Abstract—In this paper, a miniature spherical rolling robot capable of navigating over two dimensional surfaces is described. This 55 gram spherical robot consists of a 6 cm diameter external spherical shell driven by an internal two-wheeled differential drive cart. A gravity powered pendulum effect is produced as the internal device climbs up the internal surface of the shell, propelling the robot forward up to a speed of 0.16m/s. We derived its dynamic model using Lagrangian, and studied its dynamics and performance under various applied torques. The spherical robot is built and its overall mechanical, hardware and control architecture are elaborated. Experiments are conducted to evaluate the robot open and closed loop performance on a linear trajectory, captured using an optical motion capture system. We showed that with our implemented PD controller, the robot can basically follow the desired orientation.

I. INTRODUCTION

The act of rolling is one of the most efficient forms of terrestrial locomotion. In fact, pangolins (anteater like mammals) curl up into a tight ball when threatened, thereby allowing them to roll away quickly and safely as the sphere doubles as a protective shield. This process is called conglobation, and it is also present in smaller sized species such as the armadillidiidae (pill bug or roly-poly). From an engineering point of view, a spherical body not only exhibits a high degree of mobility especially in harsh environment and terrains, it has amazing capability to recover from collisions with obstacles or other entities while traversing in the environment (Armour 2006 [1]).

Spherical robots is a special class of mobile robot that has recently received significant attention (Chen et al. 2013 [2]). They offer rapid omnidirectional planar mobility with higher efficiency than other means of locomotion. The outer shell offers natural dynamic dampening in collisions as well as smooth two-dimensional surface navigation. These advantages make the design concept superior for many applications including, among others, physical gaming (Orbotix 2013 [3]) or surveillance and reconnaissance (Seeman et al, 2006 [4]).

Artificial rolling spheres at large scales have been studied by researchers over the past two decades. The crucial issues lie in the method in which the locomotion is initiated and controlled. These techniques include changing the position of the center of mass, running a small structure on the internal surface of the sphere, an internal driving mechanism which includes a set of two mutually perpendicular rotors and the use of internal actuators installed on radial spokes which induce rolling as a result of the conservation of angular momentum. For example, Bicchi et al (1997) [5] developed a spherical robot powered a small vehicle powered within. Bhattacharya and Agrawal(2000) [6] have developed a spherical robot driven by internal sphere powered by two orthogonal axis actuators. Zhan et al (2006) [7] developed a spherical mobile robot BH-1. Ming et al (2006) [8] developed a independently powered and steered spherical mobile robot HIT. Otani et al (2006) [9] develop a gyro driven spherical robot. Zhao et al (2010) [10] and Yoon (2011) [11] develop the dynamics for a two pendulum driven spherical mobile robot. Most of the developed artificial rolling spheres are relatively large scale (sizes of basketball) and almost purely academic.

Herein these extensive previous works, we strive to develop an easily fabricated miniature spherical robot, constructed from readily available components, software and materials. The motivation is to provide a novel open platform at a miniature size (baseball size) for various applications and studies. The internal device is an easily fabricated two wheeled differential drive, mounted on circuit boards and Arduino powered and controlled. A Bluetooth communication chip enables remote wireless driving from any Bluetooth enabled device. This design is highly suitable for experimental prototyping and custom extensions for experimentation, as well as educational purposes.

The paper is organised as follows. The design concept and derivation of the spherical robot dynamics model is described in Section 2. Section 3 details our prototype design. In Section 4, we evaluate the performance of the spherical robot on a straight line trajectory. Section 5 concludes our work.

II. DESIGN CONCEPT AND DYNAMIC MODELING

In this section, we described the operational principle of our miniature spherical robot and derived its equations of motion. Simulation are also performed to study the system dynamics and performance under various applied loads.

A. Design concept

Fig. 1 depicts the side and top view of the design concept of our spherical robot for omnidirectional motion. It consists of an outer shell and a 2 DOF differential drive mechanism constrained within its inner shell to roll and turn its shell about the geometric center. The driving mechanism consists...
of a body that contains the various hardware of the spherical robot, and a pair of co-axial wheels that provides omnidirectional motion.

Planar rolling motion of the robot is achieved by displacing the equivalent center of mass (COM) of its body and wheels away from its neutral equilibrium. As shown in Fig 1(A), when the wheels rotate by $\theta_w$ in the clockwise direction, a restoring force component due to gravity will exist and accelerate the displaced COM back to its equilibrium position. This will then roll the robot shell forward by $\theta_s$ about its geometric center.

Turning about the yaw (y-axis) of the robot is achieved by turning the wheels with the same angular velocity in the opposite direction. As shown in Fig 1(B), when the wheels rotate with their rotation axis pointing away from/towards each other, a counterclockwise/clockwise turning motion $\theta_{\text{turn}}$ about the spherical robot geometric center will result.

![Fig. 1. Design concept of the spherical robot.](image)

### B. Dynamic modeling of the robot

The equations of motion of the spherical robot as it rolls along a straight line are developed using the Lagrangian. The model comprises of three major parts: i) the outer shell; ii) the body; and iii) the wheel as shown in Fig. 2. Using the parameters defined, the translational kinetic energy ($K_1$), the rotational kinetic energy ($K_2$) and the potential energy ($V$) of the system can be expressed as:

$$K_1 = \frac{1}{2} m_s (\dot{x}_s^2 + \dot{y}_s^2) + \frac{1}{2} m_b (\dot{x}_b^2 + \dot{y}_b^2) + \frac{1}{2} m_w (\dot{x}_w^2 + \dot{y}_w^2),$$

$$K_2 = \frac{1}{2} I_s \dot{\theta}_s^2 + \frac{1}{2} I_b \dot{\theta}_b^2 + \frac{1}{2} I_w (\dot{\theta}_w - \dot{\theta}_w)^2,$$

$$V = m_s g y_s + m_b g y_b + m_w g y_w.$$

Assuming non-slip rolling between each of these bodies and ground, the position vectors defining the x and y coordinates of the shell, body and wheel can be written as:

$$(x_s, y_s) = (r_s \theta_s, r_s),$$

$$(x_b, y_b) = (x_s - d \sin \theta_b, y_s - d \cos \theta_b),$$

$$(x_w, y_w) = (x_s - (r_c - r_w) \sin \theta_b, y_s - (r_c - r_w) \cos \theta_b).$$

We chose the generalized coordinates $q_i$ such that $q_1 = \theta_b$, $q_2 = \theta_w$ and holonomic constraint $\phi_1$ which defines the non-slippering condition between each of these bodies as:

$$\phi_1 : r_s (\theta_s - \theta_b) - r_w \theta_w.$$

Substituting Eq. (2) into (1), the Lagrangian of the system can be derived as:

$$\frac{d}{dt} \left( \frac{\delta L}{\delta \dot{q}_i} \right) - \frac{\delta L}{\delta q_i} = Q_i,$$

where $Q_i$ are the generalized forces and found as $Q_1 = \tau_b$ and $Q_2 = \tau_w$ and $L = K_1 + K_2 - V.$
In our design, the body and the wheels are connected by a gear mechanism to the motor. Therefore, the torques $\tau_b$ and $\tau_w$ are related to the motor torque $\tau_m$ by

\[
\begin{align*}
\tau_b &= -\tau_m, \\
\tau_w &= n_r \tau_m.
\end{align*}
\] (5)

where $\tau_m = \tau_0 - k\dot{\theta}_w$. $n_r$ is the gear ratio of the motor, $\tau_0$ is the original input torque, and $k$ is a coefficient that relates $\tau_0$ and $\tau_m$. Using Eq. (2)-(5), the equations of motion of the spherical robot can now be fully described.

C. Simulation results

The effect of a constant torque input on the system dynamics is as shown in Fig. 3 and Fig. 4. Fig. 3 shows the simulation result of the pitch angular behaviour of the cart ($\theta_b$) as the spherical robot travels forward. It shows that the cart will oscillate back and forth inside the sphere. This is due to the pendulum effect as the cart climbs up the internal surface of the shell described in the earlier section and forms the driving source for the spherical shell. Fig. 4 shows the simulation result of the $x$ trajectory of the spherical shell. It exhibits oscillatory behaviour resulting in periodically fast and slow speed.

III. PROTOTYPE DESIGN

In this section, we described our spherical robot architecture and specifications as well as the various hardware available for sensing and control.
A. Robot Architecture

The overall miniature spherical robot design and interconnections are shown in Fig. 7. It consists of a spherical outer shell and a chassis that holds the drivetrain and sensors. The shell diameter is miniature with a diameter of 6 cm. The chassis is compact, with tight clearances on the front and back. The circuit boards themselves are completely attached solely through interconnects with no bulky and failure-prone wiring. The circuit boards and DC motors are housed in a compact housing which provides clearance against the shell inner surface. The wheels are designed to conform to the inner shell diameter with the aim of reducing slippage during motion.

Fig. 7. Miniature Spherical Robot Design.

The components selected in the design are either standard off the shelf components easily attained, or are custom parts easily 3D printed on a multi-material 3D printer (CON Nex350, Stratasys). These are shown in Fig. 8. The standard components included a ATmega328P Arduino micro controller, a 6552 motor driver, and RN42 Bluetooth communication chip, and an MPU-9150 accelerometer/gyroscope (InvenSense). The motors are Polulu micro geared motors. The circuit boards were held together by custom breakout boards and connected by header pins.

Fig. 8. Miniature Spherical Robot Components.

Fig. 9 shows the hardware configuration of the spherical robot. As central processing unit, the micro controller ATmega328P is used to conduct all necessary calculation and coordination with different parts of the system. The sensor MPU-9150 contains accelerometer, gyroscope and magnetometer, which is used to provide orientation information for feedback control. The communication module uses bluetooth to receive command from host computer or mobile device.

Fig. 9. Hardware configuration of the spherical robot.

B. Fabrication

Fig. 10 shows the fabricated miniature spherical rolling robot. The overall weight of the robot is 55 grams. This is the smallest and lightest so far based on our knowledge. The 3D printed components include the internal body, wheels, support frame and geared wheels. Table I shows mechanical specifications of the miniature spherical robot.

<table>
<thead>
<tr>
<th>Mechanical parameter</th>
<th>mass or dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass of sphere</td>
<td>14 g</td>
</tr>
<tr>
<td>mass of cart</td>
<td>36 g</td>
</tr>
<tr>
<td>mass of wheels</td>
<td>1 g</td>
</tr>
<tr>
<td>mass of battery and other accessories</td>
<td>4 g</td>
</tr>
<tr>
<td>diameter of sphere</td>
<td>60 mm</td>
</tr>
<tr>
<td>diameter of wheel</td>
<td>16 mm</td>
</tr>
</tbody>
</table>
IV. EXPERIMENT

In this section, experiments are conducted to quantify the performance of the spherical rolling robot. The objective is to validate the effectiveness of the mathematical model we developed, as well as to show that the spherical robot can work in practical settings.

A. Experimental setup

The platform we use for our experiments is shown in Fig. 11. It consists of a boxed extruded aluminium frame designed to support a 2\(m\times2m\) wooden board. On this frame also mounted a set of motion capture system, which consisted of four overhead cameras from OptiTrack, to record the trajectories of the robot. A snapshot of the software environment associated with the motion capture system is shown in Fig. 12. Notice that the boxed numbers 1-4 corresponds to each of the four cameras as labeled on the platform. The orange dot at the side of camera 4 denotes the current location of the spherical robot.

B. Experimental results

In the following, we compared the performance of the miniature spherical robot moving along a linear trajectory under both open-loop and proportional-derivative yaw orientation control. Fig. 13 shows the results of both methods tracked using the motion capture system.

For open-loop control, we initialized the robot initial position at \(t = 0\) to be the origin. We drive both the DC motors at 80\% maximum speed to move the robot in a linear trajectory and track the robot resulting motion. The resulting trajectory is as shown by the blue dotted line in Fig. 13. From the figure, we can see that the robot went roughly straightly for the first 0.5 meters, but deviated quite significantly from its predicted straight line upon that. As time went by, the deviation gradually grows to a notable degree. The reason for this large deviation is that though we assign the same speed command to both motors, the actual speed of the two motors may not be perfectly the same. In addition, the platform that the robot is traveling is not even, and there are uncertainties such as bumps.

In open-loop control, we obtained the pitch angle trajectory of the robot from sensor, and it is shown in Fig. 14. We see that in experiment (Fig. 14), the pitch angle moves ±20 degrees around its mean value. In simulation as shown in Fig. 3, the pitch angle moves ±30 degrees around its mean value. Also, the robot travels around 1.05 meters in simulation as shown in Fig. 4, and around 1.5
meters in experiment as shown in Fig. 13. We can see that the experimental results and experimental results are basically consistent, which validates the effectiveness of the mathematical model.

\[ e = \alpha_r - \alpha \]  \hspace{1cm} (6)

The PD control law is given as follows:

\[ u = k_p e + k_d \dot{e} \]  \hspace{1cm} (7)

where \( k_p \) and \( k_d \) are coefficients of the proportional term and derivative term, respectively. The speeds of the two wheels are updated according to:

\[ v_l(t + \Delta t) = v_l(t) - u(t)k_u \Delta t \]  \hspace{1cm} (8)

\[ v_r(t + \Delta t) = v_r(t) + u(t)k_u \Delta t \]  \hspace{1cm} (9)

where \( v_l \) and \( v_r \) are speed of the left wheel and speed of the right wheel, and \( k_u \) is a coefficient associated with \( u \). The speeds are given in this way, so that the average speed of the robot would not change as the control law rectifies the orientation error. The trajectory of the closed-loop control is the red solid line as shown in Fig. 13. We see that, whenever the robot deviates from its desired orientation/course, it will come back to its original direction, which shows the effectiveness of the PD control law.

The trajectories in Fig. 13 took 9 seconds. Thus, we can infer that the average speed of the robot is about 0.16 m/s, which is considerably fast regarding the size of the robot.

V. CONCLUSION

In this paper, the dynamic model, mechanical design and hardware implementation of a spherical rolling robot is described. Based on the formulated model, the relationship between the motion of the robot and applied torque is first investigated. From the model, we find that the applied torque is proportional to the amplitude of the internal oscillation of the cart and speed of the spherical shell. However, having large pitch oscillation is not good for maintaining the system stability. Thus, a tradeoff needs to be made. The fabricated miniature robot consists of a chassis designed to house the micro controller, sensors, wheels and other components in a compact way inside a small transparent plastic ball. Experiments are conducted on a platform with the robot trajectory tracked using a motion capture system. We showed the robot cannot move in a desired direction under open-loop control, either due to unavoidable speed differences between the two wheels or unpredictable environmental disturbances. PD control is employed to control its yaw orientation. Our experiment confirms that robot to rectify its yaw orientation error and move in its desired direction.

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