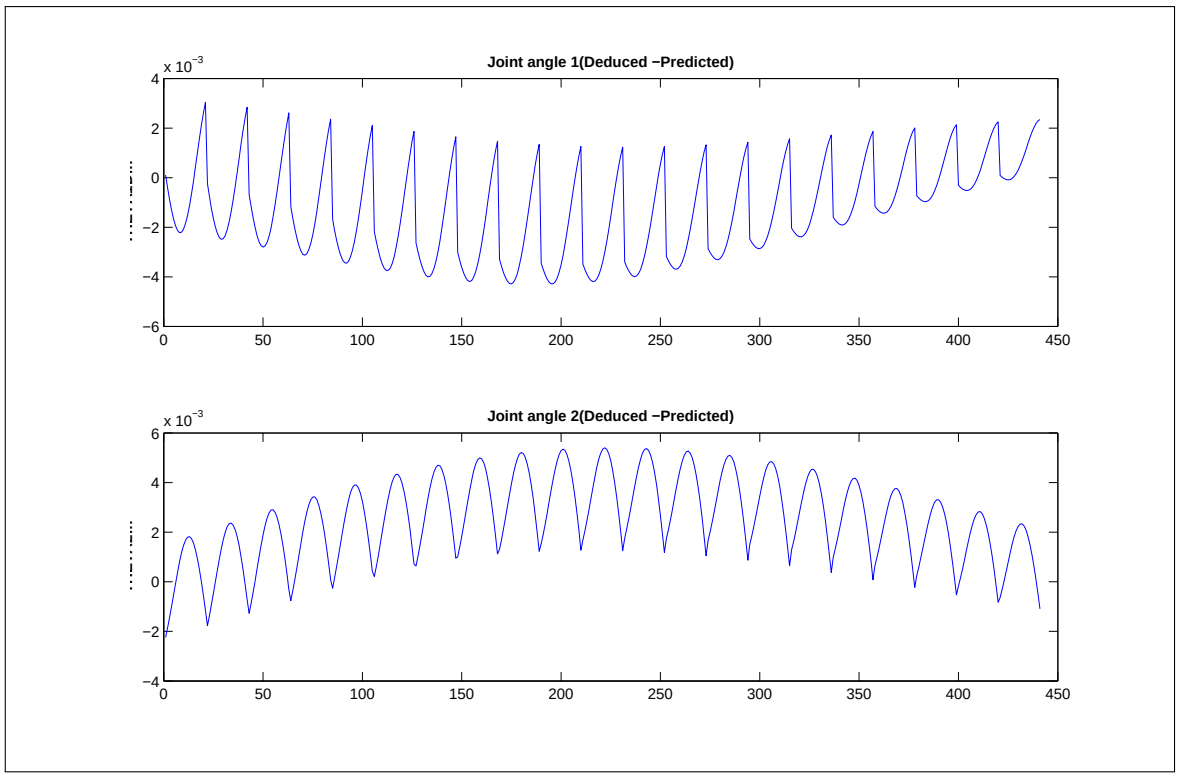

Figure 9: Difference in theta deduced and the data predicted with ANFIS trained for 2DOF manipulator

3.2 Three Degree of Freedom planar manipulator

For a 3 DOF planar redundant manipulator, the forward kinematic equations are,

$$x = l_1 cos(\theta_1) + l_2 cos(\theta_1 + \theta_2) + l_3 cos(\theta_1 + \theta_2 + \theta 3) \tag{12}$$

$$y = l_1 sin(\theta_1) + l_2 sin(\theta_1 + \theta_2) + l_3 sin(\theta_1 + \theta_2 + \theta 3) \tag{13}$$

$$\phi = \theta_1 + \theta_2 + \theta_3 \tag{14}$$

and the inverse kinematics equations are,

$$\theta_2 = atan2(sin\theta_2, cos\theta_2) \tag{15}$$

$$\theta_1 = atan2((k_1 y_n - k_2 x_n), (k_1 x_n - k_2 y_n) \tag{16}$$

$$\theta_3 = \phi - (\theta_1 + \theta_2) \tag{17}$$

where, $k_1 = l_1 + l_2 cos\theta_2$, $k_2 = l_2 sin\theta_2$, $cos\theta_2 = \frac{(x^2+y^2-l_1^2-l_2^2)}{2l_1 l_2}$, $sin\theta_2 = \sqrt{\pm(1-cos^2\theta_2)}$, $x_n = x - l_3 cos\phi$ and $y_n = y - l_3 sin\phi$.

For simulation, the length for three links are l_1 = 10, l_2 = 7 and l_3 = 5 with joint angle constraints $0 < \theta_1 < \frac{\pi}{3}, 0 < \theta_2 < \frac{\pi}{2}, 0 < \theta_3 < \pi$ coordinates of the arm are calculated for two joints using forward kinematics. Figure 10 shows the workspace for three link planar arm. The coordinates and the angles are used as training data to train ANFIS network with Gaussian membership function with hybrid learning algorithm. Figure 11, Figure 12 and Figure 13 shows the training data of three ANFIS networks for three joint angles. Figure 14 shows the difference in theta deduced analytically and the data predicted with ANFIS.

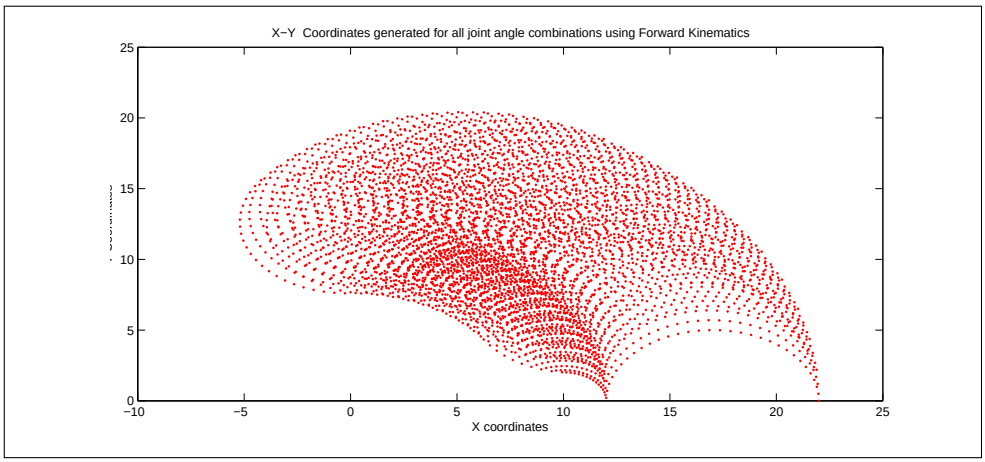

Figure 10: Workspace for three link planar arm

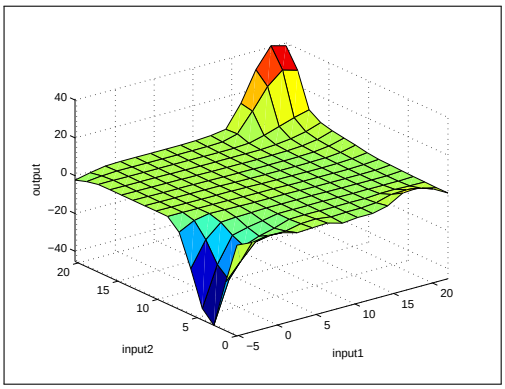

Figure 11: Training data of θ_1 for 3DOF manipulator

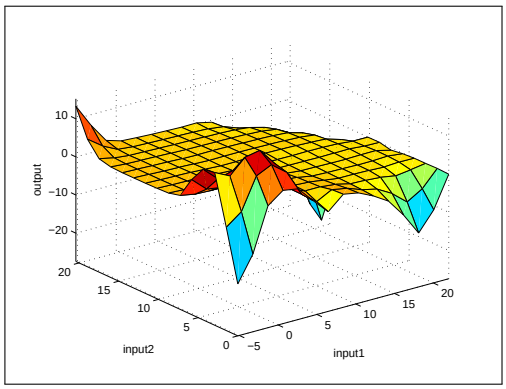

Figure 12: Training data of θ_2 for 3DOF manipulator

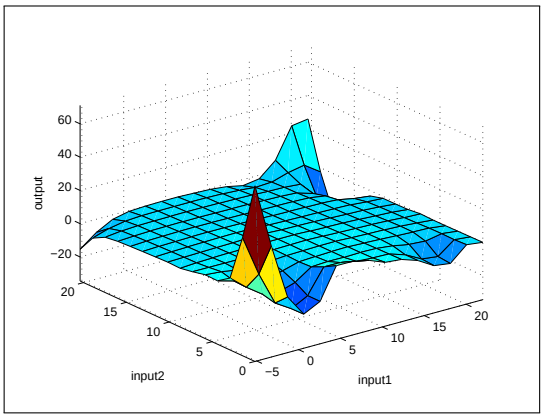

Figure 13: Training data of θ_3 for 3DOF manipulator

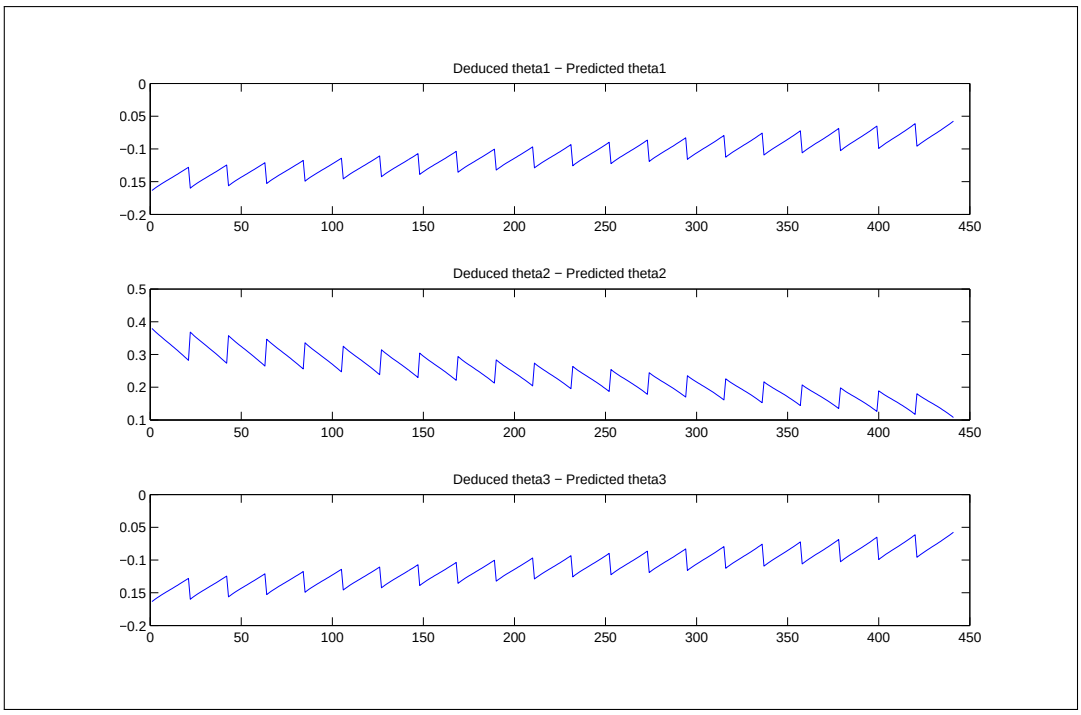

Figure 14: Difference in theta deduced and the data predicted with ANFIS trained for 3 DOF manipulator

4 Summary and Conclusions

The difference in theta deduced and the data predicted with ANFIS trained for two and three degree of freedom planar manipulator clearly depicts that the proposed method results in an acceptable error. Also the ANFIS converges with a smaller number of iteration steps with the hybrid learning algorithm. Hence trained ANFIS can be utilized to provide fast and acceptable solutions of the inverse kinematics thereby making ANFIS as an alternate approach to map the inverse kinematic solutions. Other techniques like input selection, tuning methods and alternate ways to model the problem may be explored for reducing the error further.

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