Effective Resource-Driven Loop Splitting for Large Unstructured Mesh Applications on GPUs

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University of Oxford

HiStencils'15 20 January 2015

Goal: fast, automated resolution of PDEs

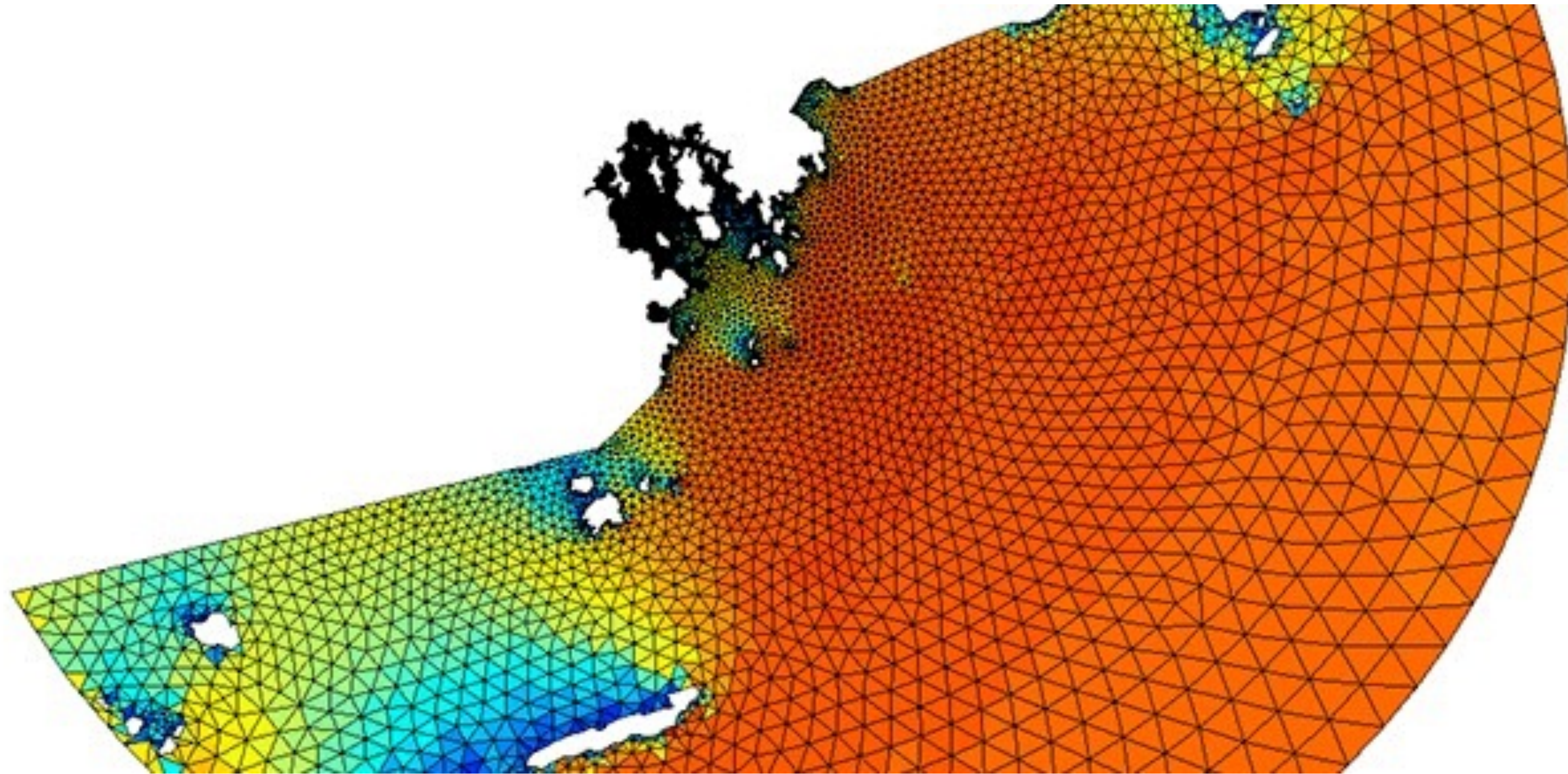

Image publicly available from http://www.bmtargoss.com/

Real-world computations we are interested in:

- weather forecast in a given time window (e.g., 1 hour)
- simulation turbo-machinery of aircraft engines (HYDRA)
- simulation of tsunami waves (VOLNA)
- ...

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+

Raise the level of abstraction

(through domain specific languages)

$$\int \nabla \cdot \rho \ p \ dx$$

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Stack of optimizing compilers

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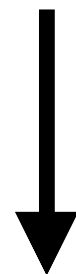
Stack of optimizing compilers



faster code than you can reasonably write “by hand”

From DSL to loop chains

Firedrake provides a DSL for finite element methods

```
phi, p = Function(mesh, …)
…
while not convergence:
{
   …
   phi -= dt / 2 * p
   if …:

      p += (assemble(dt*inner(nabla_grad(v),…))*dx)

   else:
      solve(…)
   …
   phi += dt / 2 * p
   …
}
…
```

From DSL to loop chains

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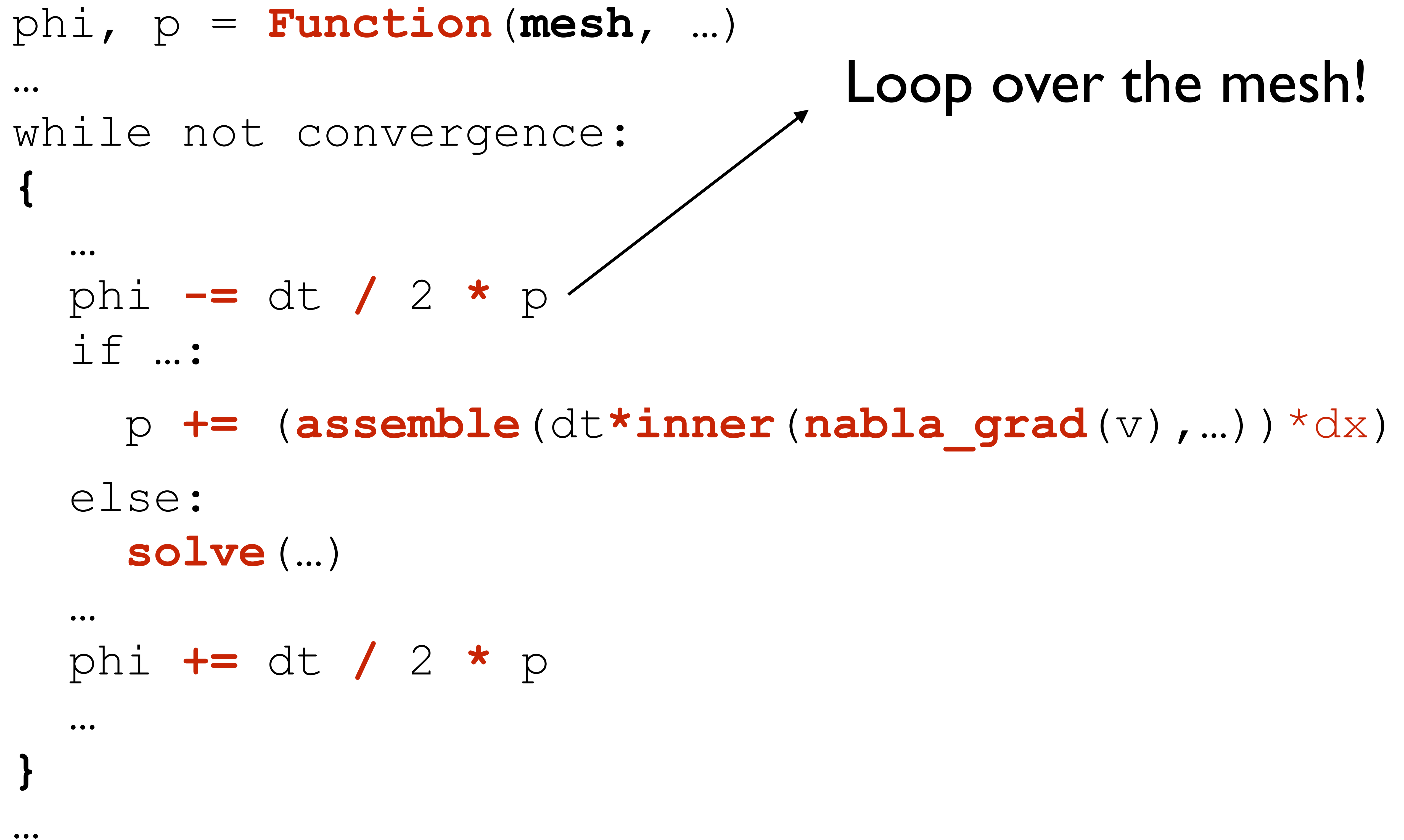

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Loop over the mesh!

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   …
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   …
}
…
```

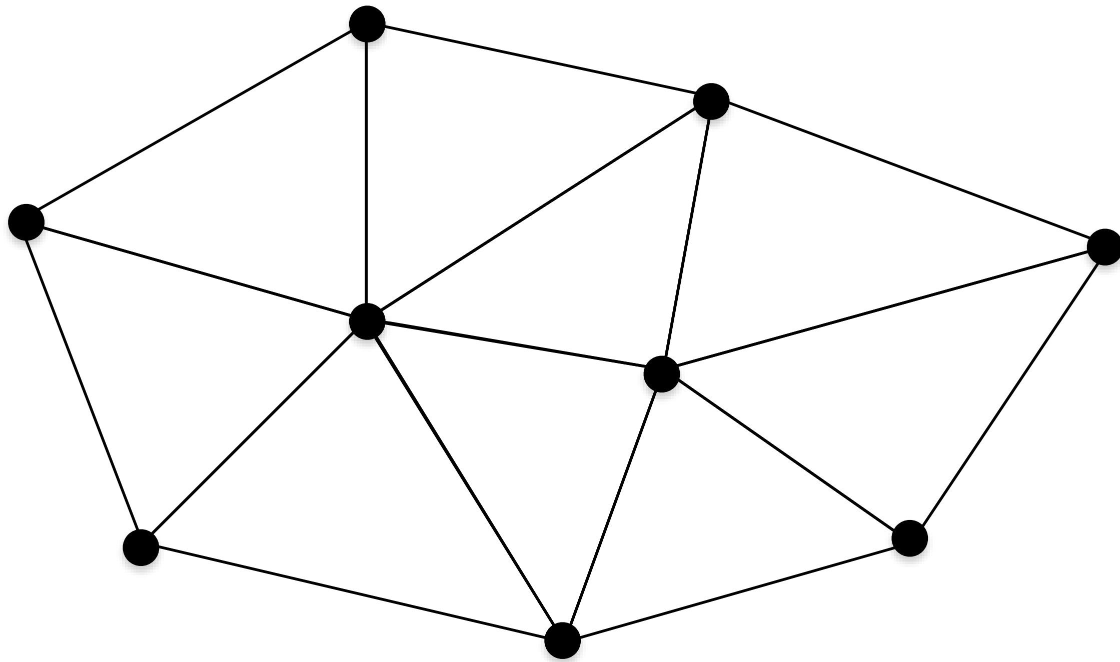
Loop over the mesh!

Loop over the mesh!

Call to third party library!

Loop over the mesh!

The OP2 programming model



```
void incrVertices (
  double* e_weight,
  double* v1,
  double* v2) {
    *v1 += f(e_weight)
    *v2 += f(e_weight)
}
```

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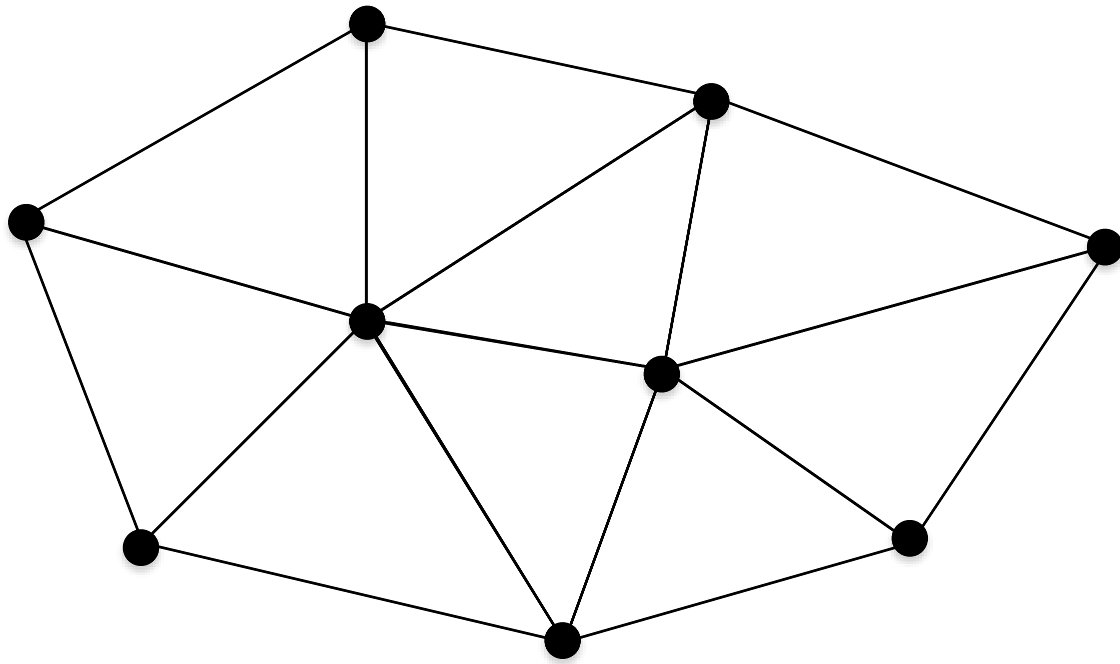

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```
op_par_loop (incrVertices, edges,
```

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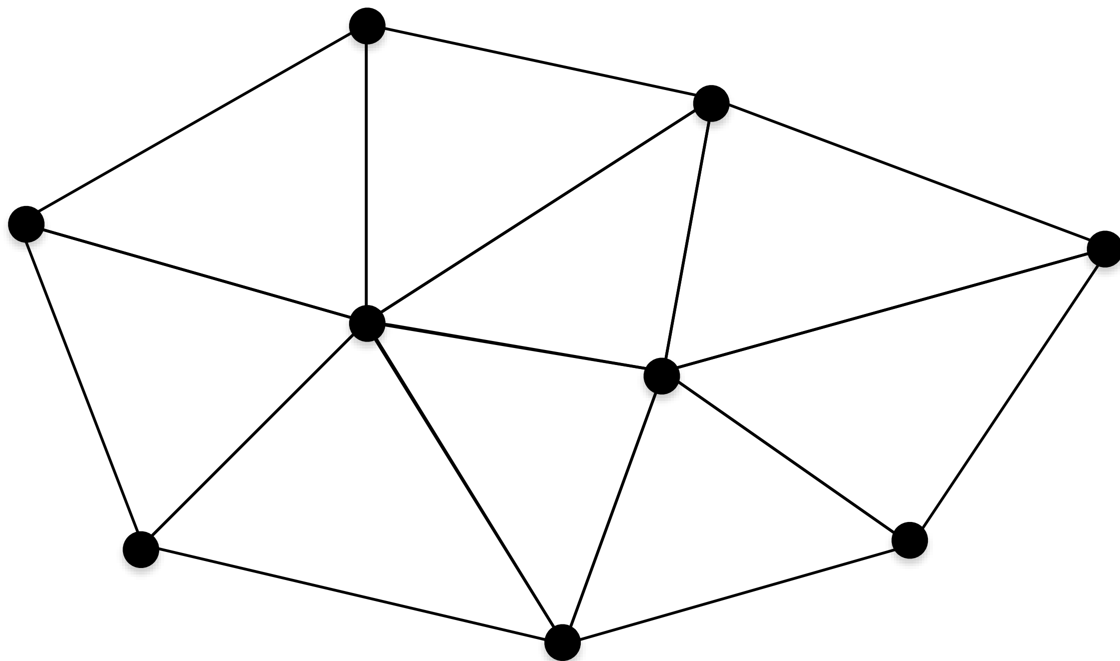

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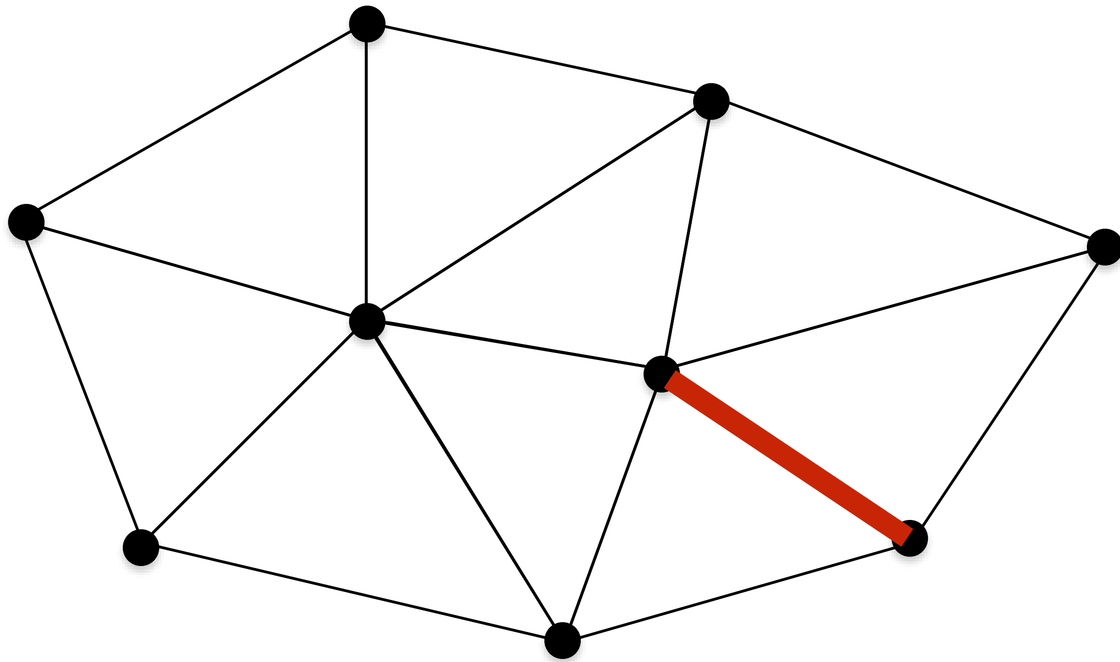

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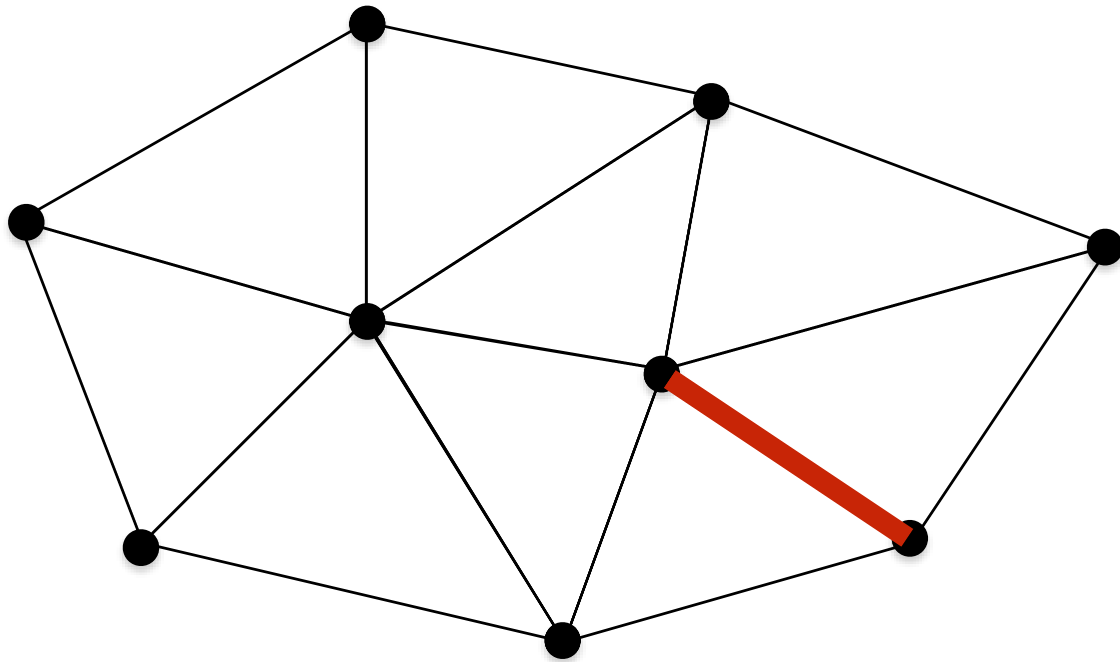

```
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      *v1 += f(e_weight)
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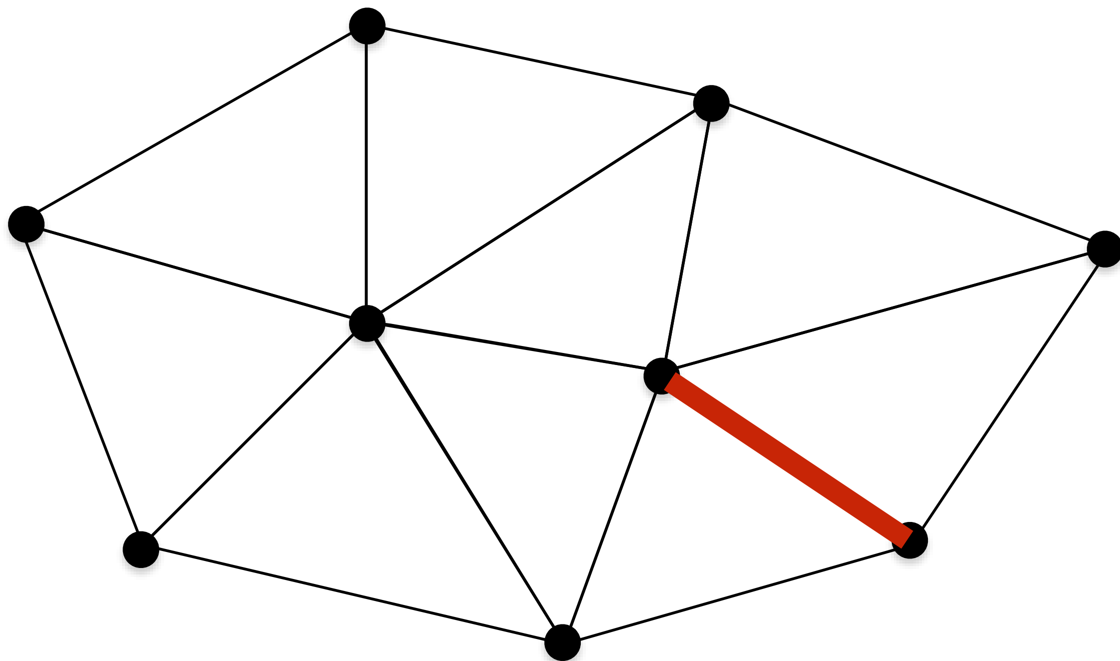
```

```
op_par_loop (incrVertices, edges,
  op_arg_dat (edgeWeight, -1, OP_ID,          OP_READ),
```

The OP2 programming model



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```
op_par_loop (incrVertices, edges,
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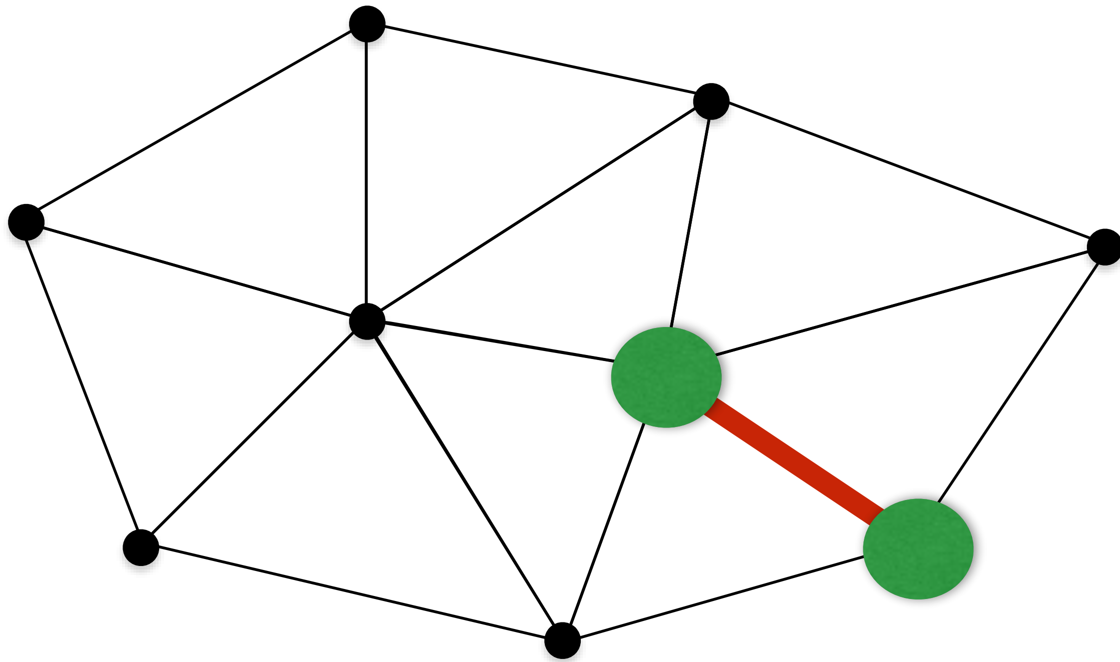

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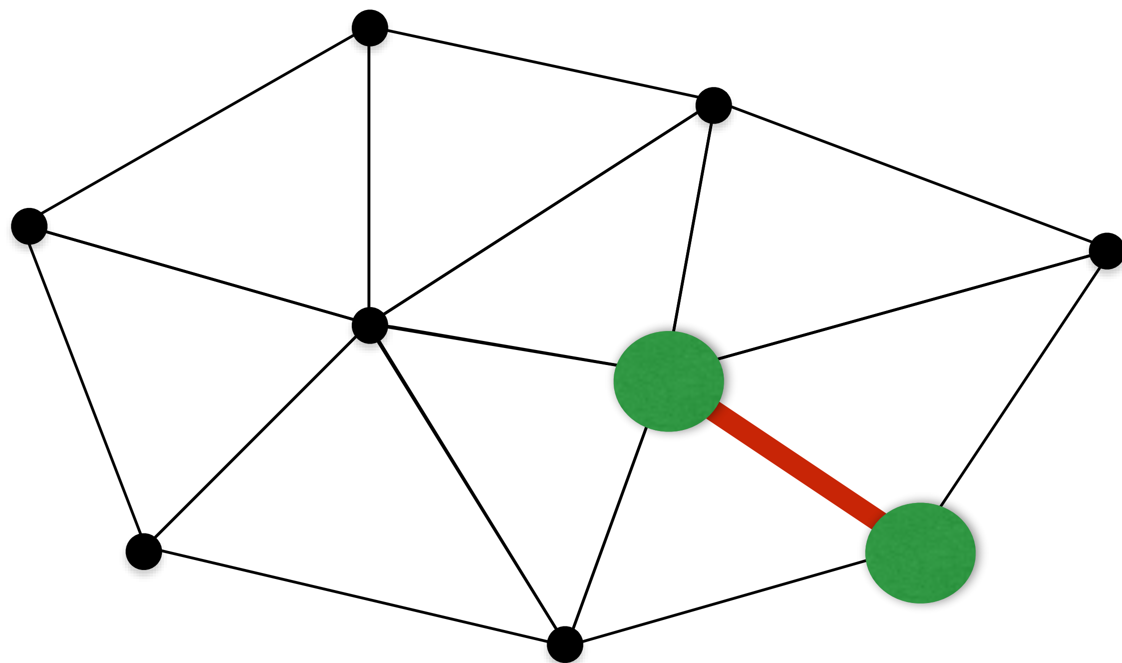

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INDIRECT MEMORY ACCESSES (A[B[i]])!

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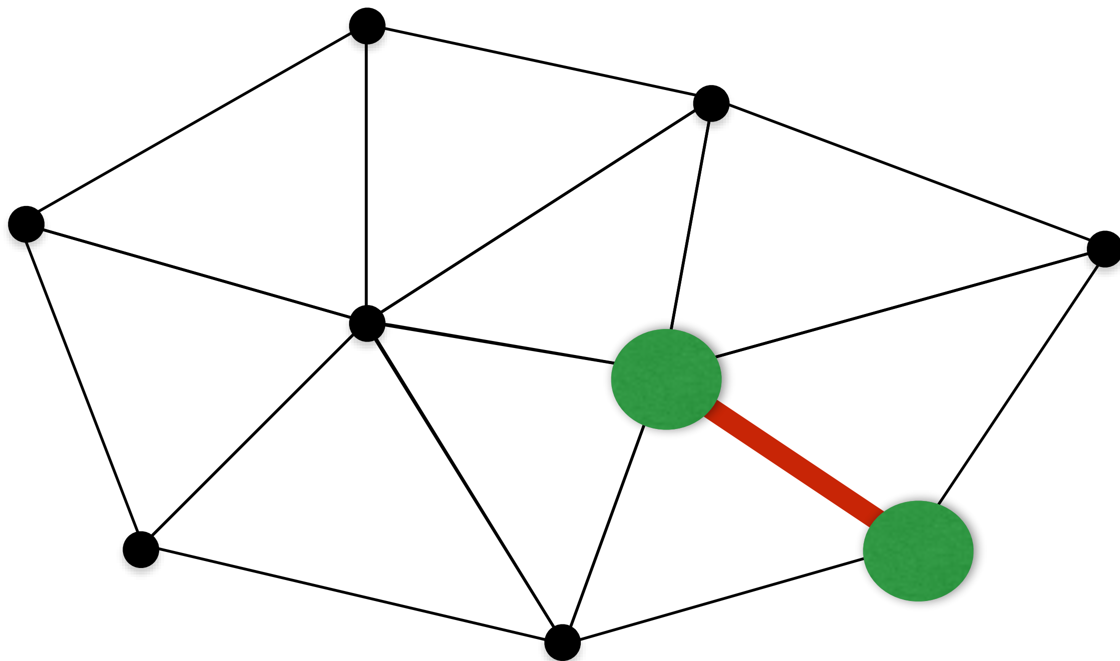

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op_par_loop (X, cells, …)
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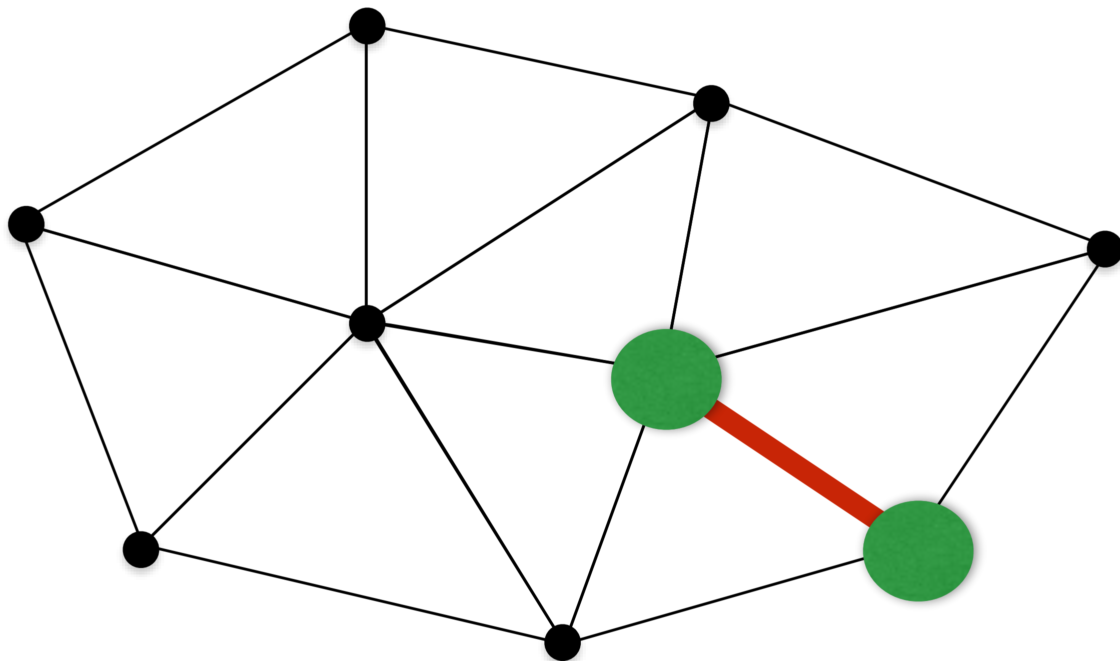

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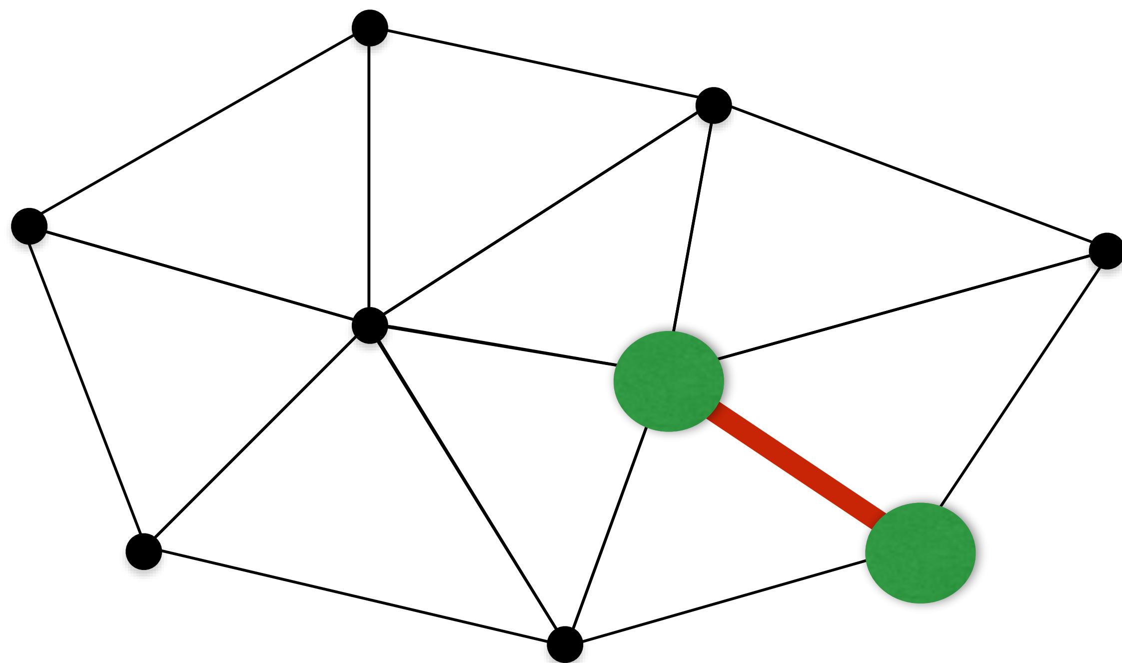
Synchronization point (function call e.g., PETSc)

The OP2 programming model



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INDIRECT MEMORY ACCESSES (`A[B[i]]`)!

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op_par_loop (X, cells, …)
Synchronization point (function call e.g., PETSc)
op_par_loop (Y, vertices, …)
```

Implementation of an op_par_loop in CUDA

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op_par_loop (incrVertices, edges,
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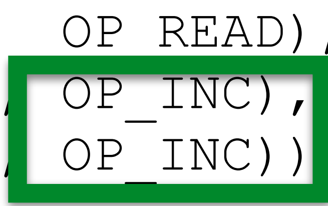

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void incrVertices (
  double* e,
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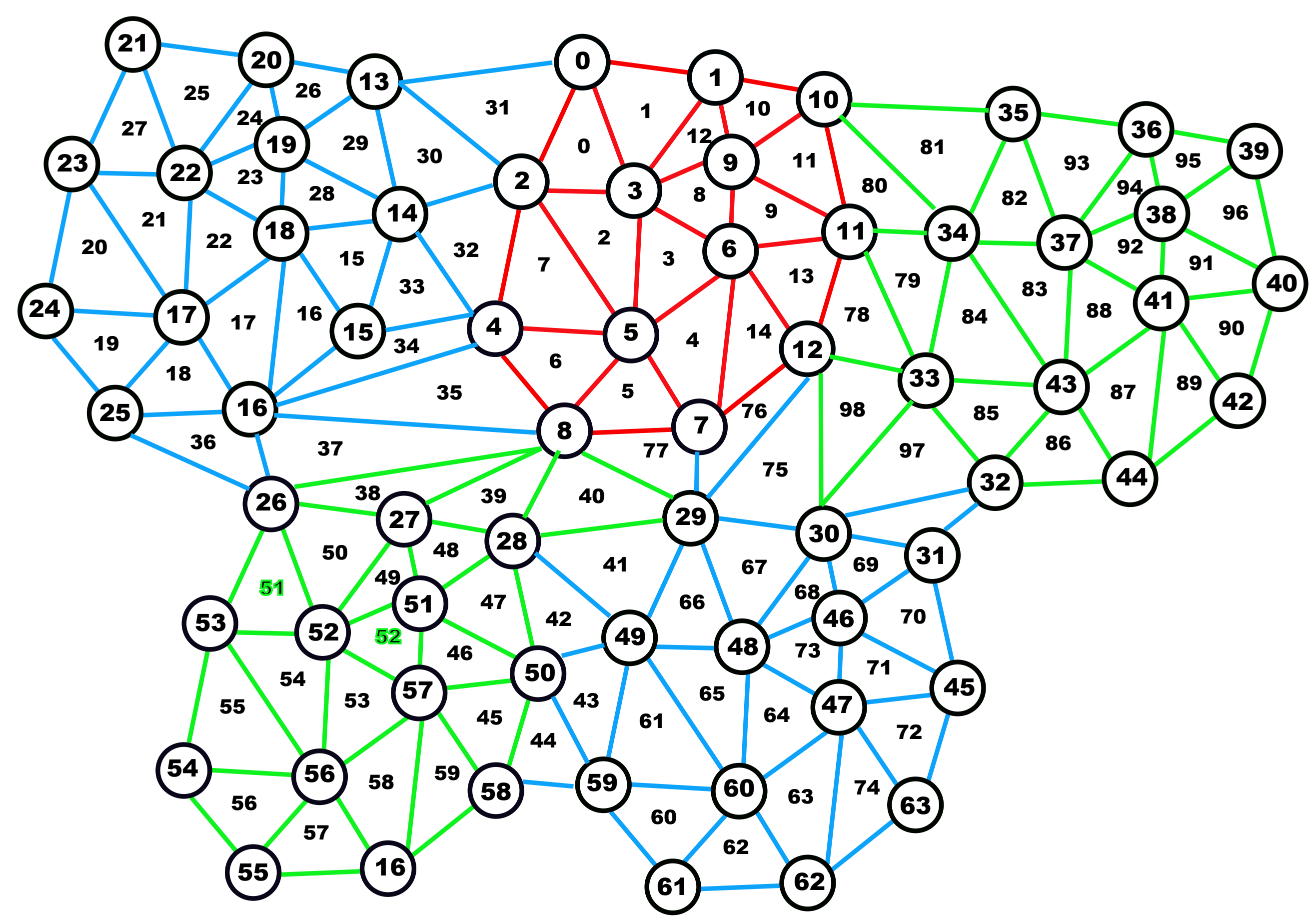
Coloring used for avoiding race conditions in shared memory parallel execution



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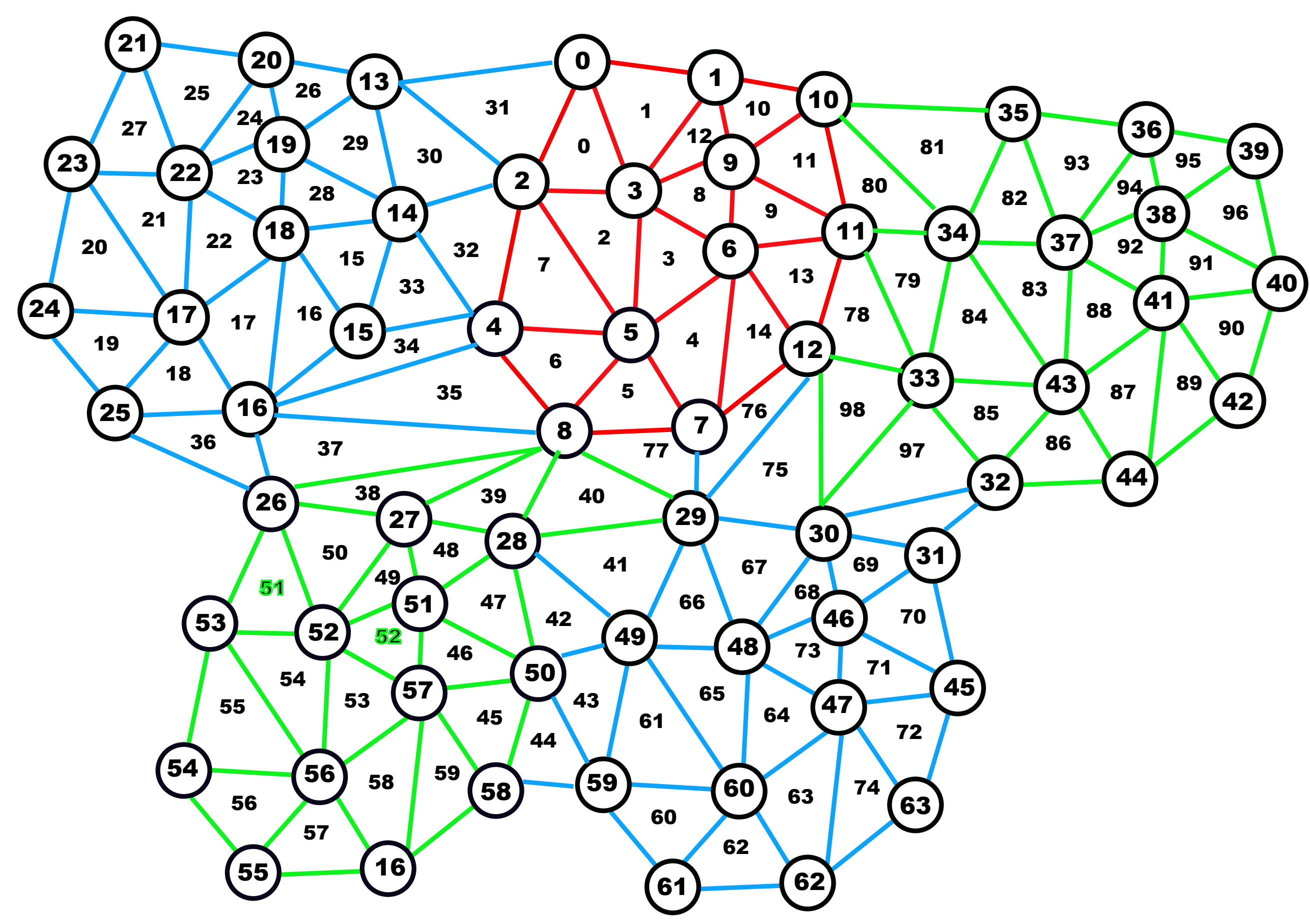

Each partition assigned to a Thread Block and further colored

Implementation of an op_par_loop in CUDA

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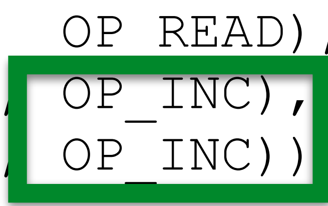

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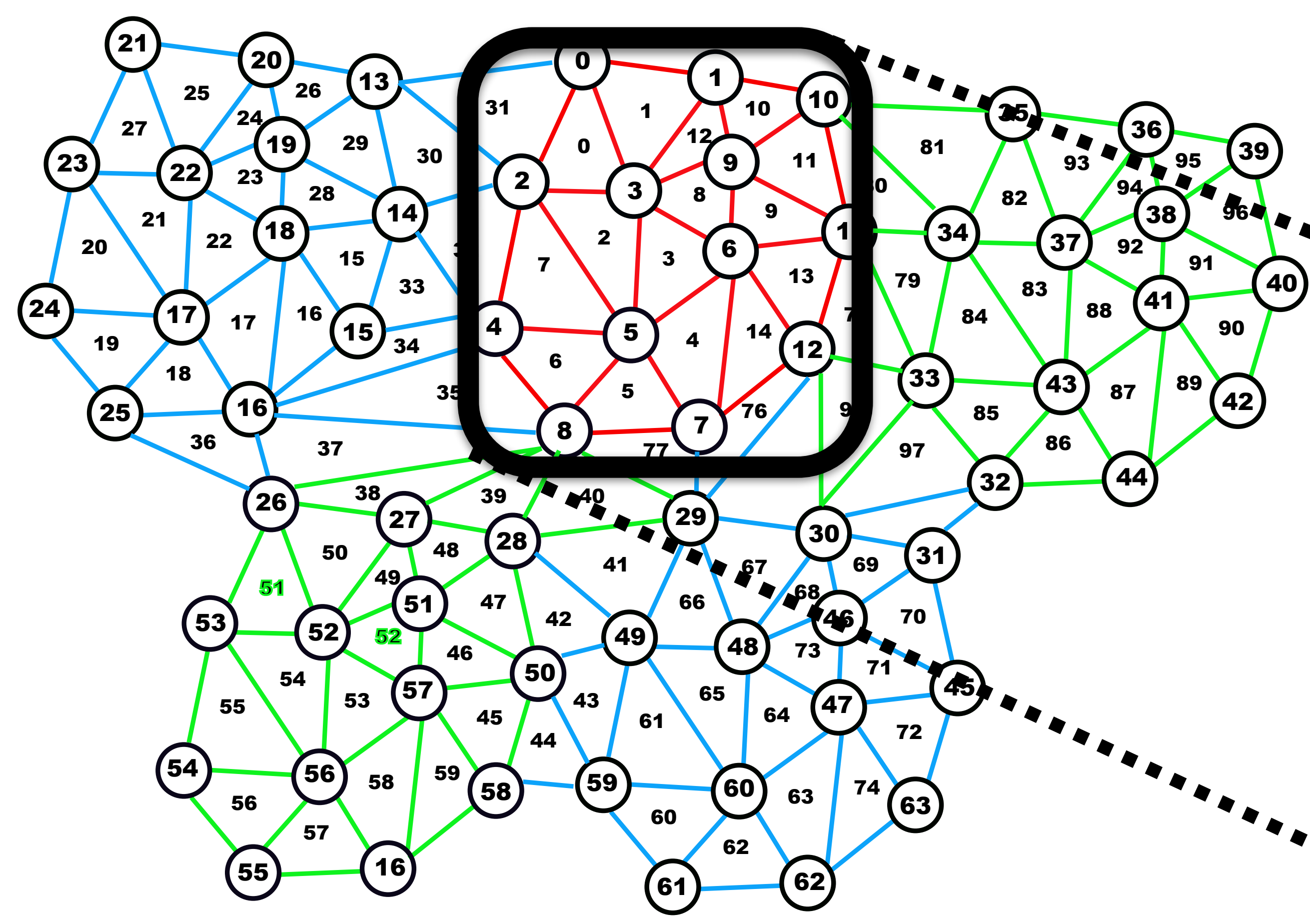

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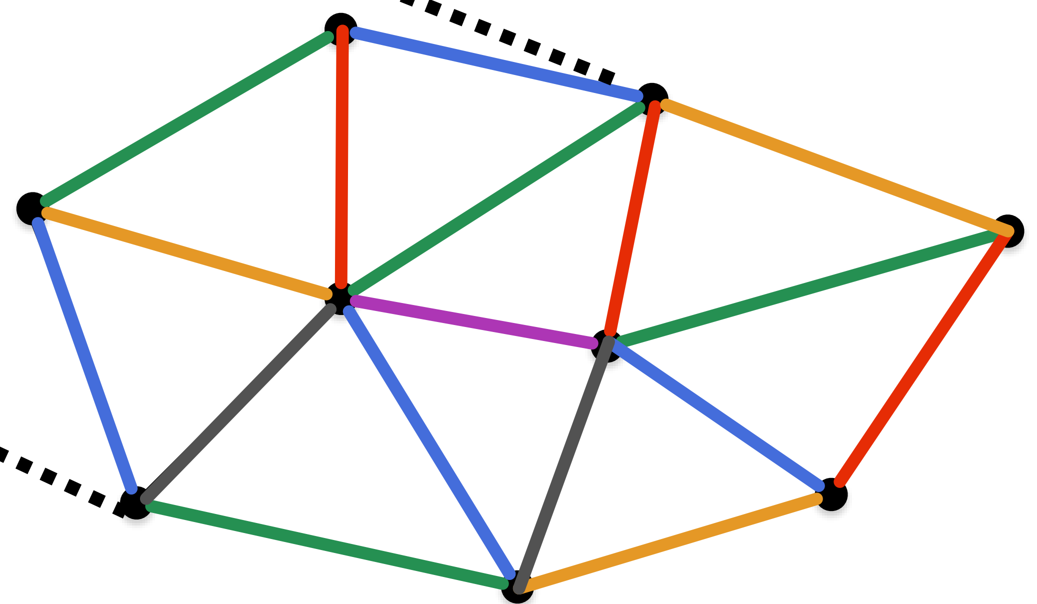

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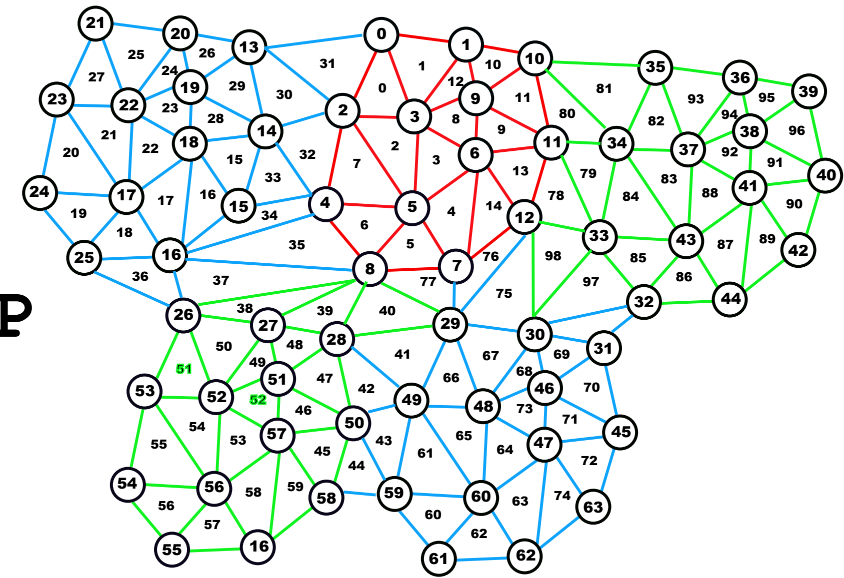

```
loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for partition p in P
    loop over edges e in P //par
      tmp = contributions (edgeWeight)
      update ( e(v1), tmp)
      update ( e(v2), tmp)
    stage out all data for partition p in P
  end loop
end loop
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Implementation of an op_par_loop in CUDA

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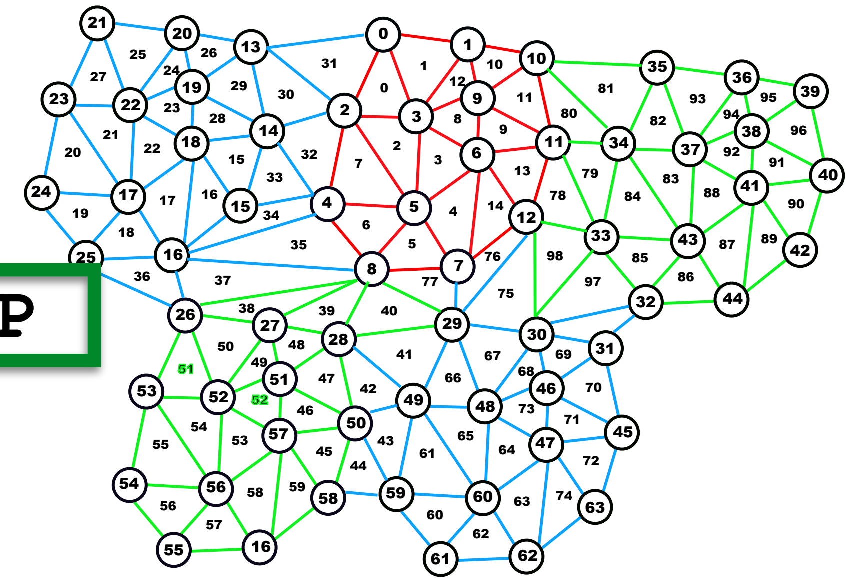
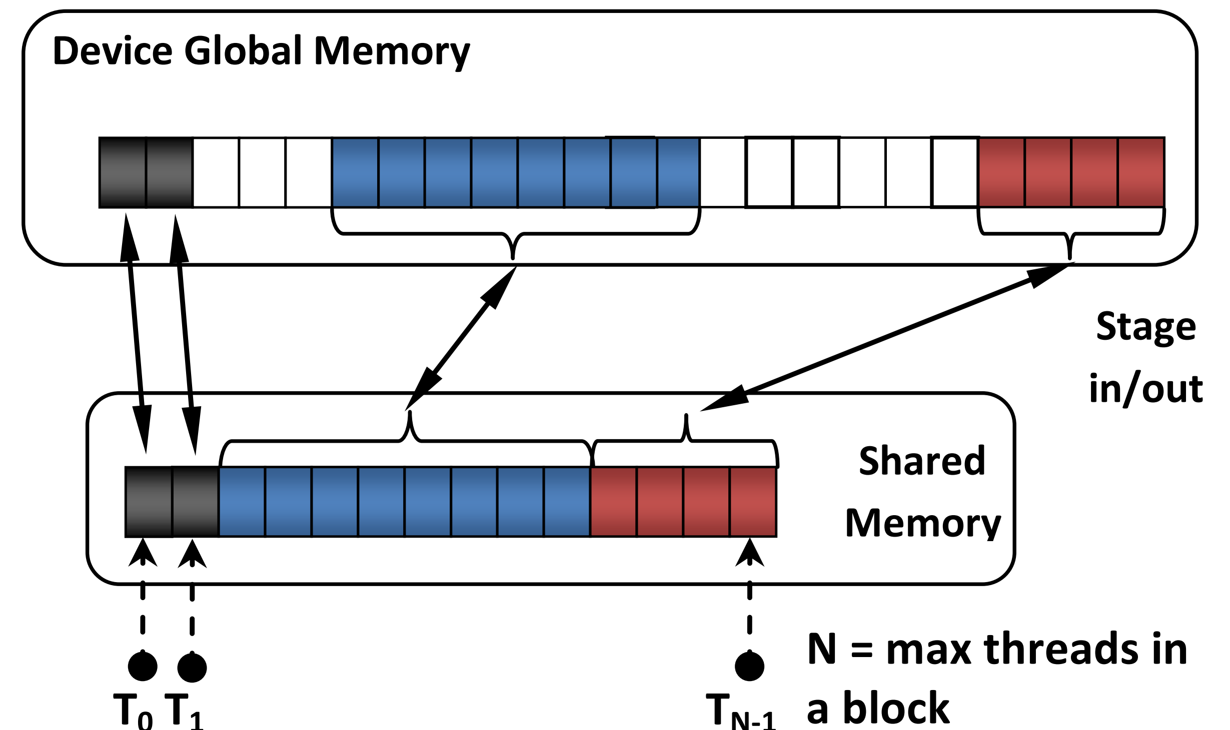


Data is <u>usually</u> **staged** in shared memory

- spatial locality (SoA)
  => coalescing
- temporal locality



Challenge for effective GPU execution

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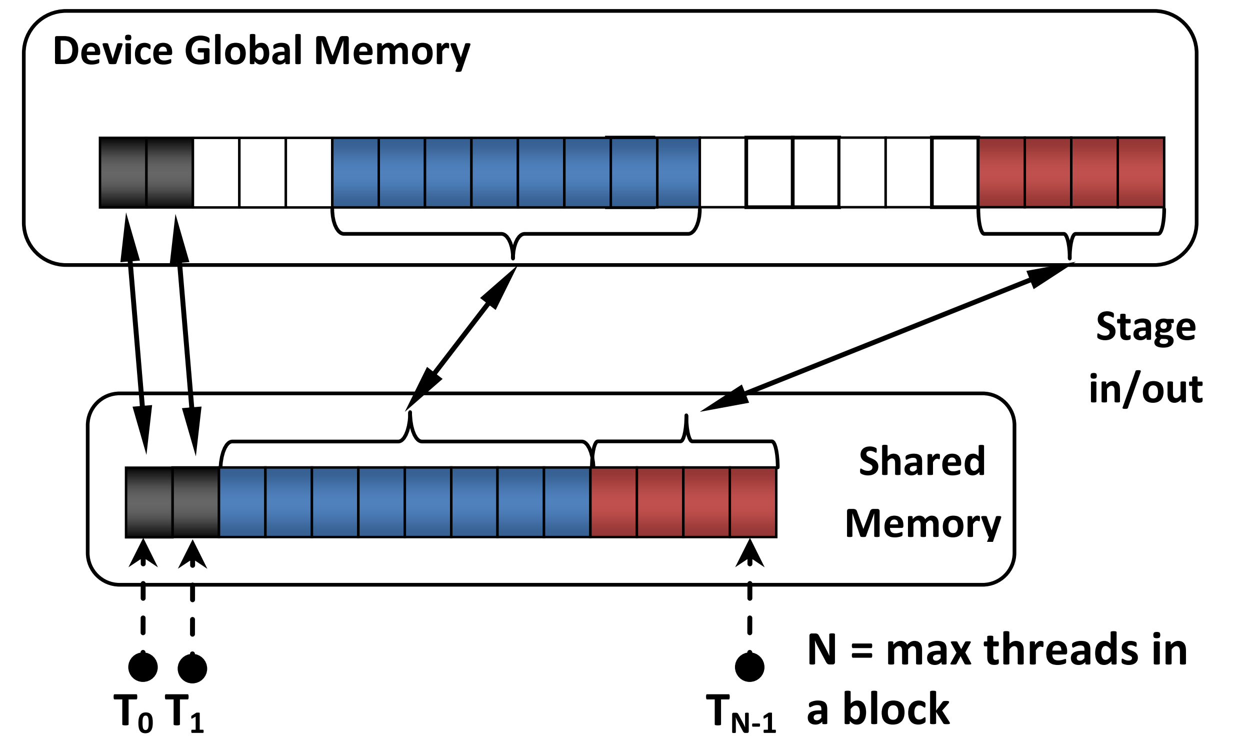

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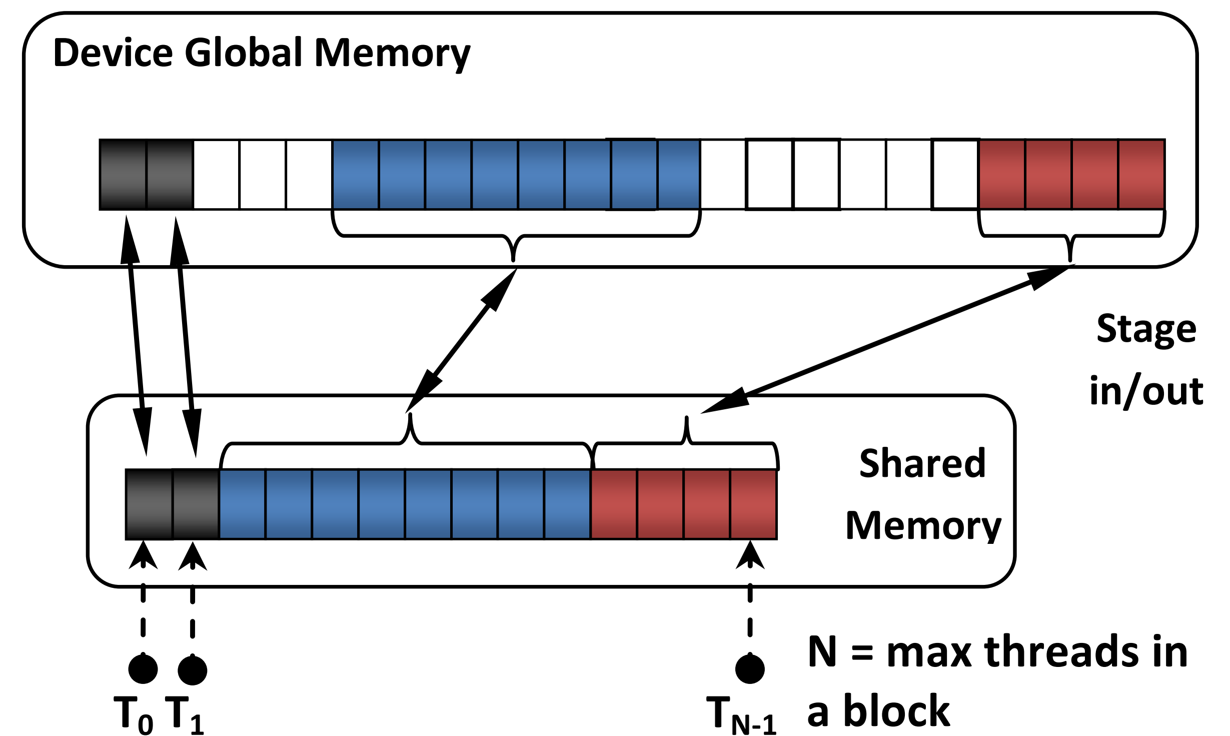

Time-expensive parallel loops in real-world computations can have a large memory footprint => Less parallelism!

Challenge for effective GPU execution

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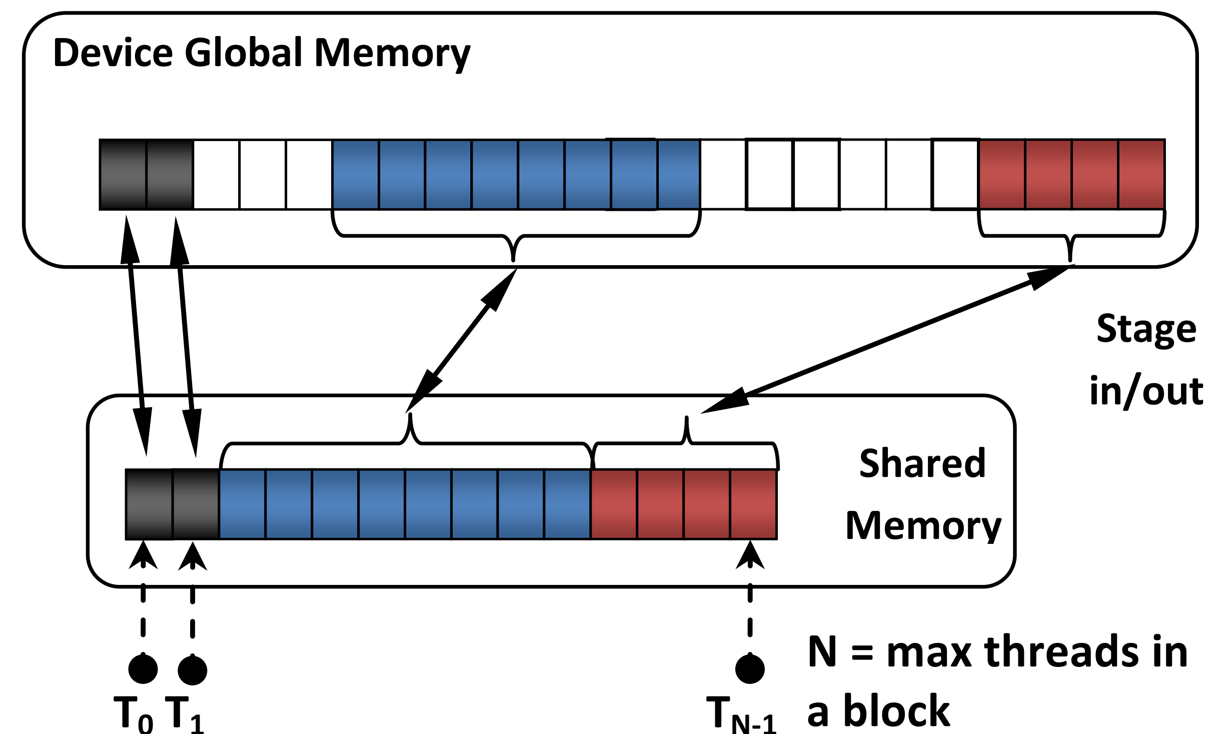
- **spatial locality (SoA)** => coalescing
- **temporal locality**



Time-expensive parallel loops in real-world computations can have a large memory footprint => Less parallelism!

HYDRA example: viscous or smoothing fluxes calculation (Vflux), which accesses indirectly **808 bytes and directly 24 bytes for each iteration**. The incremented datasets are 96 bytes in size per iteration.

Loop splitting - version 1 (simplest)

```
loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for contrib
    loop over edges e in P //par
      tmp = contributions (edgeWeight)
   stage out all tmp data
  end loop
end loop

loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for update1 (tmp)
    loop over edges e in P //par
      update ( e(v1), tmp)
   stage out all data for partition p in P
  end loop
end loop

loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for update2 (tmp)
    loop over edges e in P //par
      update ( e(v2), tmp)
   stage out all data for partition p in P
  end loop
end loop
```

Loop splitting - version 1 (simplest)

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  loop over partitions P in C //par
    stage in all data for contrib
    loop over edges e in P //par
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  loop over partitions P in C //par
    stage in all data for update1 (tmp)
    loop over edges e in P //par
      update ( e(v1), tmp)
   stage out all data for partition p in P
  end loop
end loop

loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for update2 (tmp)
    loop over edges e in P //par
      update ( e(v2), tmp)
   stage out all data for partition p in P
  end loop
end loop
```

Advantages:

- Less data required by staging, so more parallelism

Loop splitting - version 1 (simplest)

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      update ( e(v1), tmp)
   stage out all data for partition p in P
  end loop
end loop

loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for update2 (tmp)
    loop over edges e in P //par
      update ( e(v2), tmp)
   stage out all data for partition p in P
  end loop
end loop
```

Advantages:

- Less data required by staging, so more parallelism

Disadvantages:

- User intervention
- Multiple staging of same dataset (tmp)

Loop splitting - version 2

```
loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for contrib
    loop over edges e in P //par
      tmp(e) = contrib (edgeData[e:0…n])
    stage in all data for update1
    loop over edges e in P //par
      update ( e(v1), tmp(e) )
    stage out all data for update1
    stage in all data for update2
    loop over edges e in P //par
      update ( e(v2), tmp(e) )
    stage out all data for update2
  end loop
end loop
```

Loop splitting - version 2

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loop over colors C //seq
  loop over partitions P in C //par
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    stage in all data for update2
    loop over edges e in P //par
      update ( e(v2), tmp(e) )
    stage out all data for update2
  end loop
end loop
```

Unary data, so kept in global memory and coalescing

Loop splitting - version 2

```
loop over colors C //seq
  loop over partitions P in C //par
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  end loop
end loop
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Unary data, so kept in global memory and coalescing

Can overwrite previously used data

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    stage in all data for update1
    loop over edges e in P //par
      update ( e(v1), tmp(e) )
    stage out all data for update1
    stage in all data for update2
    loop over edges e in P //par
      update ( e(v2), tmp(e) )
    stage out all data for update2
  end loop
end loop
```

Unary data, so kept in global memory and coalescing

Can overwrite previously used data

Advantage: Does not require user intervention, just an alternative code synthesis

Loop splitting - version 3

Was a single loop "contrib"

```
loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for contrib1
    loop over edges e in P //par
      tmp1(e) = contrib1 (e); tmp(e) = tmp1(e);
    stage in all data for contrib2
    loop over edges e in P //par
      tmp2(e) = contrib2 (e); tmp(e) += tmp2(e);
    ...
    stage in all data for contribN
    loop over edges e in P //par
      tmpN(e) = contribN (e); tmp(e) += tmpN(e);
    ...
    stage in all data for update1
    loop over edges e in P //par
      update ( e(v1), tmp(e) )
    stage out all data for update1
    stage in all data for update2
    loop over edges e in P //par
      update ( e(v2), tmp(e) )
    stage out all data for update2
  end loop
end loop
```

Loop splitting - version 3

Was a single loop "contrib"

```
loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for contrib1
    loop over edges e in P //par
      tmp1(e) = contrib1 (e); tmp(e) = tmp1(e);
    stage in all data for contrib2
    loop over edges e in P //par
      tmp2(e) = contrib2 (e); tmp(e) += tmp2(e);
    ...
    stage in all data for contribN
    loop over edges e in P //par
      tmpN(e) = contribN (e); tmp(e) += tmpN(e);
    ...
    stage in all data for update1
    loop over edges e in P //par
      update ( e(v1), tmp(e) )
    stage out all data for update1
    stage in all data for update2
    loop over edges e in P //par
      update ( e(v2), tmp(e) )
    stage out all data for update2
  end loop
end loop
```

Relatively "simple" in HYDRA, but can also be much more complex…

How “contrib” could look like

```
loop over colors C //seq
  loop over partitions P in C //par
    stage in all data for computing A
    …
    for (int ip  = 0; ip < m; ++ip) {
      …
      for (int j  = 0; j < n; ++j) {
        for (int k  = 0; k < o; ++k) {
          A[j][k] += (det * W[ip] * B[ip][k] * B[ip][j]);
        }
      }
    }
    …
    stage out A
  end loop
end loop
```

`m, n, o` *rarely greater than 30*
typically between 3 and 15

Finite element - mass matrix operator

How “contrib” could look like

m, n, o rarely greater than 30
typically between 3 and 15

```
...
...
for (int ip  = 0; ip < m; ++ip) {
   ...
   for (int j  = 0; j < n; ++j) {
      for (int k  = 0; k < o; ++k) {
        A[j][k] += (((B[ip][k] * B[ip][j]) + (((((K[2] * B0[ip]
[k]) + (K[5] * B1[ip][k]) + (K[8] * B2[ip][k])) * ((K[2] *
B0[ip][j]) + (K[5] * B1[ip][j]) + (K[8] * B2[ip][j]))) +
(((K[1] * B0[ip][k]) + (K[4] * B1[ip][k]) + (K[7] * B2[ip]
[k])) * ((K[1] * B0[ip][j]) + (K[4] * B1[ip][j]) + (K[7] *
B2[ip][j]))) + (((K[0] * B0[ip][k]) + (K[3] * B1[ip][k]) +
(K[6] * B2[ip][k])) * ((K[0] * B0[ip][j]) + (K[3] * B1[ip]
[j]) + (K[6] * B2[ip][j])))) * F1 * F0)) * det * W[ip]);
      }
   }
}
...
```

Finite element -Helmholtz operator

How “contrib” could look like

m, n, o rarely greater than 30
typically between 3 and 15

```
…
for (int ip  = 0; ip < m; ++ip) {
  …
  for (int j  = 0; j < n; ++j) {
    for (int k  = 0; k < o; ++k) {
        A[j][k] += ((((K[2] * BC10[0][j]) + (K[5] * BC11[0][j]) + (K[8] * BC12[0][j])) * (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][k])) * (((((((K[8] * F2) + (K[5] *
F1) + (K[2] * F0)) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[7] * F8) + (K[4] * F7) + (K[1] * F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[8] * F5) + (K[5] * F4) + (K[2] *
F3)) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) + 1.0))) / 2.0)) + ((((((K[8] * F2) + (K[5] * F1) + (K[2] * F0)) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[7] * F8) + (K[4] * F7) + (K[1] *
F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[8] * F5) + (K[5] * F4) + (K[2] * F3)) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) + 1.0))) / 2.0))) * F9) + (((K[6] * F5) + (K[3] * F4) +
(K[0] * F3)) * ((((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[6] * F8) + (K[3] * F7) + (K[0] * F6))) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k]))
* ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[0] * BC00[0][k]) + (K[3] *
BC01[0][k]) + (K[6] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[0] * BC20[0][k]) + (K[3] * BC21[0][k]) + (K[6] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0))
+ (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[6] * F2) + (K[3] * F1) + (K[0] * F0) + 1.0))) / 2.0)) + ((((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0]
[k])) * ((K[6] * F8) + (K[3] * F7) + (K[0] * F6))) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k]) + (K[5] *
BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[0] * BC00[0][k]) + (K[3] * BC01[0][k]) + (K[6] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) +
(((K[0] * BC20[0][k]) + (K[3] * BC21[0][k]) + (K[6] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[6] *
F2) + (K[3] * F1) + (K[0] * F0) + 1.0))) / 2.0))) * F9) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((((((K[8] * F5) + (K[5] * F4) + (K[2] * F3)) * ((K[6] * F5) + (K[3] *
F4) + (K[0] * F3))) + (((K[6] * F8) + (K[3] * F7) + (K[0] * F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[8] * F2) + (K[5] * F1) + (K[2] * F0)) * ((K[6] * F2) + (K[3] * F1) + (K[0] *
F0) + 1.0))) / 2.0)) + ((((((K[8] * F5) + (K[5] * F4) + (K[2] * F3)) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[6] * F8) + (K[3] * F7) + (K[0] * F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] *
F6) + 1.0)) + (((K[8] * F2) + (K[5] * F1) + (K[2] * F0)) * ((K[6] * F2) + (K[3] * F1) + (K[0] * F0) + 1.0))) / 2.0))) * F9) + (((((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) *
((K[7] * F8) + (K[4] * F7) + (K[1] * F6))) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[1] * BC00[0][k]) + (K[4] * BC01[0]
[k]) + (K[7] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[1] *
BC20[0][k]) + (K[4] * BC21[0][k]) + (K[7] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[7] * F5) + (K[4]
* F4) + (K[1] * F3) + 1.0))) / 2.0)) + (((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[7] * F8) + (K[4] * F7) + (K[1] * F6))) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k])
+ (K[7] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[1] * BC00[0][k]) + (K[4] * BC01[0][k]) + (K[7] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] *
BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[1] * BC20[0][k]) + (K[4] * BC21[0][k]) + (K[7] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7)
+ (K[2] * F6) + 1.0)) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) + 1.0))) / 2.0))) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) +
1.0) * F9) + (((K[8] * F5) + (K[5] * F4) + (K[2] * F3)) * ((((((((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k])
+ (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] *
F0))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[8] *
F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0))) / 2.0)) + ((((((K[2] * BC10[0][k]) +
(K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] *
F3))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] *
F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k])
+ (K[8] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0))) / 2.0))) * F9) + (((F10) / 2.0) * ((1.0)) * ((((((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] *
F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) +
(K[8] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC20[0]
[k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7)
+ (K[2] * F6) + 1.0))) / 2.0) + ((((K[1] * BC20[0][k]) + (K[4] * BC21[0][k]) + (K[7] * BC22[0][k])) * ((K[7] * F8) + (K[4] * F7) + (K[1] * F6))) + (((K[1] * BC20[0][k]) + (K[4] * BC21[0][k]) + (K[7]
* BC22[0][k])) * ((K[7] * F8) + (K[4] * F7) + (K[1] * F6))) + (((K[1] * BC00[0][k]) + (K[4] * BC01[0][k]) + (K[7] * BC02[0][k])) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[1] * BC00[0][k]) +
(K[4] * BC01[0][k]) + (K[7] * BC02[0][k])) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][k])) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3)
+ 1.0)) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][k])) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) + 1.0))) / 2.0) + (((((K[0] * BC20[0][k]) + (K[3] * BC21[0][k]) + (K[6] *
BC22[0][k])) * ((K[6] * F8) + (K[3] * F7) + (K[0] * F6))) + (((K[0] * BC20[0][k]) + (K[3] * BC21[0][k]) + (K[6] * BC22[0][k])) * ((K[6] * F8) + (K[3] * F7) + (K[0] * F6))) + (((K[0] * BC10[0][k]) +
(K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((K[6] * F5) ….
    }
  }
}
…
```

Finite element - Hyperelasticity operator

How “contrib” could look like

m, n, o rarely greater than 30
typically between 3 and 15

```
…
for (int ip  = 0; ip < m; ++ip) {
  …
  for (int j  = 0; j < n; ++j) {
    for (int k  = 0; k < o; ++k) {
        A[j][k] += ((((K[2] * BC10[0][j]) + (K[5] * BC11[0][j]) + (K[8] * BC12[0][j])) * (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][k])) * ((((((K[8] * F2) + (K[5] *
F1) + (K[2] * F0)) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[7] * F8) + (K[4] * F7) + (K[1] * F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[8] * F5) + (K[5] * F4) + (K[2] *
F3)) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) + 1.0))) / 2.0)) + (((((K[8] * F2) + (K[5] * F1) + (K[2] * F0)) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[7] * F8) + (K[4] * F7) + (K[1] *
F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[8] * F5) + (K[5] * F4) + (K[2] * F3)) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) + 1.0))) / 2.0))) * F9) + (((K[6] * F5) + (K[3] * F4) +
(K[0] * F3)) * ((((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[6] * F8) + (K[3] * F7) + (K[0] * F6))) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k]))
* ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[0] * BC00[0][k]) + (K[3] *
BC01[0][k]) + (K[6] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[0] * BC20[0][k]) + (K[3] * BC21[0][k]) + (K[6] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0))
+ (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[6] * F2) + (K[3] * F1) + (K[0] * F0) + 1.0))) / 2.0)) + ((((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0]
[k])) * ((K[6] * F8) + (K[3] * F7) + (K[0] * F6))) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k]) + (K[5] *
BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[0] * BC00[0][k]) + (K[3] * BC01[0][k]) + (K[6] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) +
(((K[0] * BC20[0][k]) + (K[3] * BC21[0][k]) + (K[6] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[6] *
F2) + (K[3] * F1) + (K[0] * F0) + 1.0))) / 2.0))) * F9) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((((((K[8] * F5) + (K[5] * F4) + (K[2] * F3)) * ((K[6] * F5) + (K[3] *
F4) + (K[0] * F3))) + (((K[6] * F8) + (K[3] * F7) + (K[0] * F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[8] * F2) + (K[5] * F1) + (K[2] * F0)) * ((K[6] * F2) + (K[3] * F1) + (K[0] *
F0) + 1.0))) / 2.0)) + (((((K[8] * F5) + (K[5] * F4) + (K[2] * F3)) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[6] * F8) + (K[3] * F7) + (K[0] * F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] *
F6) + 1.0)) + (((K[8] * F2) + (K[5] * F1) + (K[2] * F0)) * ((K[6] * F2) + (K[3] * F1) + (K[0] * F0) + 1.0))) / 2.0))) * F9) + (((((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) *
((K[7] * F8) + (K[4] * F7) + (K[1] * F6))) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[1] * BC00[0][k]) + (K[4] * BC01[0]
[k]) + (K[7] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[1] *
BC20[0][k]) + (K[4] * BC21[0][k]) + (K[7] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[7] * F5) + (K[4]
* F4) + (K[1] * F3) + 1.0))) / 2.0)) + (((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] *              ) + (K[1] * F6))) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k])
+ (K[7] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[1] * BC00[0][k])                 k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] *
BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[7] * F2) + (K[4] * F1) + (K[1] *             BC21[0][k]) + (K[7] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7)
+ (K[2] * F6) + 1.0)) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * (             + 1.0))) / 2.0))) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) +
1.0) * F9) + (((K[8] * F5) + (K[5] * F4) + (K[2] * F3)) * ((((((((K[2] * BC10[0][k]) + (K[5] *              [8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k])
+ (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + ((              + (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] *
F0))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[             [0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[8] *
F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8]               7) + (K[2] * F6) + 1.0))) / 2.0)) + ((((((K[2] * BC10[0][k]) +
(K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((               + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] *
F3))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[             [0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] *
F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[             [2] * F6) + 1.0)) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k])
+ (K[8] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0))) / 2.0))) * F9) + (((             * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] *
F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) +          + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) +
(K[8] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) *          (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC20[0]
[k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC20[0][k]) + (K[5] * BC          [0][k])) * ((K[8] * F8) + (K[5] * F7)
+ (K[2] * F6) + 1.0))) / 2.0) + ((((K[1] * BC20[0][k]) + (K[4] * BC21[0][k]) + (K[7] * BC22[0][k])) * ((K[7] * F8) + (K[4] * F7) + (K[1]          [0][k]) + (K[4] * BC21[0][k]) + (K[7]
* BC22[0][k])) * ((K[7] * F8) + (K[4] * F7) + (K[1] * F6))) + (((K[1] * BC00[0][k]) + (K[4] * BC01[0][k]) + (K[7] * BC02[0][k])) * ((K[7] * F2) + (K[4]          [1] * F0))) + (((K[1] * BC00[0][k]) +
(K[4] * BC01[0][k]) + (K[7] * BC02[0][k])) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][          [7] * F5) + (K[4] * F4) + (K[1] * F3)
+ 1.0)) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][k])) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) + 1.0))) / 2.0) + (((((K[0] * BC20[0][       + (K[3] * BC21[0][k]) + (K[6] *
BC22[0][k])) * ((K[6] * F8) + (K[3] * F7) + (K[0] * F6))) + (((K[0] * BC20[0][k]) + (K[3] * BC21[0][k]) + (K[6] * BC22[0][k])) * ((K[6] * F8) + (K[3] * F7) + (K[0       F6))) + (((K[0] * BC10[0][k]) +
(K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((K[6] * F5    ....
    }
  }
}
…
```

• • •

Finite element - Hyperelasticity operator

How “contrib” could look like

m, n, o rarely greater than 30
typically between 3 and 15

```
...
for (int ip  = 0; ip < m; ++ip) {
  ...
  for (int j  = 0; j < n; ++j) {
    for (int k  = 0; k < o; ++k) {
```

Work in progress

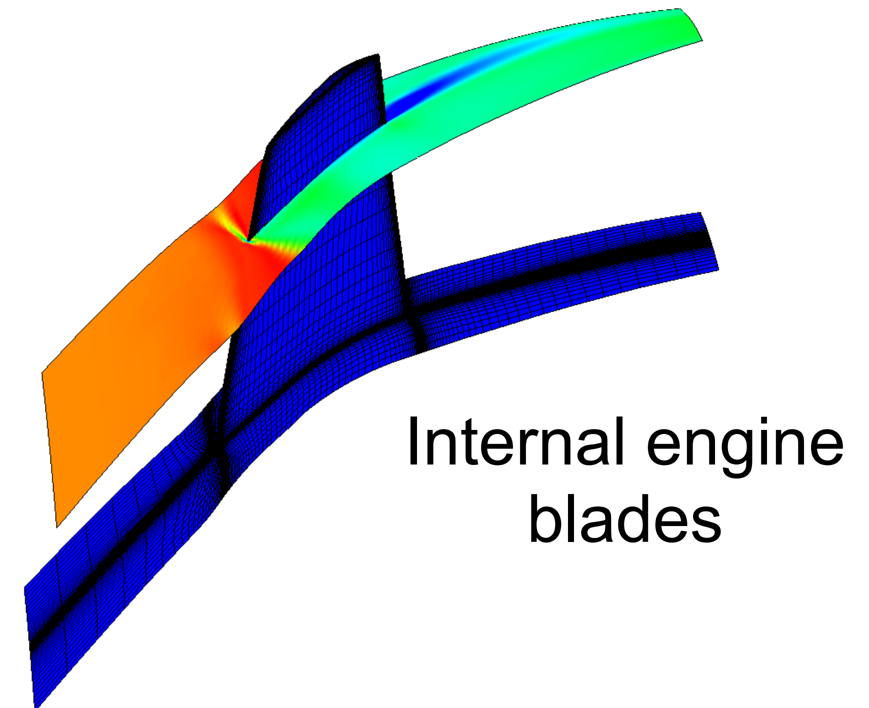
```
[j])) * ((((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][k])) * (((((((K[8] * F2) + (K[5] *
+ (K[1] * F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[8] * F5) + (K[5] * F4) + (K[2] *
F1) + (K[2] * F0)) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[7] * F8) + (K[4] * F7) + (K[1] *
* F3)) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) + 1.0))) / 2.0))) * F9) + (((K[6] * F5) + (K[3] * F4) +
* F8) + (K[3] * F7) + (K[0] * F6))) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k]))
+ (K[8] * BC12[0][k])) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[0] * BC00[0][k]) + (K[3] *
20[0][k]) + (K[3] * BC21[0][k]) + (K[6] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0))
* F1) + (K[0] * F0) + 1.0))) / 2.0)) + ((((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0]
k]) + (K[6] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k]) + (K[5] *
BC00[0][k]) + (K[3] * BC01[0][k]) + (K[6] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) +
7) + (K[2] * F6) + 1.0)) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[6] *
1[0][k]) + (K[6] * BC12[0][k])) * ((((((K[8] * F5) + (K[5] * F4) + (K[2] * F3)) * ((K[6] * F5) + (K[3] *
+ (K[2] * F6) + 1.0)) + (((K[8] * F2) + (K[5] * F1) + (K[2] * F0)) * ((K[6] * F2) + (K[3] * F1) + (K[0] *
F4) + (K[0] * F3))) + (((K[6] * F8) + (K[3] * F7) + (K[0] * F6)) * ((K[8] * F8) + (K[5] * F7) + (K[2] *
* F0) + 1.0))) / 2.0))) * F9) + (((((((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) *
[7] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[1] * BC00[0][k]) + (K[4] * BC01[0]
k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[1] *
* F6) + 1.0)) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[7] * F5) + (K[4]
) + (K[1] * F6))) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k])
k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] *
BC21[0][k]) + (K[7] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7)
+ 1.0))) / 2.0))) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) +
[8] * F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k])
+ (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] *
F0))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[ ... [0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[8] *
F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] ... 7) + (K[2] * F6) + 1.0))) / 2.0)) + (((((K[2] * BC10[0][k]) +
(K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] * F3))) + ((( ... + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + (K[2] *
F3))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] * F2) + (K[ ... [0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ((K[8] *
F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[ ... [2] * F6) + 1.0)) + (((K[2] * BC20[0][k]) + (K[5] * BC21[0][k])
+ (K[8] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0))) / 2.0))) * F9) + ((( ... * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] *
F5) + (K[5] * F4) + (K[2] * F3))) + (((K[2] * BC10[0][k]) + (K[5] * BC11[0][k]) + (K[8] * BC12[0][k])) * ((K[8] * F5) + (K[5] * F4) + ... + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) +
(K[8] * BC02[0][k])) * ((K[8] * F2) + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC00[0][k]) + (K[5] * BC01[0][k]) + (K[8] * BC02[0][k])) * ... + (K[5] * F1) + (K[2] * F0))) + (((K[2] * BC20[0]
[k]) + (K[5] * BC21[0][k]) + (K[8] * BC22[0][k])) * ((K[8] * F8) + (K[5] * F7) + (K[2] * F6) + 1.0)) + (((K[2] * BC20[0][k]) + (K[5] * BC ... [0][k])) * ((K[8] * F8) + (K[5] * F7)
+ (K[2] * F6) + 1.0))) / 2.0) + (((((K[1] * BC20[0][k]) + (K[4] * BC21[0][k]) + (K[7] * BC22[0][k])) * ((K[7] * F8) + (K[4] * F7) + (K[1] ... [0][k]) + (K[4] * BC21[0][k]) + (K[7]
* BC22[0][k])) * ((K[7] * F8) + (K[4] * F7) + (K[1] * F6))) + (((K[1] * BC00[0][k]) + (K[4] * BC01[0][k]) + (K[7] * BC02[0][k])) * ((K[7] * F2) + (K[4] ... K[1] * F0))) + (((K[1] * BC00[0][k]) +
(K[4] * BC01[0][k]) + (K[7] * BC02[0][k])) * ((K[7] * F2) + (K[4] * F1) + (K[1] * F0))) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][ ... K[7] * F5) + (K[4] * F4) + (K[1] * F3)
+ 1.0)) + (((K[1] * BC10[0][k]) + (K[4] * BC11[0][k]) + (K[7] * BC12[0][k])) * ((K[7] * F5) + (K[4] * F4) + (K[1] * F3) + 1.0))) / 2.0) + (((((K[0] * BC20[0] ... (K[3] * BC21[0][k]) + (K[6] *
BC22[0][k])) * ((K[6] * F8) + (K[3] * F7) + (K[0] * F6))) + (((K[0] * BC20[0][k]) + (K[3] * BC21[0][k]) + (K[6] * BC22[0][k])) * ((K[6] * F8) + (K[3] * F7) + (K[ ... F6))) + (((K[0] * BC10[0][k]) +
(K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((K[6] * F5) + (K[3] * F4) + (K[0] * F3))) + (((K[0] * BC10[0][k]) + (K[3] * BC11[0][k]) + (K[6] * BC12[0][k])) * ((K[6] * F5....
    }
  }
}
...
```

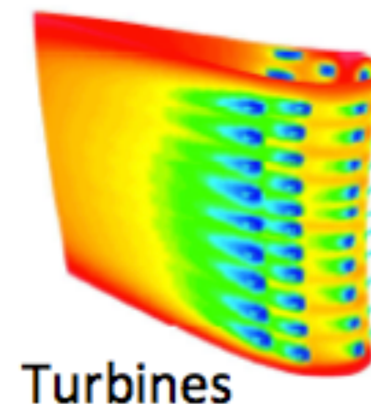
...

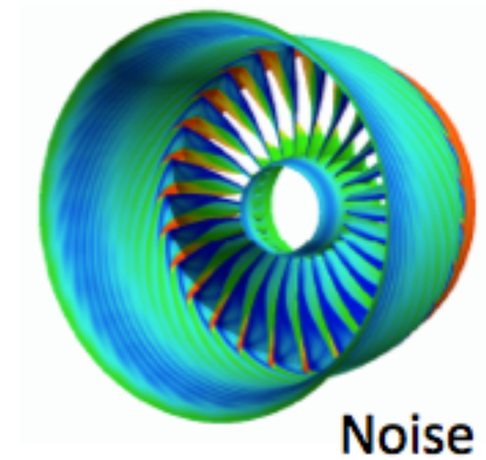
Finite element - Hyperelasticity operator

Performance results - HYDRA

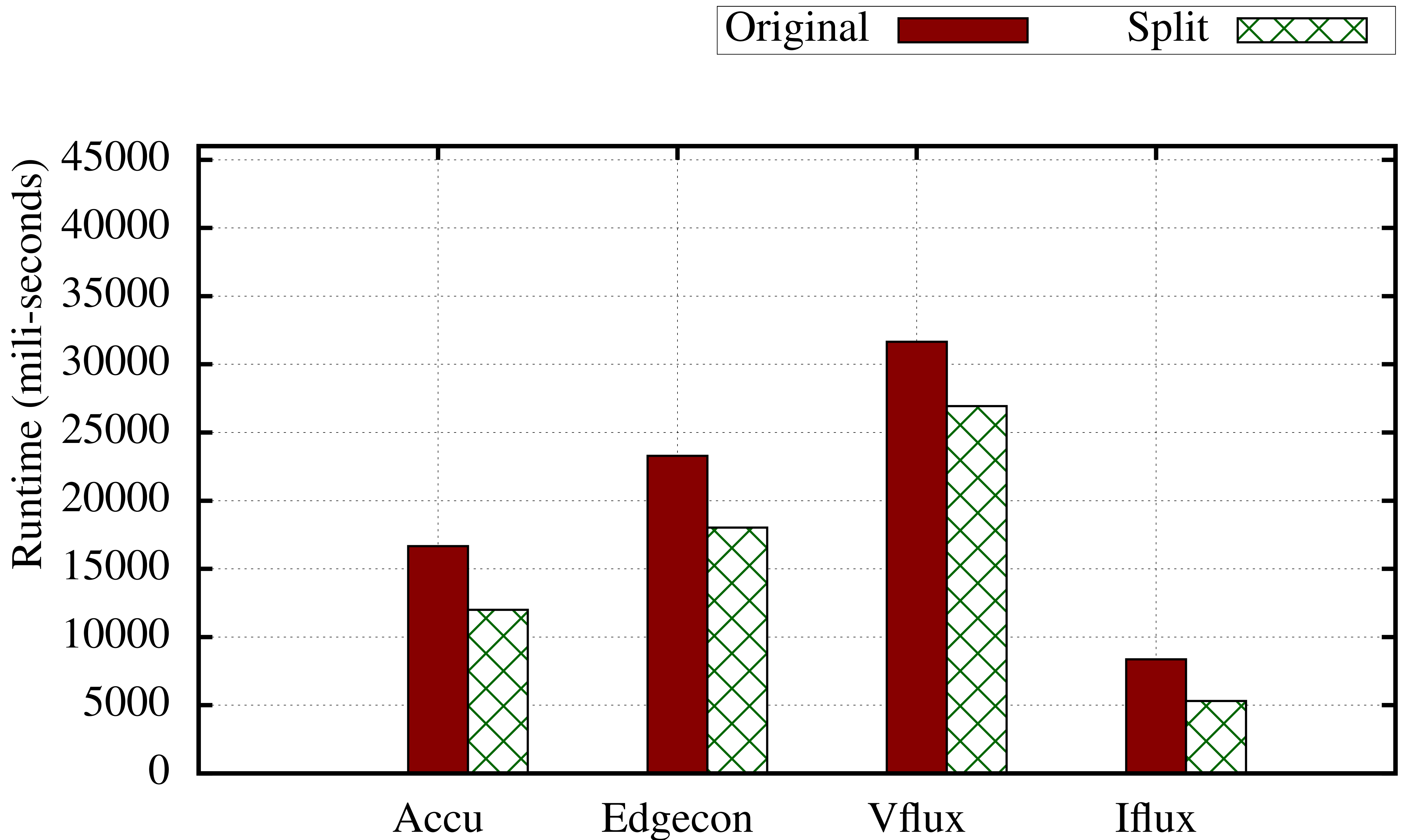
- Hydra is an unstructured mesh production CFD application used at Rolls-Royce for simulating turbo-machinery of aircraft engines
- Used for the design of turbomachinery
  - Key CFD production code
  - Steady and unsteady flow
  - Reynolds Averaged Navier-Stokes
- **In development for >15 years**
  - **Fortran 77**
  - **> 50k lines of code**
  - **> 300 computational op_par_loops**







Performance results - HYDRA



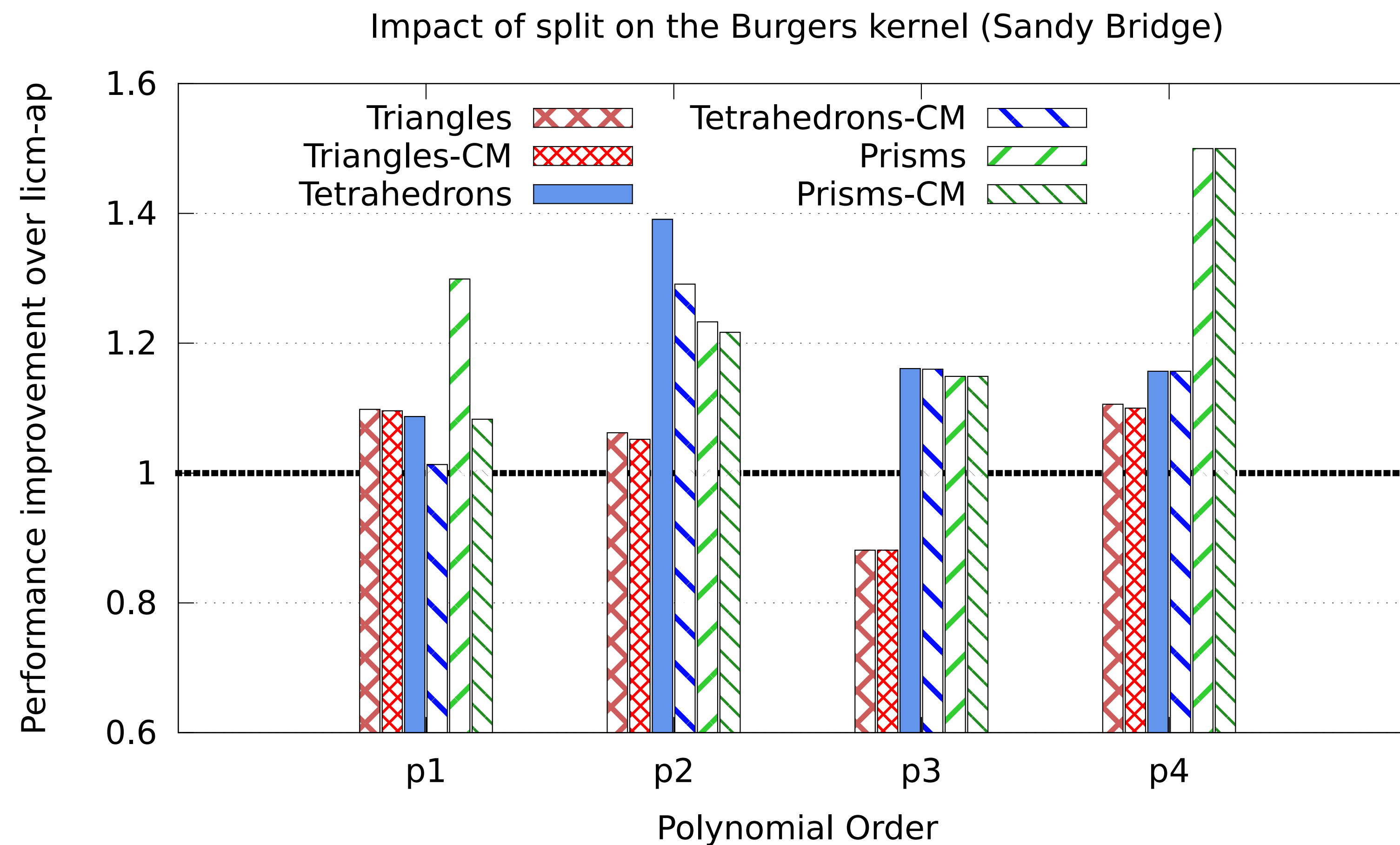
GPU: NVIDIA Tesla C2070

Conclusions and Future Work

- *"Effective Resource-Driven Loop Splitting for Large Unstructured Mesh Applications on GPUs" — "Resource-driven" is notably important*
- *Methodology:*
  1. *prototypes to assess loop splitting, at different levels*
  2. *implementation in the OP2 compiler (on going work)*
  3. *implementation in Firedrake (expression splitting)*
- *This study: driven by real computations' memory requirement*
- *Simple but effective transformations, up to ~35% speedup over the original op_par_loop implementation.*

Spare slides

Towards generalized loop splitting (finite element)



- Problem:
  - **Burgers equation**
  - polynomial orders 1 to 4
  - speedup over original (i.e., non-transformed) code
  - expression restructured (split) through COFFEE for improving register reuse
  - heuristic: minimize repeated accesses to same arrays in the split loops
- Setup:
  - Single core of an Intel Sandy Bridge (I7-2600 CPU @ 3.40GHz)
  - Intel compiler (version 13.1, -O3, -xAVX, -ip, -xHost)

Generalized loop splitting

- Remember the title of the talk:
  "*Effective Resource-Driven Loop Splitting for Large Unstructured Mesh Applications on GPUs*"
- *Resource can be shared memory in a GPU…*
- *…or even registers in a CPU!*
- *Future work: (after suitable optimization for FLOPS,) create an expression graph and find splitting points that improve memory usage*