An Improved Three-dimensional Localization Scheme based on APIT

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Abstract—In wireless sensor networks, location information is essential to its monitoring activities. In view of the “boundary effect” and lack of accuracy in APIT-3D, this paper proposes an improved three-dimensional localization scheme based on APIT named IAPIT-3D. In the scheme, the probability of misjudgment is reduced by adding the conditions of judgment. On one hand, the signal strength that all of the neighbor nodes receive should be compared with the one of the unknown node. On the other hand, two variables are set for comparison to determine the position of the node relative to the triangular pyramid. Furthermore, narrowing down the targeted area through “Perpendicular Median Surface Cutting” reduces the localization error. The results of simulation show that compared with APIT-3D, IAPIT-3D can improve the accuracy of localization.

Index Terms—wireless sensor networks; APIT; three-dimensional localization; Perpendicular Median Surface Cutting

I. INTRODUCTION

Wireless sensor network (WSN) is a self-organized network that consists of large-scale and energy constrained micro-sensor nodes with low power consumption \cite{1}. In WSN, location information is essential to its monitoring activities. The location information is an important element in the monitoring message. The message without location information is usually meaningless. Therefore, node localization is one of the most basic functions in WSN and plays a key role in the effectiveness of the WSN’s application.

Currently, the research on self-location of the node is mainly focused on the two-dimensional plane \cite{2}. But the two-dimensional plane node localization algorithm is difficult to be practiced in the three-dimensional space due to the more complicated environmental factors and the increased computational cost, which would lead to great complexity of localization. However, the practical applications of wireless sensor nodes are usually distributed in three-dimensional space. For example, in the fire control work, only when the firefighters locate the exact floor and room can they control the fire and then put out. If combat occurs in fields, the soldiers cannot cooperate well without knowing the three-dimensional positions of the comrade-in-arms. For the malfunction monitoring of large equipment, the three-dimensional coordinate of the point appearing failure must be known. In the process of environmental monitoring of ocean and lake, the parameters such as temperature, salinity, pressure, flow, dissolved oxygen and sulfur dioxide should be sensed by the nodes which are distributed at the different depth of the water. Thus, it is apparent, from all above cases, that the nodes are usually deployed randomly in the three-dimensional space, or even the adverse circumstances that the human cannot reach and predict the position of each node. Therefore, the research on the three-dimensional localization of nodes with high precision, low power consumption and low cost has become the essential part for the development and application of WSN.

In this paper, we address the issue of three-dimensional localization. Section II presents some works related to localization methods. Section III introduces APIT-3D localization scheme. Section IV analyzes the “in-to-out error” and “out-to-in error”. Section V proposes an improved three-dimensional localization algorithm based on APIT. In section VI, simulation is presented and the results are analyzed. Finally, in section VII, we present some concluding remarks.

II. RELATED WORKS

The localization mechanism in WSN is usually classified into two categories: range-based mechanism and range-free mechanism \cite{1}.

Range-based localization mechanism mainly includes TOA \cite{4} (time of arrival), TDOA \cite{4,6} (time difference of arrival), AOA \cite{7} (angle of arrival), RSSI \cite{8-10} (received signal strength indicator), etc. The principle of TOA-based localization is that use the known signal speed and its propagation time to calculate the distance between two nodes, and then the position of nodes can be computed through the fundamental approaches (trilateration / triangulation / max likelihood estimation). For the localization based on TDOA, the transmitting node

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launches two different signals with their respective speed, and thereby the distance between the transmitting node and the receiving node can be calculated according to the speed of signals and the time difference. Then the position is localized by the same method mentioned above. As to the AOA localization, through the direction, obtained by the receiving node, in which the signal arrives by the antenna arrays or several ultrasonic receivers, the angle between transmitting and receiving nodes is calculated. Then, the position is gained through triangulation. The localization based on RSSI uses the known intensity of launched signal and received signal to obtain the propagation loss which is transformed into the distance between these two nodes, through the empirical model and theoretical model. Similarly, the position is calculated through the fundamental approaches mentioned above. To sum up, the range-based localization mechanism calculates the position by measuring the actual distance or orientation between the nodes. However, nodes need additional hardware facilities to do the range measurement.

For the limitation of hardware, range-free localization mechanism which has the advantages of low cost, little environmental impact, and low requirement for the hardware is proposed. It mainly includes centroid algorithm [11], DV-HOP algorithm [12] and APIT (approximate point-in-triangulation test) algorithm, etc. With the centroid algorithm, the anchor nodes broadcast the messages of their IDs and positions to the neighbors periodically. When the number of messages received by the unknown node exceed a certain threshold, the centroid of polygon consisted of anchors is taken as the position of this unknown node. Under the DV-HOP algorithm, the minimum hops between the node and the anchors are calculated by the unknown node, and the length of every hop is estimated which is then used to obtain the distance between unknown nodes and anchors by multiplying the minimum hops. Finally, the position of the unknown node can be gained by the same approaches as previous ones.

As for the APIT algorithm, several triangulations containing the unknown node shall be determined. The overlapped regions of these triangulations form a polygon which narrows down the region including the unknown node. The centroid of the polygon is considered as the position of the unknown node. Among these algorithms, APIT algorithm has the advantages of high accuracy and stable performance under the circumstance of irregular wireless signal and random deployment of sensor nodes. Compared with the centroid algorithm whose thought is similar to APIT, the APIT algorithm gets higher accuracy and lower requirements for the distribution of anchors.

Tian He et al. firstly propose APIT localization algorithm [13]. They detail the theoretical basis and the positioning process of APIT, and discuss two kinds of misjudgment which are prone to happen in the localization mechanism. Then, by the comparison between APIT and the other range-based localizations, analysis of effects of irregular radio (DOI), anchors heard (AH), anchor to node range ratio (ANR), node density (ND) and GPS error on the performance of APIT was given. Literature [14] improves the APIT by using the idea of iteration that the located nodes are promoted to anchor nodes. The method increases the coverage of the anchors and reduces the probability of misjudgment. Literature [15] uses the concept of Volume-coordinates and integrates some thought of APIT into the three-dimensional centroid algorithm to improve the accuracy. In literature [16], the original APIT is improved by using the method of “Perpendicular Bisectors Cutting” to enhance the accuracy. Literature [17] extends the APIT into three-dimensional space and proposes an APIT-3D (Approximate Point-In-Tetrahedron) localization scheme. The scheme can achieve a relatively satisfactory accuracy without increasing the communication overhead. However, by using this scheme, misjudgment still exists so that it can’t fulfill the requirement of the applications with high accuracy.

As for the two misjudgment existed in APIT-3D, IAPIT-3D (Improved Approximate Point-In-Tetrahedron) is proposed in this paper. It improves the localization accuracy in three-dimensional space effectively while it nearly does not increase the locating time.

III. APIT-3D LOCALIZATION SCHEME

A. APIT Localization Scheme

1) The theoretical basis of APIT

The theoretical basis of APIT is PIT (perfect point-in-triangulation test). The schematic diagram of PIT is shown in Fig.1.

![Figure 1. The schematic diagram of PIT](image)

**Perfect P.I.T Test Theory:** If there exists a direction such that a point adjacent to M is further/closer to points A, B, and C simultaneously, then M is outside of ΔABC. Otherwise, M is inside ΔABC.

2) The basic flow of APIT

However, in WSN, the nodes are usually static. APIT uses the relatively high density of the nodes to simulate the movement of the nodes. It uses the characteristics of the signal transmission to judge whether the node is close to or away from the anchors. Generally, the closer the node is to another, the stronger the signal is received. The node judges whether it is close to or away from the anchor through exchanging the strength of received signal between the neighbors in order to simulate the movement of the nodes in PIT. As shown in Fig.2 (a), M exchanges...
message with node 1. It can be known that the signal strengths that M receives from anchor B, C are larger than that node 1 receives and the signal strength that M receives from anchor A is smaller than that node 1 receives. According to the comparison of signal strength, if M moves to the position of node 1, it will be close to the anchor A and away from anchor B, C. Node 2, 3, 4 can be judged in the same way successively. Then it can be known that M is inside \( \triangle ABC \). As shown in Fig. 2(b), if M moves to the position of neighbor2, it will be far away from the anchor A, B, C simultaneously. According to PIT, M is outside of \( \triangle ABC \). Finally, as shown in Fig. 2(c), the centroid of the overlapped region is taken as the position of the unknown node.

The main process of APIT is described as follows: initially, unknown nodes collect the information of the neighbor anchors. Then three anchor nodes are selected from the set of these neighbor anchors arbitrarily. Assuming that there are n elements in the set, then, \( c^1_n \) different selections can be made to constitute \( c^1_n \) different triangles. Each triangle needs to be tested for deciding whether the unknown node is inside the triangle in order to achieve the required positioning accuracy. Finally, the centroid of the triangles’ overlapped region will be calculated as the position of unknown node.

### B. APIT-3D \(^{[17]}\)

APIT-3D is a three-dimensional localization algorithm based on APIT whose process is similar to APIT. Firstly, each unknown nodes receive the messages and measure the RSSI. Then, the unknown nodes exchange the message received from the anchors (including the value of RSSI) and judge whether they are inside the triangular pyramids which are constituted by the anchors. Finally, the centroid of all the triangular pyramids’ intersection is calculated as the position of the unknown nodes. The same as APIT, APIT-3D also adopts the method of simulating dynamic state from static state to judge whether the unknown node is inside the triangular pyramids or not. The principle diagrams are shown in Fig.3.

![Figure 2. Approximate Point-In-Triangulation Test](image)

(a) M is inside \( \triangle ABC \)  
(b) M is outside of \( \triangle ABC \)

![Figure 3. Schematic diagram of APIT-3D](image)

(a) The unknown node is inside the triangular pyramid  
(b) The unknown node is outside of the triangular pyramid

In Fig. 3(a), node 1, 2, 3, 4, and 5 are the neighbors of unknown node N. The signal strength that node 1 receives from the anchor A is larger than N receives. The signal strength that node 1 receives from anchor B, C and D is smaller than that N receives. By analogy, the node 2, 3, 4 and 5 can also simulate movement of N. Then it can be known that N is inside the tetrahedron. As shown in Fig. 3(b), the signal strength that neighbor node 1,2 and 5 receive from anchor A,B,C and D is larger than that N receives. On the opposite, the signal strength that neighbor node 3 and 4 receive from anchor A, B, C and D is smaller than that N receives. That means that N is outside the tetrahedron.

In summary, determination conditions of APIT-3D are described as follows: as long as there is one neighbor M of N that the signal strength M receives from anchor A, B, C and D is smaller or larger than that N receives, N is determined to be outside of the triangular pyramid. Otherwise, N is inside the triangular pyramid.

### IV. "IN-TO-OUT ERROR" AND "OUT-T-O-IN ERROR"

Whether the node is inside the triangular pyramid or not is judged via exchanging information with its neighbor nodes. So, the correctness of the APIT-3D test is closely related to the distribution of nodes. In actual practice, the “out-to-in error” and “in-to-out error” are commonly seen. The two scenes which are usually called “boundary effect” are shown in Fig.4.

It can be seen from Fig.4, (a) that N is outside of the triangular pyramid but near the plane ABD. The signal strength that node N receives from the anchor A, B, and D is larger than that N’ receives. The signal strength that node N receives from the anchor C is smaller than that N’ receives. According to the determination conditions in APIT-3D, if N’ is the unique neighbor of node N, N is judged to be inside the triangular pyramid. It can be seen from Fig.4, (b), the strength of signal that node N
receives from the anchor A, B, C, and D is larger than that N’ receives. If N’ is the unique neighbor node of N, according to the determination conditions in APIT-3D, N is outside of the triangular pyramid.

![Image](image1)

(a) “out-to-in error”  (b) “in-to-out error”  
Figure 4. The Error Scenarios for the APIT-3D Test

Literature [13] analyzes the relation between node density and the error percentage in APIT as shown in Fig. 5. It can be known from the figure that the occurrence probability of “out-to-in error” is high and obviously higher than that of “in-to-out error”. The errors are particularly serious when the node density is low. Therefore, the modification of these two errors is expected to significantly improve system performance.

![Image](image2)

Figure 5. The occurrence probability of the two errors

V. THE IMPROVED THREE-DIMENSIONAL LOCATING ALGORITHM BASED ON APIT

As an improvement of APIT-3D, IAPIT-3D adopts the “Perpendicular Median Surface cutting” to narrow down targeted area in order to decrease the error of localization. It reduces the probability of misjudgment by adding the conditions of judgment.

A. Perpendicular Median Surface Cutting

In order to improve the positioning accuracy of APIT-3D, this paper proposes an improved localization scheme named IAPIT-3D.

According to the theorem of space geometry that the distances between any points on the perpendicular median surface and the two endpoints of the line segment are equal, the points distributed on both sides of the perpendicular median surface satisfy the following condition:

$$\left| \frac{\times}{2} - x_0 \right| + \left| \frac{\times}{2} - x_0 \right| + \left| \frac{\times}{2} - x_0 \right| > \left( x_0 - x_0 \right)$$

Given that the endpoints of line segment are A (x_a, y_a, z_a) and B (x_b, y_b, z_b), if N(x, y, z) satisfies AN > BN, then

$$\left( x_0 - x_0 \right) + \left( x_0 - x_0 \right) + \left( x_0 - x_0 \right) > 0$$

As shown in Fig.6, the triangular pyramid can be divided into two parts by the perpendicular median surface. Plane EFG is the perpendicular median surface of BC. So the distances from any point on EFG to B and C should be equal. It can be seen from Fig.6 that the distance between unknown node N and B is shorter than the distance between N and C. The signal strength that N receives from B is greater than the signal strength that N receives from C. So, it can be determined which part of the triangular pyramid the unknown node belongs to after the cutting.

![Image](image3)

Figure 6. The perpendicular median surface cutting

It can be known from Fig.7 that every triangular pyramid is constituted by six sides. That means the triangular pyramid can be cut six times. After being cut six times, the region which contains the unknown node is narrowed down dramatically. That means the accuracy of localization will be improved.

![Image](image4)

Figure 7. The triangular pyramid after cutting

B. The Solutions of the “Boundary Effect”

In the view of the two kinds of misjudgment which is usually called “boundary effect” in APIT-3D, the additional determination conditions used for reducing the misjudgments are introduced as follows:

- The signal strengths that received by all of the neighbor nodes (including the neighbor anchors) should be compared with that the unknown node receives for deciding whether the unknown is inside the triangular pyramid or not. In essence, it increases the directions of simulated movement.
- Two variables are defined to determine whether the unknown node is inside the triangular pyramid or not. One variable represents the frequency of the node being determined to be inside the triangular pyramid. While the other represents the frequency of that the unknown node is determined outside of the triangular pyramid. That the unknown node is inside or outside of the pyramid.
is determined by the variable with higher frequency.

C. The Flow of IAPIT-3D

To sum up, the basic flow of IAPIT-3D is delineated as the following 8 steps:

Step 1: Initialize the configuration of the network. That means the sensor nodes are distributed randomly and all the counters reset to 0.

Step 2: The anchors transmit their information of locations and IDs.

Step 3: The nodes receive and record information from the anchors (including signal intensity).

Step 4: The neighbor nodes exchange information from anchors with each other.

Step 5: Determine whether the nodes have more than 3 neighbor anchors.

Step 6: If the node has more than 3 neighbor anchors, the APIT-3D with Perpendicular Median Surface Cutting is executed. Otherwise, determine whether there is any neighbor node.

Step 7: If the APIT-3D is executed, the localization has been finished. If the number of neighbor anchors is less than 4, the centroid of the graph consisted of the anchors is regarded as the position of the unknown node. If there is no neighbor anchor, the node is considered to be the one which cannot be localized. Thus the position of the unknown node has to be the centroid of the whole space.

Step 8: The localization is finished.

The flow chart of IAPIT-3D is shown in Fig.8. The difference between APIT-3D and IAPIT-3D is that IAPIT-3D adds two conditions to limit the probability of misjudgment and uses “Perpendicular Median Surface Cutting” to improve the accuracy.

D. Grid SCAN

IAPIT-3D adopts the method of “Grid SCAN” [13]. It divides the whole space into a number of small cubes with the same volume and each of them corresponds to a counter. If the cube is localized within the triangular pyramid that contains the node, the counter increases, meaning the number on the cube increases. If the cube is localized within the triangular pyramid that does not contain the node, the counter decreases. The max value of all counters then will be figured out and the ones with the max value are selected to constitute a 3D-graph. The centroid of this 3D-graph is the position of the unknown node. The 2D schematic diagram is showed in Fig. 9.

VI. SIMULATION

In order to verify the validation of IAPIT-3D, the scientific tool MATLAB is adopted for the simulation. It creates relatively ideal environment without complicated factors, such as geomagnetic interference and obstacle etc. There is only white Gaussian noise existing in the process of the experiment.

A. Experimental Parameters

200 unknown nodes and some anchors are randomly deployed in a space of size 50m × 50m × 50m. The number of anchor nodes is increased from 10 to 100 with step of 10. The average localization error (normalized to the ratio of locating error to radio range) is calculated by the experiments with different radio range (R=10m, R=12m, R=15m, R=18m).

B. Experimental Results and Analysis

Fig.10 describes the relationship between the number of anchors and the average error under different radio ranges. There is a characteristic of the curves: although the general trend of locating error is downward with the increment of anchors, the phenomenon that the locating accuracy is decreasing with the increment of anchors exists in short-term. This is caused by the Self-defect of APIT-3D. In APIT-3D, when the unknown node is being located, the number of the triangular pyramids which contain the unknown node is increasing with the increment of the anchors. That means the region of reconstructed overlap area is reduced. But this may make the estimated position deviating from the actual position of the unknown node. Fig.11 describes the Self-defect of APIT in the two-dimensional space [16].

Figure 8. The flow chart of IAPIT-3D

Figure 9. Grid SCAN [13]
The performance and characters of the IAPIT-3D are revealed by analyzing the relationship and comparison between parameters as follows:

1) The relationship between radio range and localization error

Taking the occasion that the number of anchor nodes is 80 for example, the errors of APIT-3D and IAPIT-3D under different radio ranges are shown in Table 1.

<table>
<thead>
<tr>
<th>R(m)</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error (%)</td>
<td>APIT-3D</td>
<td>130.25</td>
<td>75.65</td>
<td>44.38</td>
</tr>
<tr>
<td></td>
<td>IAPIT-3D</td>
<td>127.70</td>
<td>71.12</td>
<td>31.71</td>
</tr>
</tbody>
</table>

It can be known from Fig.10 and Table.1 that the localization error is decreasing with the increment of radio range because the number of unknown nodes’ neighbor is increasing with the increment of radio range. This brings two advantages:

- More simulated moving nodes can be used to reduce the “in-to-out error” and “out-to-in error”.
- More triangular pyramid can be formed to shrink the region that the nodes might be inside in order to improve the positioning accuracy.

2) The relationship between radio range and improvement of locating accuracy

Taking the occasion that the number of anchor nodes is 80 as an example, it can be seen from Fig.10 and Table.1 that with the increment of radio range, the improvements effect of IAPIT-3D is more and more apparent.
3) The relationship between the number of anchor heard and improvement of locating accuracy

The relationship between the anchors heard and the number of deployed anchors when the radio range R equals 15 is shown in Fig.12. It can be analyzed from Fig.10 and Fig.12 that the average number of anchor heard (AH) is 4.01 when the number of anchors is 50. And the number of anchor heard is sufficient to form a triangular pyramid for APIT-3D localization. Therefore, compared with APIT-3D, the accuracy of IAPIT-3D is improved.

![Figure 12. The relationship between number of anchors and number of anchors heard when R=15](image)

4) Comparison of locating time

The locating time of APIT-3D and IAPIT-3D when the radio range R is 15 are compared as shown in Fig.13. Where, abscissa represents the number of anchor nodes, and ordinate represent the time of localization. It can be known from Fig.13 that the two curves almost coincide. So, compared with APIT-3D, IAPIT-3D almost has no increment of the locating time.

![Figure 13. The comparison diagram of locating time](image)

VII. CONCLUSION

APIT-3D is a range-free localization scheme, and can be applied under the case that the hardware is relatively simple. The special property of perpendicular median surface in the Geometry and approximate point-in-triangulation test are the theoretical foundation of IAPIT-3D. The main idea of IAPIT-3D is described as follows: the region in which the unknown nodes may exist should be determined at first. Then the region is narrowed down according to the property of perpendicular median surface. Finally, the centroid of the region is calculated as the position of the unknown node. The results of the simulation reveal that IAPIT-3D improves the estimation accuracy of the unknown nodes' position in three-dimensional space effectively. The locating time of IAPIT-3D is nearly the same as APIT-3D.

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