Contribution: This paper makes the following contributions.

- We show that, in contrast to the flat fading case, for frequency–selective fading channels beamforming filters (BFFs) instead of simple beamforming weights are beneficial. Thereby, we consider both finite impulse response (FIR) and infinite IR (IIR) BFFs.
- In case of IIR BFFs, we obtain a closed–form solution for the optimum BFFs which maximize the signal–to–noise ratio (SNR). For FIR BFFs we provide an efficient numerical method for calculation of the optimum BFFs.
- We show that transmit beamforming with perfect CSI results in significant performance gains for frequency–selective channels and can also guide the design of beamforming with imperfect CSI.

Organization: In Section II, the considered system model is presented. The optimization of IIR and FIR BFFs is discussed in Sections III and IV, respectively. Simulation results are provided in Section V, and some conclusions are drawn in Section VI.

I. INTRODUCTION

In recent years, the application of multiple antennas in wireless communication systems has received considerable interest from academia and industry. In particular, transmit beamforming has received much attention as a simple yet efficient technique to exploit the benefits of multiple transmit antennas, cf. e.g. [1] and references therein.

Beamforming generally requires channel state information (CSI) at the transmitter. Since perfect CSI may not be available at the transmitter in practical systems, recent research in this field has focused on the impact of noisy and/or quantized CSI, cf. e.g. [2], [3], [4]. However, most of the existing literature on transmit beamforming with perfect or imperfect CSI has assumed frequency–nonselective fading. In practice, this assumption may not be realistic, especially for high–rate transmission. For systems employing orthogonal frequency–division multiplexing (OFDM) to cope with frequency–selective fading channels, effective beamforming techniques were proposed in [5], [6]. However, these techniques cannot be applied to single–carrier systems, which are the focus of this paper. Beamforming for single–carrier transmission is especially relevant for the upgrade of existing systems such as the Global System for Mobile Communications (GSM) and the Enhanced Data Rates for GSM Evolution (EDGE) system.

In this paper, we consider transmit beamforming for frequency–selective fading channels with perfect CSI at the transmitter. Due to the intersymbol interference (ISI) caused by the frequency selectivity of the channel, equalization is necessary at the receiver and the optimum beamformer depends on the equalizer used. Here, we adopt decision–feedback equalization (DFE) because of its low complexity and good performance.

Contribution: This paper makes the following contributions.

- In this paper, we propose beamforming schemes for frequency–selective channels with decision–feedback equalization (DFE) at the receiver. We consider both finite impulse response (FIR) and infinite impulse response (IIR) beamforming filters (BFFs). In case of IIR beamforming, we are able to derive closed–form expressions for the optimum BFFs. Simulation and numerical results for typical GSM/EDGE channels confirm the significant performance gains achievable with beamforming compared to single–antenna transmission and optimized delay diversity.

Notation: In this paper, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$, $I_X$, $0$, $\mathbb{E}\{\cdot\}$, and $\otimes$, denote transpose, Hermitian transpose, complex conjugate, the $X \times X$ identity matrix, the all–zero column vector, expectation, and discrete–time convolution, respectively. Furthermore, $X(f) \triangleq \mathcal{F}\{x[k]\}$ is the discrete–time Fourier transform of $x[k]$ and $|x|^+ \triangleq \max(0, x)$.

II. SYSTEM MODEL

Let us consider a multiple–input multiple–output (MIMO) system with $N_T$ transmit and $N_R$ receive antennas. The block diagram of the discrete–time overall transmission system in complex baseband representation is shown in Fig. 1. The modulated symbols $b[k]$ are taken from a scalar symbol alphabet $A$ such as phase–shift keying (PSK) or quadrature amplitude modulation (QAM), and have variance $\sigma_b^2 = \mathbb{E}\{|b[k]|^2\} = 1$. The transmit BFF impulse response coefficients of antenna $n_t$, $1 \leq n_t \leq N_T$, are denoted by $g_{n_t}[k]$, $-\infty < k < \infty$, and their energy is normalized to $\sum_{n_t=1}^{N_T} \sum_{k=-\infty}^{\infty} |g_{n_t}[k]|^2 = 1$. The signal transmitted over antenna $n_t$ at time $k$ is given by

$$s_{n_t}[k] = g_{n_t}[k] \otimes b[k].$$

The discrete–time received signal at receive antenna $n_r$, $1 \leq n_r \leq N_R$, can be modeled as

$$r_{n_r}[k] = \sum_{n_t=1}^{N_T} h_{n_t,n_r}[k] \otimes s_{n_t}[k] + n_{n_r}[k],$$

where $n_{n_r}[k]$ denotes additive (spatially and temporally) white Gaussian noise (AWGN) with variance $\sigma_n^2 = \mathbb{E}\{|n_{n_r}[k]|^2\} = N_0$, and $N_0$ denotes the single–sided power spectral density of the underlying continuous–time passband noise process.

$$h_{n_t,n_r}[k], 0 \leq k \leq L-1,$$ denotes the overall channel impulse
response (CIR) between transmit antenna \( n_t \) and receive antenna \( n_r \) of length \( L \). In our model, \( h_{n_t,n_r}[k] \) contains the combined effects of transmit pulse shaping, wireless channel, receive filtering, and sampling. We assume a block fading model, i.e., the channel is constant for the duration of at least one data burst before it changes to a new independent realization. In general, the \( h_{n_t,n_r}[k] \) are spatially and temporally correlated because of insufficient antenna spacing and transmit/receive filtering, respectively.

By substituting Eq. (1) into Eq. (2) we obtain

\[
\begin{align*}
r_{n_r}[k] &= h^\text{eq}_{n_r}[k] \otimes b[k] + n_{n_r}[k],
\end{align*}
\]

where the equivalent CIR \( h^\text{eq}_{n_r}[k] \) corresponding to receive antenna \( n_r \) is defined as

\[
\begin{align*}
h^\text{eq}_{n_r}[k] &= \sum_{n_t=1}^{N_T} h_{n_t,n_r}[k] \otimes g_{n_t}[k].
\end{align*}
\]

Eq. (3) shows that a MIMO system with beamforming can be modeled as an equivalent single-input multiple-output (SIMO) system. Therefore, at the receiver the same equalization, channel estimation, and channel tracking techniques as for single-antenna transmission can be used.

III. BEAMFORMING WITH IIR FILTERS

We first consider IIR beamforming. For convenience the frequency responses of the IIR BFFs \( \tilde{G}_{n_t}(f) \equiv \mathcal{F}\{g_{n_t}[k]\} \) are collected in vector \( \mathbf{G}(f) \equiv [G_1(f)\ G_2(f)\ \ldots\ G_{N_T}(f)]^T \). The unbiased SNR for DFE with optimum IIR feedforward filtering is given by \[7\]

\[
\text{SNR}(\mathbf{G}(f)) = \frac{\sigma^2_{\chi}}{\sigma^2_e} - \chi,
\]

where \( \chi = 0 \) and \( \chi = 1 \) for zero-forcing (ZF) and minimum mean-squared error (MMSE) DFE filter optimization, respectively. In Eq. (5), the DFE error variance is given by \[8\]

\[
\sigma^2_e = \sigma^2_t \exp \left\{ -\frac{1}{2} \int \ln \left[ \xi + \sum_{n_t=1}^{N_T} |H^\text{eq}_{n_t}(f)|^2 \right] df \right\},
\]

where \( \xi = 0 \) and \( \xi = \sigma^2_t/\sigma^2_n \) for ZF–DFE and MMSE–DFE, respectively. The equivalent channel frequency response

\[
H^\text{eq}_{n_r}(f) \triangleq \mathcal{F}\{h^\text{eq}_{n_r}[k]\}
\]

is given by

\[
H^\text{eq}_{n_r}(f) = \sum_{n_t=1}^{N_T} G_{n_t}(f) H_{n_t,n_r}(f)
\]

with \( H_{n_t,n_r}(f) \triangleq \mathcal{F}\{h_{n_t,n_r}[k]\} \).

The optimum BFF vector \( \mathbf{G}_{\text{opt}}(f) \triangleq [G^\text{opt}_1(f)\ G^\text{opt}_2(f)\ \ldots\ G^\text{opt}_{N_T}(f)]^T \) shall maximize \( \text{SNR}(\mathbf{G}(f)) \) subject to the transmit power constraint

\[
\int_{-1/2}^{1/2} \mathbf{G}^H(f)\mathbf{G}(f) \, df = 1.
\]

A convenient approach for calculating \( \mathbf{G}_{\text{opt}}(f) \) is the classical Calculus of Variations method \[9\]. Following this method, we model the BFF of antenna \( n_t \) as \( G_{n_t}(f) = G^\text{opt}_{n_t}(f) + \varepsilon_n B_{n_t}(f) \), where \( B_{n_t}(f) \) and \( \varepsilon_n \) denote an arbitrary function of \( f \) and a real-valued variable, respectively. The optimization problem can now be formulated in terms of its Lagrangian

\[
L(\varepsilon) = \text{SNR}(\mathbf{G}(f)) + \mu \int_{-1/2}^{1/2} \mathbf{G}^H(f)\mathbf{G}(f) \, df,
\]

where \( \varepsilon \equiv [\varepsilon_1\ \varepsilon_2\ \ldots\ \varepsilon_{N_T}]^T \) and \( \mu \) is the Lagrange multiplier. The optimum BFF vector \( \mathbf{G}_{\text{opt}}(f) \) has to fulfill the condition \[9\]

\[
\frac{\partial L(\varepsilon)}{\partial \varepsilon^*} \bigg|_{\varepsilon=0} = 0,
\]

for arbitrary \( B_{n_t}(f) \), \( 1 \leq n_t \leq N_T \). Combining Eqs. (9) and (10), it can be shown that vector \( \mathbf{G}_{\text{opt}}(f) \) has to fulfill

\[
\mathbf{S}(f)\mathbf{G}_{\text{opt}}(f) = \bar{\mu} [\mathbf{G}^H_{\text{opt}}(f)\mathbf{S}(f)\mathbf{G}_{\text{opt}}(f) + \xi] \mathbf{G}_{\text{opt}}(f),
\]

where \( \bar{\mu} \) is a constant and \( \mathbf{S}(f) \) is an \( N_T \times N_T \) matrix given by \( \mathbf{S}(f) \triangleq \sum_{n_t=1}^{N_T} H_{n_t}(f) H^H_{n_t}(f) \). \( \mathbf{H}_n(f) \triangleq [H_{1,n}(f)\ H_{2,n}(f)\ \ldots\ H_{N_T,n}(f)]^T \). Eq. (11) is a typical eigenvalue problem and \( \mathbf{G}_{\text{opt}}(f) \) can be expressed as

\[
\mathbf{G}_{\text{opt}}(f) = X(f) \mathbf{E}(f),
\]

where \( \mathbf{E}(f) \equiv [E_1(f)\ E_2(f)\ \ldots\ E_{N_T}(f)]^T \) is that unit-norm eigenvector of \( \mathbf{S}(f) \) which corresponds to its largest eigenvalue \( \lambda_{\text{max}}(f) \), and \( X(f) \) is a scalar factor. Eq. (12) shows that in general the optimum IIR BFFs can be viewed as concatenation of two filters: A filter \( X(f) \) which is common to all transmit antennas and a filter \( E_{n_t}(f) \) which is transmit antenna dependent. In the following, we derive filter \( X(f) \) for ZF–DFE and MMSE–DFE.

A. ZF–DFE

In this case, \( \xi = 0 \) is valid and combining Eqs. (11) and (12) results in

\[
|X(f)| = \frac{1}{\sqrt{\bar{\mu}}}
\]

Furthermore, from the power constraint in Eq. (8), we obtain \( \bar{\mu} = 1 \). Therefore, the optimum IIR BFFs for ZF–DFE are given by

\[
\mathbf{G}_{\text{opt}}(f) = \mathbf{E}(f) e^{j\phi(f)},
\]
where \( \varphi(f) \) is the phase which can be chosen arbitrarily. Replacing \( G(f) \) in Eq. (5) by \( G_{\text{opt}}(f) \) from Eq. (14) yields the maximum SNR for ZF–DFE

\[
\text{SNR}(G_{\text{opt}}(f)) = \frac{\sigma_n^2}{\sigma_n^2} \exp \left\{ \frac{1}{2} \int_{-1/2}^{1/2} \ln(\lambda_{\text{max}}(f)) \, df \right\}.
\]  

(15)

B. MMSE–DFE

For MMSE–DFE, \( \xi = \frac{\sigma_n^2}{\sigma_n^2} \) holds. Therefore, it can be shown that in this case the magnitude of the optimum \( X(f) \) is given by

\[
|X(f)| = \left[ \frac{1}{\mu_{\text{opt}}} - \frac{\xi}{\lambda_{\text{max}}(f)} \right]^+,
\]

where we took into account that \( |X(f)| \) has to be non-negative. Finding the optimum \( \mu \) is a typical Water Filling problem [10]. In particular, motivated by the power constraint in Eq. (8) we define

\[
P(\mu) = \frac{1}{2} \int_{-1/2}^{1/2} \left[ \frac{1}{\mu} - \frac{\xi}{\lambda_{\text{max}}(f)} \right]^+ \, df.
\]

(17)

The optimum \( \mu_{\text{opt}} \) can be found by slowly increasing a very small starting value \( \mu = \mu_0 \) which yields \( P(\mu_0) > 1 \) until \( P(\mu = \mu_{\text{opt}}) = 1 \) is achieved. The optimum IIR BFFs for MMSE–DFE are given by

\[
G_{\text{opt}}(f) = \sqrt{\left[ \frac{1}{\mu_{\text{opt}}} - \frac{\xi}{\lambda_{\text{max}}(f)} \right]^+} E(f) e^{j\varphi(f)},
\]

(18)

where \( \varphi(f) \) is again the phase which can be chosen arbitrarily. The corresponding maximum SNR is

\[
\text{SNR}(G_{\text{opt}}(f)) = \frac{\sigma_n^2}{\sigma_n^2} \exp \left\{ \frac{1}{2} \int_{-1/2}^{1/2} \ln(\lambda_{\text{max}}(f) - \xi) \, df \right\} - 1.
\]

(19)

Comparing the BFFs and the maximum SNRs for ZF–DFE and MMSE–DFE, we make the following interesting observations.

1) For \( \sigma_n^2 \to 0 \) (i.e., \( \xi \to 0 \)) the optimum IIR BFFs for MMSE–DFE approach those for ZF–DFE.

2) In case of ZF–DFE, the optimum BFF frequency response vector \( G_{\text{opt}}(f) \) at frequency \( f = f_0 \) is just the dominant eigenvector of matrix \( S(f) \) at frequency \( f = f_0 \). Since \( S(f_0) \) only depends on the channel frequency responses \( h_{n,n}(f) \) at frequency \( f = f_0 \), \( G_{\text{opt}}(f_0) \) is independent of the \( h_{n,n}(f) \). \( f \neq f_0 \). This is not true for MMSE–DFE, where the optimum frequency response vector \( G_{\text{opt}}(f) \) at frequency \( f = f_0 \) also depends on the channel frequency responses \( h_{n,n}(f) \) at frequencies \( f \neq f_0 \), because of the constraint \( P(\mu) = 1 \), cf. Eq. (17). In fact, for MMSE–DFE \( X(f) \) may be interpreted as a power allocation filter which allocates more power to frequencies with large eigenvalues \( \lambda_{\text{max}}(f) \).

3) If the underlying channel is frequency–nonselective, \( S(f) = S \) for all \( f \). In this case, it is easy to see that the optimum BFFs have only one non–zero tap and are identical to the well–known beamforming weights for frequency–nonselective channels, cf. e.g. [1]. In this case, the DFE structure collapses to a simple threshold detector, of course.

IV. BEAMFORMING WITH FIR FILTERS

Although the IIR BFFs derived in the previous section achieve a higher performance than FIR BFFs, IIR BFFs may not be feasible in practice due to their high computational cost. This motivates the investigation of FIR BFFs whose impulse responses \( g_n[k] \), \( 0 \leq k \leq L_q-1 \), \( 1 \leq n \leq N_T \), have \( L_q \) non–zero taps. As a consequence, the equivalent overall CIR \( h_{\text{eq}}[k] \) has length \( L_{\text{eq}} = L + L_q - 1 \). Introducing the BFF vector \( g \triangleq [g_1^T, g_2^T, \ldots, g_N^T]^T \), \( g_n \triangleq [g_n[0], g_n[1], \ldots, g_n[L_q-1]^T \), we can now express the frequency response of the equivalent channel as

\[
H_{\text{eq}}[f] = d^H(f) H_n r, \quad (20)
\]

where \( d(f) \triangleq [1, e^{j2\pi f}, \ldots, e^{j2\pi(f-L_{\text{eq}}-1)}]^T \), \( H_n \triangleq [H_{n,1}, H_{n,2}, \ldots, H_{N_T,n}] \), and \( H_{n,n} \) is an \( L_{\text{eq}} \times N_T \) matrix defined as

\[
H_{n,n} \triangleq \begin{bmatrix}
h_{n,n}[0] & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & h_{n,n}[0] \\
\vdots & \ddots & \vdots \\
0 & \cdots & h_{n,n}[L_q-1]
\end{bmatrix}.
\]

(21)

Since we still assume that the DFE at the receiver side employs IIR feedforward filters, Eqs. (5) and (6) remain valid. Therefore, the SNR for ZF–DFE and MMSE–DFE can be expressed as

\[
\text{SNR}(g) = \frac{\sigma_n^2}{\sigma_n^2} \exp \left\{ \frac{1}{2} \int_{-1/2}^{1/2} \ln(g^H B(f) g + \xi) \, df \right\} - 1 - \chi,
\]

(22)

with \( N_T L_q \times N_T L_q \) matrix \( B(f) \triangleq \sum_{n=1}^{N_T} H_{n,n}^H d(f) \cdot d^H(f) H_n \). The optimum BFF vector \( g_{\text{opt}} \) shall maximize \( \text{SNR}(g) \) subject to the power constraint \( g^H g = 1 \). Thus, the Lagrangian of the optimization problem can be formulated as

\[
L(g) = \text{SNR}(g) + \mu g^H g, \quad (23)
\]

where \( \mu \) denotes again the Lagrange multiplier. The optimum vector \( g_{\text{opt}} \) has to fulfill \( \partial L(g)/\partial g \neq 0 \), which leads to the non–linear eigenvalue problem

\[
\left[ \int_{-1/2}^{1/2} \frac{B(f) + \xi I_{N_T L_q}}{g_{\text{opt}}^H B(f) + \xi I_{N_T L_q}} g_{\text{opt}} \, df \right] g_{\text{opt}} = g_{\text{opt}},
\]

(24)

where \( \mu \) does not appear since Eq. (24) already includes the constraint \( g_{\text{opt}}^H g_{\text{opt}} = 1 \). However, Eq. (24) does not seem to have a closed–form solution. In fact, this is not surprising since
by discretizing the integral in Eq. (22), it can be shown that the optimization problem in Eq. (23) is equivalent to maximizing a product of Rayleigh quotients

\[
\tilde{L}(g) = \prod_{i=1}^{S} \frac{g_i^H (B(f_i) + \xi I_{N_T L_g}) g_i}{g_i^H g_i},
\]

where \( f_i \triangleq -1/2 + (i - 1)/(S - 1) \) and \( S \) is a large integer. Note that Eq. (25) is scale invariant (i.e., \( \tilde{L}(g) \) does not depend on the magnitude of \( g \)) and the resulting solution has to be scaled to meet \( g_i^H g_i = 1 \). It is well-known that the maximization of a product of Rayleigh quotients is a difficult mathematical problem which is not well understood and a closed-form solution is not known except for the trivial case \( S = 1 \), cf. e.g. [11], [12]. In [12] a gradient method is advocated to search for the optimum vector \( g_{opt} \). However, gradient methods usually suffer from a slow convergence.

Here, we avoid a gradient approach and propose a novel fast converging numerical method based on Eq. (24). If the denominator under the integral in Eq. (24) was absent, \( g_{opt} \) would simply be the maximum eigenvalue of matrix \( \int_{-1/2}^{1/2} (B(f) + \xi I_{N_T L_g}) df \), which could be calculated efficiently using the so-called Power Method [10]. Motivated by this observation, we propose a Modified Power Method (MPM) for recursive calculation of \( g_{opt} \):

1) Let \( i = 0 \) and initialize the BFF vector with some \( g_0 \) fulfilling \( g_0^H g_0 = 1 \).
2) Update the BFF vector

\[
\tilde{g}_{i+1} = \left[ \int_{-1/2}^{1/2} \frac{B(f)}{g_i^H (B(f) + \xi I_{N_T L_g}) g_i} df \right] g_i.
\]

3) Normalize the BFF vector

\[
g_{i+1} = \frac{\tilde{g}_{i+1}}{\sqrt{g_{i+1}^H g_{i+1}}}
\]

4) If \( |g_{i+1}^H g_{i+1}| < \epsilon \), goto Step 5), otherwise increment \( i \rightarrow i + 1 \) and goto Step 2).
5) \( g_{i+1} \) is the desired BFF vector.

For the termination constant \( \epsilon \) in Step 4) a suitably small value should be chosen, e.g. \( \epsilon = 10^{-4} \). Because of its involved nature, we are not able to prove global convergence of the MPM algorithm to the maximum SNR. However, in our simulations for \( \xi > 0 \) the algorithm always achieved high SNR values for different initializations \( g_0 \). The convergence time of the algorithm depends on \( L_g \). For example, for \( L_g = 2 \) and \( L_g = 5 \) the algorithm typically terminated after less than 20 and 100 iterations, respectively.

V. SIMULATION AND NUMERICAL RESULTS

In this section, we present simulation and numerical results for the SNR and the bit error rate (BER) of DFE with transmit beamforming. As relevant example, we consider the equalizer test (EQ) channel of the GSM system [13]. As is usually done for GSM, we model Gaussian minimum-shift keying (GMSK) modulation as filtered binary PSK (BPSK). For all results we assume \( N_T = 3 \) transmit and \( N_R = 1 \) receive antennas and a maximum channel length of \( L = 7 \). The correlation coefficient between all transmit antennas is \( \rho = 0.5 \).

Before we show SNR and BER results, we illustrate the convergence behavior of the proposed MPM algorithm for calculation of the FIR BFF vector \( g \). We assume BFFs of length \( L_g = 5 \) and two different initializations: \( g_{n_1}^{(1)} = \frac{1}{\sqrt{5}} [1 0 0 0 0]^T \) and \( g_{n_1}^{(2)} = \frac{1}{\sqrt{13}} [1 1 1 1 1]^T \), \( 1 \leq n_1 \leq 3 \). Fig. 2 shows the angle distance measure \( |g_{i+1}^H g_i| \) [4], [3] vs. iteration number \( i \) for one random realization of the EQ channel and \( 10 \log_{10}(E_b/N_0) = 10 \) dB, where \( E_b \) is the average received energy per bit. As can be observed, the algorithm converges in both cases to the steady-state value 1 after less than 20 iterations.

For the FIR beamforming results shown in Figs. 3 and 4 we calculated the BFFs for each channel realization using the MPM algorithm with \( \epsilon = 10^{-4} \). The BFF vector was initialized with the normalized all-ones vector and the algorithm was stopped after 100 iterations if it had not terminated before.

In Fig. 3, we show the average SNR (SNR) vs. \( E_b/N_0 \) of ZF–DFE with IIR BFFs and MMSE–DFE with FIR and IIR BFFs. SNR was obtained by averaging the respective SNRs in Eqs. (15), (19), and (22) over 500 independent realizations of the EQ channel. For comparison we also show the average SNR of ZF–DFE and MMSE–DFE for single–antenna transmission \( (N_T = 1, N_R = 1) \) in Fig. 3. As can be observed, even FIR beamforming with \( L_g = 1 \) leads to performance gains of 2–3 dB compared to single–antenna transmission. These gains increase with increasing \( L_g \) but FIR BFFs with \( L_g = 5 \) achieve practically the same performance as IIR BFFs. This observation is also useful for the design of beamforming with imperfect CSI and can guide the search for suitable BFF lengths \( L_g \) in that case.

Fig. 4 shows the simulated BER (averaged over 50,000 channel realizations) of MMSE–DFE with FIR BFFs of different lengths \( L_g \). For the simulation we implemented MMSE–DFE with feedforward filter length 4L which causes negligible performance degradation compared to IIR feedforward filters. For comparison, Fig. 4 also includes simulation results for transmission with just one antenna \( (N_T = 1, N_R = 1) \) and optimized delay diversity (ODD) for DFE as proposed in [14]. The ODD filters of length \( L_g = 3 \) are optimized based on the channel statistics, i.e., in contrast to the BFFs the ODD filters do not depend on the particular realization of the channel. Furthermore, we show in Fig. 4 the BER of MMSE–DFE with IIR BFFs obtained by numerically averaging \( Q(2\text{SNR}(G_{opt}(f))) \) over the channel realizations, where \( Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-t^2/2) dt \) denotes the Gaussian Q–function and \( \text{SNR}(G_{opt}(f)) \) is obtained from Eq. (19). We note that this numerical method does not take into account the effect of error propagation in the DFE feedback filter, whereas the simulated BER performances of beamforming with FIR filters, ODD, and single–antenna transmission include this effect, of course. This explains why the performance difference between MMSE–DFE with IIR BFFs and MMSE–DFE with FIR BFFs is larger in Fig. 4 than expected from Fig. 3 (which does not include the effect of error propagation for either scheme). However, Fig. 4 also clearly shows that FIR transmit...
beamforming achieves a large performance gain compared to single–antenna transmission and ODD. For example, at BER = $10^{-3}$ FIR beamforming with $L_g = 3$ achieves a gain of more than 3 dB over ODD with $L_g = 3$. Of course, this gain comes at the cost of an increased complexity as beamforming requires instantaneous CSI whereas ODD only needs average CSI.

VI. CONCLUSION

In this paper, we have considered beamforming with perfect CSI for single–carrier transmission over frequency–selective fading channels with ZF–DFE and MMSE–DFE at the receiver. We have derived closed–form solutions for IIR BFFs and we have provided an efficient numerical method for calculation of FIR BFFs of arbitrary lengths $L_g$. The FIR BFFs approach the performance of the IIR solution as $L_g$ increases. However, significant performance gains over single–antenna transmission and delay diversity can be achieved already with very short FIR BFFs. The derived SNRs for beamforming with perfect CSI can also serve as valuable upper bounds for the performance of beamforming with imperfect CSI.

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