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Renormalizability of $\mathcal{N} = \frac{1}{2}$ Wess-Zumino model in superspace

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ABSTRACT

In this letter we use the spurion field approach adopted in hep-th/0307099 in order to show that by adding F and F^2 terms to the original lagrangian, the $\mathcal{N} = \frac{1}{2}$ Wess-Zumino model is renormalizable to all orders in perturbation theory. We reformulate in superspace language the proof given in the recent work hep-th/0307165 in terms of component fields.

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It has been recently shown [1, 2] that the IIB superstring in the presence of a graviphoton background defines a superspace geometry with nonanticommutative spinorial coordinates. This deformation of superspace was previously considered in [3]. Field theories defined over $\mathcal{N} = 1/2$ superspace (i.e. $\mathcal{N} = 1$ euclidean superspace deformed by a nonanticommutativity parameter $\{\theta^\alpha, \theta^\beta\} = 2C^{\alpha\beta}$ with C a nonzero constant) have been considered in [1, 4, 5, 6, 7].

In this non(anti)commutative superspace we study the Wess-Zumino model

$$S = \int d^8z\bar{\Phi}\Phi - \frac{m}{2}\int d^6z\Phi^2 - \frac{\bar{m}}{2}\int d^6\bar{z}\bar{\Phi}^2 - \frac{g}{3}\int d^6z\Phi * \Phi * \Phi - \frac{\bar{g}}{3}\int d^6\bar{z}\bar{\Phi} * \bar{\Phi} * \bar{\Phi} \quad (1)$$

that in [1] was shown to reduce to the usual WZ augmented by a nonsupersymmetric component term $\frac{g}{6}\int d^4xC^2F^3$ (with $C^2 = C^{\alpha\beta}C_{\alpha\beta}$).

In [6], by introducing a spurion field [8], $U = C^2\theta^2\bar{\theta}^2$, to represent the supersymmetry breaking term F^3, the divergence structure and renormalizability of the $\mathcal{N} = 1/2$ WZ model have been studied systematically in superspace through two loops.

In this approach the classical action reads

$$S = \int d^8z\bar{\Phi}\Phi - \frac{m}{2}\int d^6z\Phi^2 - \frac{\bar{m}}{2}\int d^6\bar{z}\bar{\Phi}^2 - \frac{g}{3}\int d^6z\Phi^3 - \frac{\bar{g}}{3}\int d^6\bar{z}\bar{\Phi}^3 + \frac{g}{6}\int d^8zU(D^2\Phi)^3 \,. \quad (2)$$

It has been proven [6] that, up to this order, divergences are at most logarithmic, that divergent terms have at most one U-insertion (i.e. there is at most one power of C^2) and they are of the form $F^\alpha\bar{G}^k$, with $\bar{G} = \bar{m}\bar{\phi} + \bar{g}\bar{\phi}^2$ and $\alpha \geq 1$, $\alpha + k \leq 3$ (here $\Phi| = \phi$, $D_\alpha\Phi| = \psi_\alpha$, $D^2\Phi| = F$ and analogous relations for the antichiral superfield). Finally, a counterterm of the form $F^\alpha\bar{G}^k$ has been shown to be completely equivalent to a counterterm of the form $F^{\alpha+k}$. After adding by hand the terms $\int d^8zU(D^2\Phi)^2$ and $\int d^8zU(D^2\Phi)$, the model is renormalizable up to two loop order.

In the recent paper [7] it has been shown that the same results hold to all orders in perturbation theory: in particular the authors of [7], working in terms of component fields, constrain the form of divergent terms in the effective action using the two global $U(1)$ (pseudo)symmetries of the theory [4] and making general considerations on the structure and combinatorics of the Feynman diagrams.

In this short letter we reformulate in superspace formalism the discussion of [7], since this approach is usually more suitable when some supersymmetry is left. We use the conventions of [9].

We parametrize the terms F and F^2 in the classical lagrangian as

$$\lambda_1 g^3\bar{m}^4\int d^8zU(D^2\Phi) + \lambda_2 g^2\bar{m}^2\int d^8zU(D^2\Phi)^2 \,. \quad (3)$$

We consider the two global $U(1)$ (pseudo)symmetries of the theory, the $U(1)_\Phi$ flavor symmetry and $U(1)_R$ R-symmetry [4]. In superspace language we have the charge assignment given in table 1.

In particular with the parametrization (3) the coefficients λ_1 and λ_2 are charge neu-

	dim	$U(1)_R$	$U(1)_\Phi$		dim	$U(1)_R$	$U(1)_\Phi$
Φ	1	1	1	$\bar\Phi$	1	-1	-1
U	-4	4	0	$d^4\theta$	2	0	0
D_α	1/2	-1	0	$\bar D_{\dot\alpha}$	1/2	1	0
D^2	1	-2	0	$\bar D^2$	1	2	0
g	0	-1	-3	$\bar g$	0	1	3
m	1	0	-2	$\bar m$	1	0	2
λ_1	0	0	0	λ_2	0	0	0

Table 1: Global $U(1)$ charge assignment in superspace

tral under both $U(1)$ (pseudo)symmetries [7].

The most general divergent term in the effective action has the form

$$\int d^4x\Gamma_{\mathcal{O}} = \lambda \int d^4x d^4\theta\, (D^2)^\gamma\, (\bar D^2)^\delta\, (D_\alpha \partial^{\alpha\dot\alpha} \bar D_{\dot\alpha})^\eta\, \Box^\zeta\, U^\rho\, \Phi^\alpha\, \bar\Phi^\beta \tag{4}$$

with γ, δ, η, ζ, ρ, α, β non-negative integers. It is understood that every D^2, $\bar D^2$, D_α, $\bar D_{\dot\alpha}$, $\Box$, $\partial^{\alpha\dot\alpha}$ is acting on U, Φ, $\bar\Phi$ superfields, taking into account that

$$\begin{aligned} D_\alpha \bar\Phi &= 0\,, & \bar D_{\dot\alpha}\Phi &= 0 \\ \left[D^\alpha, \bar D^2\right] &= i\partial^{\alpha\dot\alpha}\bar D_{\dot\alpha}\,, & \left[\bar D^{\dot\alpha}, D^2\right] &= i\partial^{\dot\alpha\alpha} D_\alpha \\ D^2\bar D^2 D^2 &= \Box D^2\,, & \bar D^2 D^2 \bar D^2 &= \Box \bar D^2\,. \end{aligned} \tag{5}$$

In our notation the coefficient λ, with dimension d and charges $q_R = R$ and $q_\Phi = S$, is

$$\lambda \sim \Lambda^d g^{x-R} \bar g^x \left(\frac{m}{\Lambda}\right)^y \left(\frac{\bar m}{\Lambda}\right)^{y+\frac{S-3R}{2}} \lambda_2^{\omega_2} \tag{6}$$

where Λ is an ultraviolet momentum cutoff. λ cannot be a function of λ_1 since we cannot form a 1PI connected diagram with a $\int U\,(D^2\Phi)$ term. Moreover $\omega_2 \leq \rho$ since λ_2 appears only in terms with a U insertion (see (3)).

Since the term $\Gamma_{\mathcal{O}}$ has dimension 4 and zero charge, we have

$$\begin{aligned} d &= 2 + 4\rho - \alpha - \beta - \gamma - \delta - 2\eta - 2\zeta \\ R &= \beta - \alpha + 2\gamma - 2\delta - 4\rho \\ S &= \beta - \alpha\,. \end{aligned} \tag{7}$$

The overall power of Λ in $\Gamma_{\mathcal{O}}$ is

$$P = d - 2y - \frac{S-3R}{2} \tag{8}$$

and using eq. (7)

$$P = 2 + 2\gamma - 2\rho - 2\alpha - 4\delta - 2y - 2\eta - 2\zeta\,. \tag{9}$$

Obviously we have a divergent contribution iff $P \geq 0$. We consider the different cases:

- $\rho = 0$

  It is the ordinary Wess-Zumino case.

- $\rho = 1$

  We have

$$\gamma - \alpha - 2\delta - y - \eta - \zeta \geq 0\,. \tag{10}$$

  Since the U superfield has only the $\theta^2\bar{\theta}^2$ component, the $d^4\theta$ integration acts on it. Moreover the covariant D^2 derivatives can act only on Φ superfields (in fact $D^3 = 0$, $D^2\bar{D}_{\dot{\alpha}}\bar{\Phi} = 0$ and $D^2\bar{D}^2\bar{\Phi} = \Box\bar{\Phi}$), so we have

$$\gamma \leq \alpha\,. \tag{11}$$

  Therefore the only possibility to satisfy (10) is for

$$\gamma = \alpha, \quad \delta = y = \eta = \zeta = 0 \tag{12}$$

  and we find the general divergent term

$$\int d^8z U(D^2\Phi)^\alpha\bar{\Phi}^\beta\,. \tag{13}$$

  With the assignment (12) we have $P = 0$ showing that there is at most a logarithmic divergence.

- $\rho = 1 + n$, $n > 0$

  Since the U superfield has only the $\theta^2\bar{\theta}^2$ component, we need at least n D^2 and n $\bar{D}^2$. Therefore

$$\gamma = n + \gamma_1\,, \qquad \delta = n + \delta_1 \tag{14}$$

  and then

$$\gamma_1 - \alpha - 2n - 2\delta_1 - y - \eta - \zeta \geq 0\,. \tag{15}$$

  Since $\gamma_1 \leq \alpha$ (as in the previous case), and $n > 0$, we see that eq. (15) cannot be satisfied.

In conclusion, we have only (logarithmic) divergent terms of the form (13). Now we look for constraints on the coefficients α and β in order to show that there are only finitely many divergent terms.

As seen in the discussion above, we have

$$\rho = 1\,, \qquad \gamma = \alpha\,, \qquad \delta = y = \eta = \zeta = 0 \tag{16}$$

therefore λ takes the form

$$\lambda \sim g^{x-R}\;\bar{g}^x\;\bar{m}^{\frac{S-3R}{2}}\;\lambda_2^{\omega_2} \tag{17}$$

where

$$\frac{S-3R}{2} = -\beta - 2\alpha + 6\,. \tag{18}$$

If we look only at the UV divergent part of a diagram, the evaluation of the integral cannot depend on the mass parameter (in fact in dimensional regularization the divergences appear just as poles in $1/\epsilon$). Therefore powers of $\bar{m}$ in the coupling constant λ can appear:

- from the vertex $\lambda_2 g^2 \bar{m}^2 \int d^8 z U (D^2\Phi)^2$
- from the propagators $\langle \Phi\Phi \rangle = -\frac{\bar{m}\ D^2}{p^2(p^2+m\bar{m})}\ \delta^{(4)}(\theta-\theta')$.

Then, if we consider that the number of propagators $\langle \Phi\Phi \rangle$ is always nonnegative and that $\omega_2 \leq \rho$, we have

- $\omega_2 = 0 \quad \rightarrow \quad -\beta - 2\alpha + 6 \geq 0 \quad \rightarrow \quad \beta + 2\alpha \leq 6$
- $\omega_2 = 1 \quad \rightarrow \quad -\beta - 2\alpha + 4 \geq 0 \quad \rightarrow \quad \beta + 2\alpha \leq 4$

We have also the condition $\alpha \geq 1$: in fact, after D-algebra at least one D^2 survives and then, using (12), there must be at least one chiral Φ superfield.

To summarize, we have found that at any loop order the logarithmic divergent terms have the form

$$\int d^8 z U (D^2\Phi)^\alpha \bar{\Phi}^\beta\,, \quad \alpha \geq 1\,, \qquad \beta + 2\alpha \leq 6 - 2\omega_2\,, \quad \omega_2 = 0, 1\,. \tag{19}$$

Now we show that we can repackage them into the form

$$\int d^8 z U (D^2\Phi)^\alpha \bar{\mathcal{G}}^k \tag{20}$$

with $\bar{\mathcal{G}} = \bar{m}\bar{\Phi} + \bar{g}\bar{\Phi}^2$ and $0 \leq k \leq 3 - \omega_2 - \alpha$.

The condition $y = 0$ implies that in a divergent diagram the coupling constant does not contain m factors and so that there are not propagators $\langle \bar{\Phi}\bar{\Phi} \rangle$ (by the same observations done for $\bar{m}$). Therefore we have divergent contributions only from diagrams without adjacent $\bar{\Phi}^3$ vertices. Then a divergent diagram with $\bar{\Phi}$ external legs is analogous to a yet divergent diagram with the insertion of $\bar{\Phi}^3$ vertices on $\langle \Phi\Phi \rangle$ propagators. In fact, this operation does not modify the divergence of the diagram, since $\langle \Phi\Phi \rangle \sim \Lambda^{-4} \sim (\langle \Phi\bar{\Phi} \rangle)^2$ and since the D-algebra is not modified if we look only at divergent contributions. The only differences are the substitution $\bar{m} \rightarrow \bar{g}\bar{\Phi}$ for every insertion and a combinatorial factor $\binom{q}{k} 2^k$ that takes into account the $\binom{q}{k}$ ways to insert k vertices in q $\langle \Phi\Phi \rangle$ propagators and a symmetry factor $2 = 3 \cdot \frac{1}{3} \cdot 2$ for every vertex.

Therefore, with this operation, it is possible to start with divergent diagrams that give contributions to terms $\int U(D^2\Phi)^\alpha$ at a given loop order, and build all possible diagrams that give contributions to terms $\int U(D^2\Phi)^\alpha\bar{\Phi}^\beta$ at the same order.

If we start with a divergent base diagram with fixed ω_2 and α, and with symmetry factor S (that we can understand to include also the poles in $1/\epsilon$), the sum of all the divergent contributions with $k \geq 1$ is

$$Sg^{x-\alpha+4}\bar{g}^x\lambda_2^{\omega_2}\bar{m}^{6-2\alpha}\sum_{k=1}^{q} 2^k \binom{q}{k}\left(\frac{\bar{g}}{\bar{m}}\right)^k \int d^8zU(D^2\Phi)^\alpha\bar{\Phi}^k\,. \tag{21}$$

Since

$$\begin{aligned}\sum_{k=1}^{q} 2^k \binom{q}{k}\left(\frac{\bar{g}}{\bar{m}}\right)^k \bar{\Phi}^k &= \left(1+2\frac{\bar{g}}{\bar{m}}\bar{\Phi}\right)^q - 1 \\ &= \left(1+4\frac{\bar{g}}{\bar{m}}\bar{\Phi}+4\frac{\bar{g}^2}{\bar{m}^2}\bar{\Phi}^2\right)^{\frac{q}{2}} - 1 \\ &= \left(1+4\frac{\bar{g}}{\bar{m}^2}\bar{\mathcal{G}}\right)^{\frac{q}{2}} - 1 \end{aligned} \tag{22}$$

and observing that for a diagram without $\bar{\Phi}$ external legs ($\beta = 0$) $q = 6 - 2\omega_2 - 2\alpha$, we can finally rewrite eq. (21) as

$$Sg^{x-\alpha+4}\bar{g}^x\lambda_2^{\omega_2}\bar{m}^{6-2\alpha}\sum_{k=1}^{3-\omega_2-\alpha} 4^k \binom{3-\omega_2-\alpha}{k}\left(\frac{\bar{g}}{\bar{m}^2}\right)^k \int d^8zU(D^2\Phi)^\alpha\bar{\mathcal{G}}^k \tag{23}$$

wich agrees with the two loop results of [6].

Taking into account that $\alpha = 1, 2, 3$ we can conclude, in agreement with [7], that to all orders in perturbation theory, the divergent terms generated are (in component fields, with $\bar{\mathcal{G}}| = \bar{G}$)

$$\begin{aligned}\omega_2 = 0 \quad &\rightarrow \quad F, F^2, F^3, F\bar{G}, F^2\bar{G}, F\bar{G}^2 \\ \omega_2 = 1 \quad &\rightarrow \quad F, F^2, F\bar{G}\end{aligned} \tag{24}$$

Now we show that the counterterms F, F^2, F^3 are sufficient to renormalize the theory. We can follow the argument of [6] to claim that a contraction of any field with $\bar{G}$ is equivalent to its contraction with F. This is possible also in completely superspace language and translates into the equivalence $\bar{\mathcal{G}} \rightarrow D^2\Phi$. In fact, let us consider for example the effect of a superfield factor $U(D^2\Phi_b)^2[D^2\Phi - \bar{m}\bar{\Phi}]$ as compared to $\bar{g}U(D^2\Phi_b)^2\bar{\Phi}^2$ (here Φ_b is the background superfield). The superfield propagators are

$$\begin{aligned}\langle\Phi\bar{\Phi}\rangle &= \frac{1}{p^2+m\bar{m}}\delta^{(4)}(\theta-\theta') \\ \langle\Phi\Phi\rangle &= -\frac{\bar{m}D^2}{p^2(p^2+m\bar{m})}\delta^{(4)}(\theta-\theta') \\ \langle\bar{\Phi}\bar{\Phi}\rangle &= -\frac{m\bar{D}^2}{p^2(p^2+m\bar{m})}\delta^{(4)}(\theta-\theta') \end{aligned} \tag{25}$$

and from the Feynman rules for each chiral (antichiral) field there is an extra $\bar{D}^2$ (D^2) derivative on each line leaving a vertex (except for one of the lines at a (anti)chiral vertex).

In the Wick expansion, the operator $U(D^2\Phi_b)^2[D^2\Phi - \bar{m}\bar{\Phi}]$ can be contracted either with a Φ^3 vertex, or with a $\bar{\Phi}^3$ vertex. Taking in account the D-algebra (and in particular $D^2\bar{D}^2D^2 = -p^2D^2$), and given the form of the propagators, in the first case the result is zero. In the second case, the D-algebra is analogous and, given the form of the propagators, the result is $\bar{g}U(D^2\Phi_b)^2\bar{\Phi}^2$. In this last case there is a little subtlety: when we contract $D^2\Phi$ with $\bar{\Phi}$, after using $D^2\bar{D}^2D^2 = -p^2D^2$, we remain with a D^2 on the propagator $\langle\Phi\bar{\Phi}\rangle$; this D^2 can be integrated by parts onto the $\bar{\Phi}^3$ vertex, to give the exact Feynman rules for a vertex $U(D^2\Phi_b)^2\bar{\Phi}^2$. We can treat in a similar way the operators $U(D^2\Phi_b)\bar{\mathcal{G}}$ and $U(D^2\Phi_b)\bar{\mathcal{G}}^2$, thus showing the equivalence of the two forms of counterterms when inserted into diagrams.

Therefore the counterterms

$$\int U(D^2\Phi), \int U(D^2\Phi)^2, \int U(D^2\Phi)^3 \tag{26}$$

are sufficient to renormalize the theory at any order of perturbation theory.

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