Analysis and Design of a 300-W 500 000-r/min Slotless Self-Bearing Permanent-Magnet Motor

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Abstract—Active magnetic bearings enable contactless operation and can therefore be used for supporting rotors spinning at high speeds. However, the rotational speed in conventional reluctance-force-based magnetic bearing topologies is limited, which is mainly due to high rotor losses and limited force control bandwidths. In this paper, a prototype of a self-bearing motor is presented, which overcomes several limitations of state-of-the-art high-speed magnetically levitated electric drive systems. Due to the employed magnetic bearing, the motor can be operated in high-purity or vacuum environments. An analytical mechanical and electrical bearing model is introduced and verified by measurements. Furthermore, a bearing inverter system is designed, and its controller performance is shown. Measurements of spinning levitated rotors up to speeds of 505 000 r/min verify the functionality of the overall system. To the authors’ knowledge, this is the highest speed achieved by magnetically levitated electrical drive systems so far.

Index Terms—Active magnetic bearing (AMB), bearing inverter system, bearingless drive, deceleration test, Lorentz actuator, permanent-magnet synchronous motor (PMSM), rotor bending resonance measurement, slotless magnetic bearing, vacuum.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B, \vec{B}, \hat{B} )</td>
<td>Magnetic flux density, vector, amplitude.</td>
</tr>
<tr>
<td>( B_{\text{rem}} )</td>
<td>Remanent flux density.</td>
</tr>
<tr>
<td>( e )</td>
<td>Euler’s number.</td>
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<tr>
<td>( e )</td>
<td>Control error.</td>
</tr>
<tr>
<td>( F )</td>
<td>Force vector.</td>
</tr>
<tr>
<td>( H )</td>
<td>Magnetic field vector.</td>
</tr>
<tr>
<td>( i, \tilde{i}, \hat{i} )</td>
<td>Winding current, space vector, amplitude.</td>
</tr>
<tr>
<td>( j )</td>
<td>Imaginary unit.</td>
</tr>
<tr>
<td>( J, \tilde{J}, \hat{J} )</td>
<td>Current density vector, space vector, amplitude.</td>
</tr>
<tr>
<td>( k )</td>
<td>Inductance coupling factor.</td>
</tr>
<tr>
<td>( K_p )</td>
<td>Proportional controller gain.</td>
</tr>
<tr>
<td>( L )</td>
<td>Axial winding length.</td>
</tr>
<tr>
<td>( L_S )</td>
<td>Transformed stator phase inductance.</td>
</tr>
<tr>
<td>( m )</td>
<td>Phase number.</td>
</tr>
<tr>
<td>( N )</td>
<td>Number of winding turns.</td>
</tr>
<tr>
<td>( p_{\text{pm}} )</td>
<td>Rotor pole-pair number.</td>
</tr>
<tr>
<td>( p_w )</td>
<td>Winding pole-pair number.</td>
</tr>
<tr>
<td>( (r, \theta, z) )</td>
<td>Cylindrical coordinates.</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>Permanent-magnet radius.</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>Rotor outer radius.</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>Bearing winding inner radius.</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>Bearing winding outer radius.</td>
</tr>
<tr>
<td>( R_S )</td>
<td>Stator-core inner radius.</td>
</tr>
<tr>
<td>( s )</td>
<td>Laplace variable.</td>
</tr>
<tr>
<td>( T )</td>
<td>Torque vector.</td>
</tr>
<tr>
<td>( T_{\text{del}} )</td>
<td>Delay time.</td>
</tr>
<tr>
<td>( T_I )</td>
<td>Integral controller time constant.</td>
</tr>
<tr>
<td>( u, \bar{u} )</td>
<td>Voltage, space vector.</td>
</tr>
<tr>
<td>( (x, y, z) )</td>
<td>Cartesian coordinates.</td>
</tr>
<tr>
<td>( x_{\text{B1}}, y_{\text{B1}} )</td>
<td>Rotor displacements.</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Angular orientation of the remanent flux density.</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Phase of the winding current space vector.</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Permeability.</td>
</tr>
<tr>
<td>( \chi_{\text{pm}} )</td>
<td>Radial bearing constant.</td>
</tr>
<tr>
<td>( \psi_{\text{pm}}, \hat{\psi}_{\text{pm}} )</td>
<td>Flux linkage, space vector.</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Angular frequency of the rotor.</td>
</tr>
<tr>
<td>Subscripts</td>
<td></td>
</tr>
<tr>
<td>( d )</td>
<td>Direct-axis component.</td>
</tr>
<tr>
<td>( L )</td>
<td>Lorentz force.</td>
</tr>
<tr>
<td>( \text{ph} )</td>
<td>Phase.</td>
</tr>
<tr>
<td>( q )</td>
<td>Quadrature-axis component.</td>
</tr>
<tr>
<td>( R )</td>
<td>Reluctance force.</td>
</tr>
<tr>
<td>( \text{ref} )</td>
<td>Reference.</td>
</tr>
<tr>
<td>( S )</td>
<td>Stator.</td>
</tr>
</tbody>
</table>

I. INTRODUCTION

ULTRAHIGH-SPEED electrical drive systems are developed for new emerging applications, such as turbo compressor systems, ultracentrifuges, rotating mirrors in optical applications, and milling spindles. Several studies on the design and speed limitations of ultrahigh-speed electrical drive systems have been recently published [1]–[3]. Most systems built today rely on ball bearings for rotor support. In many applications, the ball bearings are limiting the lifetime of a system [1]. Several applications, such as optical scanning systems or reaction wheels for attitude control of satellites in orbit, require a lifetime of several thousand hours and the operation in vacuum. These requirements exclude bearing technologies such as ball bearings or gas bearings. The use of active magnetic
bearings (AMBs) enables contactless operation and the operation in vacuum. Furthermore, rotor dynamics can be actively controlled, allowing canceling or damping of instabilities.

Most AMB systems built today rely on reluctance force, which is generated by guiding a magnetic flux over the air gap between the rotor and the stator. This flux is actively controlled by controlling the bearing winding currents. In the present paper, this bearing topology will be referred to as reluctance-force-based AMB. In contrast, Lorentz-force-based AMB rely on an external magnetic flux density (typically generated by a set of permanent magnets on the rotor) and electric current densities in bearing windings. The windings are placed in the magnetic air gap of the external flux density. Thus, the bearing force generation is mainly based on Lorentz force in the bearing windings.

There are several AMB designs proposed in literature for speeds beyond 100,000 r/min. However, there are only few publications with experimental results. Reluctance-force-based homopolar bearings are presented in [4] and [5]. In [4], a magnetically levitated milling spindle that achieved a speed of 150 kr/min is presented. In [5], a 2-kW machine was operated up to a speed of 120 kr/min. However, all reluctance-force-based AMB systems suffer from iron losses on the rotor [6]. Even homopolar bearings, which are optimized for low rotor losses, suffer from speed-dependent slot-harmonic induced losses [4], [5]. Due to the poor thermal coupling of the levitated rotor to the rest of the drive system, these losses are crucial. In many AMB applications, the maximum rotational speed is limited by the rotor losses [6]. Furthermore, eddy currents in the magnetic circuit limit the achievable actuator bandwidth [6].

One possibility to overcome the problem of high rotor losses is to rely on Lorentz forces for bearing force generation. Although the achievable maximum bearing forces are small compared with reluctance-force-based bearings, the concept of Lorentz-force-based bearings enables high-speed rotation when properly designed [7]. A slotless and, thus, Lorentz-force-based high-speed self-bearing motor is presented in [8]. In contrast to the design proposed in this paper, a short (slice) rotor was chosen in [8]. Three degrees of freedom (DOFs) of the rotor are actively controlled by a combined motor-bearing winding, whereas three DOFs are passively stabilized by reluctance forces [8]. The achieved speed of 115 kr/min is limited by the mechanical construction of the rotor [8]. One possibility to reach higher speeds is by implementing a scaled-down version of the slice motor. However, the downsizing is limited by the passive stabilization of three DOFs. More applications on slotless self-bearing motors designed for lower speeds are presented in [9] and [10] for positioning applications. Low-speed self-bearing slice motors for mixing applications are presented in [11] and [12]. A single-DOF actively positioned self-bearing motor is presented in [13]. In [14], a nonsalient permanent-magnet axial-gap self-bearing motor is proposed, combining the axial bearing and the motor.

In 1946, a rotational speed of 23 million r/min was achieved by a small magnetically levitated steel ball [15]. The highest rotational speed of any macroscopic object was achieved in [16] by spinning graphene flakes at rotational speeds of up to 60 million r/min. In this paper, the physical experiments presented in [15]–[17] will not be considered electric drive systems because they have no possibility of driving an application.

A. Contributions and Outline of the Paper

In this paper, a self-bearing machine designed for rotational speeds of up to 500 kr/min and a rated machine power of 300 W is presented, and measurement results are shown. In contrast to state-of-the-art designs, the ironless construction of the rotor and the slotless design of the stator enable ultrahigh-speed rotation and very high bearing actuator bandwidths. Thus, the presented prototype overcomes several limitations of previously presented AMB systems. All six DOFs of the rotor are actively controlled by Lorentz forces generated by air-gap windings.

The machine is designed for driving optical components, such as mirrors in laser scanning applications. In this application, a lifetime of several thousand hours at speeds of 200–500 kr/min is required. Furthermore, the rotor has to be operated in a hermetically sealed containment to prevent defilement of the optical components. Due to the long lifetime requirement, only contactless bearing technologies come into consideration. Static gas bearings are not considered as they require an external pressure supply. Thus, the only bearing technologies that can be applied in this application are the dynamic gas bearing and the magnetic bearing. Typically, gas bearings can be realized very compactly with high load capacity and stiffness. However, poor dynamic stability has been limiting their use at high rotational speeds [18]. In this paper, AMBs were selected since they fulfill the application requirements and allow actively controlling the dynamics of the rotor. The drawback, when compared with dynamic gas bearings, is the higher complexity of the drive system.

The rotor of one possible application of the presented prototype is depicted in Fig. 1. The six-faced mirror head is used for the periodic deviation of a laser beam. With this setup, a laser beam of a maximum diameter of 3 mm can be periodically deviated over a scanning angle of 60°. A rotor dynamical analysis shows that a rotor of these dimensions can be operated up to 480 kr/min. Thus, a very high repetition frequency of 48 kHz can be achieved. The ultrahigh rotational speed is chosen in order to increase the scanning speed of the laser scanner.

A preliminary version of this paper was presented in [19] at the 38th Annual Conference of the IEEE Industrial Electronics Society (IECON 2012). The maximum speed achieved in [19]
was limited by the rotor construction. The results presented in this paper are based on an improved rotor construction and control strategy. This allows operating the prototype up to its rated speed.

II. MACHINE DESIGN AND OPERATING LIMITATIONS

The concept of the implemented prototype is based on a magnetically levitated permanent-magnet synchronous motor. The integrated machine and bearing concept was proposed in [7], and a working prototype was presented in [19].

The cross section of the machine is shown in Fig. 2. The dimensions and properties of the implemented prototype can be found in Table I. A CAD sectional view of the machine is depicted in Fig. 3, and a picture of the implemented test bench is shown in Fig. 4.

A rotor containing an axially magnetized permanent magnet and two diametrically magnetized permanent magnets is inserted into a slotless amorphous-iron stator core. The magnetic air gap between the rotor and the stator core is used to place a motor winding and three bearing windings. The radial bearing windings are manufactured as skewed air-gap windings, as defined in [20]. A grooved tube of thermally conductive but electrically isolating polymer is inserted between the bearing and the motor winding, allowing cooling the windings with pressurized air during the operation. The rotor is encapsulated by a sealed PEEK tube to operate the rotor in vacuum (not shown in Fig. 2). The vacuum sealing and the ISO-KF vacuum flanges are shown in Fig. 3. A picture of the machine prototype with vacuum connections is shown in Fig. 5.

For the control of the magnetic bearing, printed-circuit-board (PCB)-based rotor position sensors are placed on both sides of the motor. The radial rotor position is measured based on eddy-current sensors, whereas the axial and angular positions are obtained by Hall-effect-based stray-field measurements. A cascaded control scheme, as proposed in [19], is employed for the AMB control. The rotor position is controlled in an outer control loop, whereas the winding currents are tracked in an inner field-oriented control loop, as described in Section IV-B. The design of the rotor position controller is beyond the scope of this paper.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{\text{max}}$</td>
<td>Maximum rotational speed</td>
<td>500,000 rpm</td>
</tr>
<tr>
<td>$P_{\text{rated}}$</td>
<td>Rated motor power</td>
<td>300 W</td>
</tr>
<tr>
<td>$\psi_{\text{pm, motor}}$</td>
<td>Motor flux linkage</td>
<td>$0.40 \cdot 10^{-3} \text{Vs}$</td>
</tr>
<tr>
<td>$R_1$</td>
<td>Permanent-magnet radius</td>
<td>3 mm</td>
</tr>
<tr>
<td>$R_2$</td>
<td>Rotor outer radius</td>
<td>3.65 mm</td>
</tr>
<tr>
<td>$R_6$</td>
<td>Stator-core inner radius</td>
<td>12 mm</td>
</tr>
<tr>
<td>$L_{\text{rotor}}$</td>
<td>Total rotor length</td>
<td>55 mm</td>
</tr>
<tr>
<td>$m_{\text{rotor}}$</td>
<td>Rotor mass</td>
<td>12.3 g</td>
</tr>
<tr>
<td>$B_{\text{rem}}$</td>
<td>Remanent flux density</td>
<td>1.07 T</td>
</tr>
</tbody>
</table>
of this paper. An analysis of the implemented controller is presented in [21].

The proposed machine topology features a set of advantages and disadvantages when compared with state-of-the-art reluctance-force-based AMB designs. The main advantage is the low high-frequency power loss. In particular, low rotor losses, resulting from the slotless construction of the stator and the ironless construction of the rotor, enable ultrahigh-speed rotation. Due to the slotless winding design, the winding fields are small when compared with slotted designs. This yields negligible rotor eddy-current losses. In contrast to reluctance-force-based AMB, the proposed bearing actuators are based on Lorentz force, i.e., the achievable actuator bandwidth is not limited by eddy currents in the rotor iron. Furthermore, small winding inductances follow from the slotless design. The concept of using separate bearing and motor windings yields low induced voltages in the bearing windings. It is shown in Section IV that these features enable to control the bearing currents and, thus, also the bearing forces up to high bandwidths with low reactive power consumption. The inverters can be implemented using low-voltage power electronics, enabling ultrafast switching frequencies. The drawback of the proposed Lorentz-force-based bearing actuators is the comparably low load capacity of the magnetic bearing.

The maximum speed of the machine is limited by the mechanical design and the dynamics of the rotor. In particular, the material stresses and the frequency of the rotor bending modes limit the rotational speed. When a rotor for a certain speed is designed, the rotor diameter is limited by the maximum tolerable stresses at the maximum speed. Similarly, the rotor length is limited by the minimum tolerable frequency of the first bending mode. Rotor unbalances excite the rotor system at the frequency of rotation [6]. In the implemented prototype, the unbalance excitation cannot be compensated by the magnetic bearing, and the rotor cannot be accelerated fast enough through the bending resonance. Therefore, the maximum rotational frequency must not exceed the frequency of the first bending mode. Fig. 6 shows the result of a finite-element modal analysis calculated using the commercial software ANSYS. The result features a frequency gap between the first bending mode frequency of 11.9 kHz and the frequency of the unbalance excitation of 8.33 kHz at the rated rotational speed. The modal analysis is verified by measurements in Section V-C.

The limitation of the machine power is given by the thermal design. The limitation is a result of the power loss in the motor winding, the temperature rating of the used materials, the thermal resistance to the stator housing, and the cooling capacity of the stator housing. The implemented prototype machine was designed for a machine power of 300 W. The maximum winding temperature for this load situation of 120 °C is calculated by a finite-element method (FEM) simulation. The electromagnetic and thermal design of the motor winding is beyond the scope of this paper. The interested reader is referred to [1] for a detailed analysis. The motor flux linkage of the machine is given in Table I. It is calculated using the analytical solution presented in [22].

III. RADIAL BEARING DESIGN

In the following analysis, vectors, such as 3-D force vectors, will be denoted by bold letters (e.g., \( \vec{F} \)), whereas stator-fixed space vectors, which are transformed by the amplitude-invariant Clarke transformation, will be denoted by underlined letters (e.g., \( \hat{\vec{F}} \)).

The electromagnetic force and torque on a rotor of an electric machine can be calculated by integrating the local stator reaction force density \( dF_S \) over the volume \( V \) of the stator, i.e.,

\[
F = - \iiint_V dF_S = - \iiint_V dF_{L, S} + dF_{R, S}.
\]

The minus sign follows from the third of Newton’s laws of motion. The force density \( dF_S \) can be further split up into a Lorentz force component

\[
dF_{L, S} = J \times B \cdot dV \tag{2}
\]

and a reluctance force component

\[
dF_{R, S} = \frac{1}{2} H^2 \cdot \nabla \mu \cdot dV \tag{3}
\]

where \( J \) is the current density, \( B \) is the magnetic flux density, \( H \) is the magnetic field, and \( \nabla \mu \) is the gradient of the local permeability. In regions with constant permeability \( \mu \), the reluctance force density \( dF_{R, S} = 0 \) vanishes. Therefore, in many slotless machine designs, the only area where reluctance forces occur is the inner surface of the stator core.

In [20], an analysis of Lorentz forces generated in slotless skewed bearing and motor windings is presented. However, in [20], the reluctance force resulting from high-permeability stator cores is not considered.

A. Permanent-Magnet Flux Density Distribution

The permanent-magnet flux density in the air gap of a slotless machine for a radially centered rotor can be described in cylindrical coordinates \( (r, \theta, z) \) by Fourier series for both the radial and azimuthal field components [23]. For the scope of this paper, the harmonic components of the permanent-magnet field are omitted. Thus, a purely sinusoidal field distribution

\[
B_r = \hat{B}_r (r) \cdot \cos (p_{pm} (\theta - \gamma)) \tag{4}
\]

\[
B_\theta = \hat{B}_\theta (r) \cdot \sin (p_{pm} (\theta - \gamma)) \tag{5}
\]

is assumed, where \( p_{pm} \) is the rotor pole-pair number, and \( \gamma \) is the angular orientation of the remanent flux density of the permanent magnet in the \( xy \) plane. In the following subsections,
the configuration \( p_{pm} = 1 \) is analyzed in more details. This field distribution is achieved with the use of a cylindrical diametrically magnetized permanent magnet [1]. The field in axial direction is assumed to be \( B_z = 0 \).

### B. Analytic Lorentz Force Calculation

Here, analytical results are presented for the Lorentz force vector generated by skewed bearing windings. The analysis is based on the integration of the Lorentz force density over the stator volume. Thereby, the reluctance force caused by the stator core and the armature reaction is neglected. Although the presented results are only correct for ironless designs, they still illustrate the generation of bearing force what will be the basis of the current and position controllers used to operate the motor. Results, including reluctance force, which are presented in Section III-C, show that the influence of reluctance force can be neglected for sufficiently large ratios of \( R_6 / R_4 \).

The winding parameterization and detailed calculations for this type of winding are given in [22]. A winding is defined by the number of pole pairs \( p_w \) and the number of phases \( m \). For the radial bearing winding, the configuration \( p_w = 2, m = 3, \) and \( p_{pm} = 1 \) is chosen. According to [22], the amplitude-invariant current density space vector

\[
J_S = J_S e^{i \varepsilon} = \left( \frac{3 \pi N \cdot i_S e^{i \varepsilon}}{\sqrt{4L^2 + \pi^2 R_4^2} - \sqrt{4L^2 + \pi^2 R_3^2}} \right)
\]

can be defined, with \( N, L, \) and \( j \) being the number of winding turns per phase and pole, the axial winding length, and the imaginary unit, respectively. \( i_S \) denotes the amplitude of the winding current space vector, and \( \varepsilon \) denotes its phase. In [20], it was shown that, for a radially centered rotor, the integration of the Lorentz force density over the phase belt of one phase and a subsequent summation over \( m \) phases yields a vanishing torque \( T_L = 0 \) and the force vector

\[
F_L = \frac{2m \sin \left( \frac{\pi}{m} \right) L^2}{\pi} \int_{R_3}^{R_4} \frac{J_S \cos (\varepsilon - \gamma)}{\sqrt{4L^2 + \pi^2 r^2}} d(\hat{B}_r + \hat{B}_\theta) r dr
\]

given in Cartesian coordinates \((x, y, z)\). The integration over \( r \) is solved using the field solution for \( B_r \) and \( B_\theta \) presented in [1]. Finally, the bearing force can be expressed by

\[
F_L = \frac{3}{2} \chi_{pm,L} \begin{bmatrix} i_S \cos (\varepsilon - \gamma) \\ i_S \sin (\varepsilon - \gamma) \\ 0 \end{bmatrix}
\]

with the Lorentz bearing constant

\[
\chi_{pm,L} = \frac{3\sqrt{3}N K_{B2} R_6^2}{\sqrt{4L^2 + \pi^2 R_4^2} - \sqrt{4L^2 + \pi^2 R_3^2}} \cdot K_1
\]

and \( K_1 \) defined as

\[
K_1 = \log \left( \frac{2L + \sqrt{4L^2 + \pi^2 R_3^2}}{2L + \sqrt{4L^2 + \pi^2 R_4^2}} \right) \cdot \frac{2L - \sqrt{4L^2 + \pi^2 R_3^2}}{2L - \sqrt{4L^2 + \pi^2 R_4^2}}
\]

C. Three-Dimensional FEM Calculations of the Bearing Forces

The bearing force calculation presented in the previous subsection is based on Lorentz force solely. With the presence of an iron stator core, the analysis must also include reluctance forces. Therefore, a magnetostatic FEM calculation is used to determine the magnetic field generated by the winding.

The 3-D winding field distribution is calculated using the commercial FEM software COMSOL. Thereby, the cylindrical stator core is modeled as a linear material with a relative permeability \( \mu_{r,Fe} = 1000 \). The total flux density in the air gap of the bearing is given by the superposition of the winding flux density and the flux density caused by the permanent magnet, for which again the analytical 2-D model in [1] is used. As proposed in [22], the bearing force is then obtained by numerical integration of the Maxwell stress tensor evaluated for the total flux density on the cylindrical surface with radius \( R_2 \).

The FEM calculation yields an increased bearing force with the presence of an iron stator core. The increase in the total force compared with the analytically calculated Lorentz force is shown in Fig. 7 for different bearing geometries. In the presented prototype, the ratio is \( R_6 / R_4 = 2 \). Thus, it is expected that the total bearing constant \( \chi_{pm} \) is approximately 7% greater than \( \chi_{pm,L} \) calculated by (9). Note that this is not the same geometry as the measured geometry shown in Fig. 7.
D. Electrical Bearing Model

In order to control the bearing force \( \mathbf{F} \), the bearing currents have to be controlled. For the design of the bearing inverter and the corresponding current controller, the electric model of the bearing has to be analyzed in more detail. In operation of the magnetic bearing, small radial rotor displacements \( x_B \) and \( y_B \) may occur. The permanent-magnet flux linkage space vector in the magnetic bearing winding is given by

\[
\psi_{pm} = \chi_{pm}(x_B + jy_B)e^{j\gamma}
\]

(12)

for small rotor displacements. The bearing has a symmetrical topology, as depicted in Fig. 8. The simple topology and, in contrast, for the bearing application at hand, relative current orientation \( \varepsilon - \gamma \) lead to (normally) insignificant torque amplitude errors. In contrast, for the bearing application at hand, relative current orientation errors result in wrong bearing force directions [cf. (8)]. Simulations have shown that such direction errors can severely compromise the performance of the position controller and cause instability. Consequently, as the current control actions are performed in \( dq \) coordinates, it is crucial that the employed coordinate transformations between \( dq \) and stator-fixed coordinates are based on precise current and angular rotor position measurements. Furthermore, these transformations must be updated at high sampling rates to keep the orientation errors small.

\[
\begin{align*}
\text{IV. POWER ELECTRONICS} \\
A. Specifications
\end{align*}
\]

The electrical bearing model (14), (15), (17) implies the need for three-phase sinusoidal bearing currents to control the rotor position. Hence, a suitable inverter system is required, which meets the following specifications.

- Extensive simulations have been performed to determine the range of applicable bearing forces \( \mathbf{F} \), which are required to stabilize and adequately control the rotor up to the target rotational speed of 500 kr/min (\( \Omega = 2\pi \cdot 8.33 \) kHz). Using (9), (10), and (17), the number of winding turns \( N \) and the geometry of the bearing winding have been chosen to obtain currents in the low ampere-range. Finally, using (14) and (15) yields that a peak output voltage \( |u_S|_{\text{max}} \) of 6 V is sufficient for the entire range of operation. The chosen current and voltage ratings allow for a greatly simplified inverter implementation, as further described in Section IV-B.

- Ripple currents deviating from the desired reference values do not contribute to a useful net bearing force but increase the ohmic losses in the windings. Therefore, such currents must be minimized in order not to compromise the maximum applicable forces.

- In motor drive applications, errors in the relative current orientation \( \varepsilon - \gamma \) lead to (normally) insignificant torque amplitude errors. In contrast, for the bearing application at hand, relative current orientation errors result in wrong bearing force directions [cf. (8)]. Simulations have shown that such direction errors can severely compromise the performance of the position controller and cause instability. Consequently, as the current control actions are performed in \( dq \) coordinates, it is crucial that the employed coordinate transformations between \( dq \) and stator-fixed coordinates are based on precise current and angular rotor position measurements. Furthermore, these transformations must be updated at high sampling rates to keep the orientation errors small.

- A controller architecture with the rotor position controller in the outer and subordinate bearing current controllers in the inner loop is proposed [19]. As a consequence of this cascaded approach, a minimum current controller bandwidth \( f_{\text{bw}} = 12 \) kHz was identified in simulations to achieve satisfactory control performance. This bandwidth requirement is equivalent to a loop delay of several microseconds in case of a standard proportional–integral (PI) current control scheme.

B. Implementation

1) Inverter Topology: The inverter system was implemented using a three-phase two-level voltage source inverter (2LVSI) topology, as depicted in Fig. 8. The simple topology and, in
rotor position $\dot{\gamma}$ for the transformation from stator-fixed to $dq$ coordinates, i.e.,

$$\dot{\gamma} = \gamma + \Omega(T_{\text{del},\gamma} - T_{\text{del},i}).$$

The back transformation into stator-fixed coordinates is analogously implemented considering the delay of the control algorithm.

3) Control: A fully field-programmable gate array (FPGA)-based (Cyclone IV) controller is proposed, which offers the advantage of running several current controllers in a simultaneous and time-synchronous manner at high sampling rates. A subharmonic sinusoidal pulsedwidth modulation (PWM) scheme is employed. A tradeoff between the number of discrete voltage levels of the associated carrier signal and the achievable switching frequency has been identified due to the high but limited maximum clock rate of the FPGA (300 MHz). The current controllers are implemented as decoupled PI controllers with feedforward terms, i.e.,

$$u_{\text{ref}} = \frac{K_p(1 + s T_1)}{s^2 T_1} \begin{bmatrix} e_d \\ e_q \end{bmatrix} + L_S \Omega \begin{bmatrix} -i_d \\ -i_q \end{bmatrix} + \chi_{\text{pm}} \Omega \begin{bmatrix} -y_B \\ x_B \end{bmatrix}.$$

The main difference to standard synchronous frame PI controllers in motor drive applications, e.g., in [24], is the feedforward terms due to the back electromotive force (EMF) voltage. In the case at hand, not only the rotational speed but also the radial rotor displacements and speeds must be known to correctly compensate the induced voltages $u_{\text{EMF}} = (\partial/(\partial t))\psi_{\text{pm}}$ [cf. (14) and (15)]. It was observed in simulations that the radial-speed-dependent induced voltages lie in the range of the target voltage quantization of the implemented PWM scheme. For simplicity reasons, these voltages are thus neglected. In contrast, the induced voltages due to radial displacements $x_B$, $y_B$ can reach up to 1 V and are, hence, compensated in (20).

With a total adjustable loop delay between 2 and 6 $\mu$s (depending on the current filtering), the current controller can achieve a bandwidth of up to $f_{\text{low}} = 20$ kHz without significant overshoots. Eventually, the high switching frequency in conjunction with the high sampling rate of the back transformation into stator-fixed coordinates reduces the error in the relative current orientation $\varepsilon - \gamma$ to a minimum.

V. MEASUREMENTS

Here, a set of measurements that are relevant to the machine design and that show the performance of the overall system is presented. In Section V-A, the mechanical bearing model is verified by force measurements. In Section V-B, the controller performance of the bearing inverter system is shown. Section V-C revises the presented modal rotor analysis. The 500-kr/min measurements and no-load power loss measurements are presented in Sections V-D and V-E, respectively. Except for the force measurements in Section V-A, all measurements are performed on the prototype whose mechanical and electrical parameters are given in Tables I and II.
A. Measurement of Bearing Force

In order to verify the presented bearing force calculations, a set of 3-D force and torque measurements have been performed. The permanent magnet used for this measurement is mounted on a rotational stage. The rotor is then inserted without mechanical contact into the stator. The stator is mounted on a piezoelectric multicomponent load cell (Kistler 9256C1) to measure the winding reaction force and torque components resulting from the injected winding currents. A Spitzenberger&Spies DM3000 power supply is used to generate the three-phase symmetric sinusoidal voltages applied to the star-connected winding at a frequency of $\omega = 2\pi \cdot 1$ Hz.

Measurement results are shown in Fig. 10 for an ironless stator $R_6 \rightarrow \infty$. Measurement setup parameters: $R_1 = 7.1$ mm, $R_3 = 8$ mm, $R_4 = 10$ mm, $L = 27.5$ mm, $N = 12$, and $B_{rem} = 1.1$ T.

The force measurement including an iron stator core $R_6 = R_4$ is marked with a cross in Fig. 7. The measured bearing force amplitude $|\hat{F}_{xy}|$ range of $(593 \pm 1.5\%)$ mN corresponds well with the FEM-based value of 601 mN.

B. Power Electronics

Fig. 11 shows a bearing current step response for $\Omega = 2\pi \cdot 8.33$ kHz of the bearing inverter system, as shown in Fig. 8. For this measurement, the rotor was removed from the motor. This allows for an isolated performance evaluation of the PI controllers and decoupling scheme [cf. (20)] implemented in the inner current control loop of the cascaded control architecture. $\gamma$ was artificially generated by a sine lookup table and could thus be used to obtain the step response in $dq$ coordinates by means of postprocessing of the measured currents. The analysis shows that the current controller features the required bandwidth and accuracy at the target speed. Furthermore, a current ripple can be observed, which comprises a low- and a high-frequency content. Whereas the high-frequency current ripple has a fundamental frequency equal to $f_{sw} = 1$ MHz and is an inevitable consequence of the use of a switched-mode power converter, the low-frequency current ripple is caused by the phase asymmetry of the bearing winding (cf. Table II). This finding is consistent with the observed variation of the bearing force amplitude $|\hat{F}_{xy}|$, as shown in Fig. 10, where a similar skewed air-gap winding was used. The low-frequency ripple shows a fundamental frequency of three times the rotational frequency. For high rotor speeds ($\Omega \gtrsim 2\pi \cdot 4$ kHz), this frequency lies outside the controller bandwidth $f_{bw} = 12$ kHz and cannot thus be controlled. However, the occurring ripple currents will solely moderately increase the losses in the

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**TABLE II**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_S$</td>
<td>Stator phase resistance</td>
<td>$1.1,\Omega$</td>
</tr>
<tr>
<td>$L_S$</td>
<td>Transformed stator phase inductance</td>
<td>$(3.75 \pm 10%)\mu$H</td>
</tr>
<tr>
<td>$\chi_{DM}$</td>
<td>Radial bearing constant</td>
<td>$71.6 \cdot 10^{-3},\text{Vs/m}$</td>
</tr>
<tr>
<td>$U_{DC}$</td>
<td>DC-link voltage</td>
<td>$12,\text{V}$</td>
</tr>
<tr>
<td>$f_{sw}$</td>
<td>Switching frequency</td>
<td>$1,\text{MHz}$</td>
</tr>
<tr>
<td>$f_{bw}$</td>
<td>Current controller bandwidth</td>
<td>$12,\text{kHz}$</td>
</tr>
</tbody>
</table>

---

Fig. 11. Measured radial-bearing current step response due to a reference step in the $d$-direction for an angular frequency $\Omega = 2\pi \cdot 8.33$ kHz. The upper plot shows the phase currents $i_A$, $i_C$ and the FPGA signal of the system-measured phase current $i_{A,FPGA}$. The lower plot shows the step response transformed into $dq$ coordinates and references $i_d,ref$ and $i_q,ref$. The observed current ripple with a fundamental frequency of three times the rotational frequency is due to phase asymmetries of the bearing winding (cf. Table II).
winding but not affect the mechanical system. The latter is a consequence of the high frequency, which is far beyond the mechanical bandwidth [21].

C. Rotor Bending Resonance Measurement

A fundamental limitation of the maximum rotational speed is the frequency of the first rotor bending mode. Due to the stiff rotor construction, the high frequency of the resonance, and the high internal damping of the rotor, exciting the bending resonance by a reference position step input was not possible. Therefore, a sine wave excitation was used for the measurement. A rotating radial force is applied to the levitated rotor by the magnetic bearing. The resulting rotor tip displacement is measured by an external optical displacement sensor (Keyence LK-H022). A measurement of the transfer function from distorting force to rotor displacement can be obtained by sweeping the frequency of the distortion while measuring the rotor tip displacement [25]. Increased displacements at the resonance frequency follow from the bending of the rotor. A measurement of the transfer function around the first bending mode is shown in Fig. 12. The measured resonance frequency of 10.7 kHz is about 10% lower than the calculated frequency of 11.9 kHz. This difference might be caused by an aberration from the specified material properties or by manufacturing tolerances.

D. Closed-Loop System Measurements

Fig. 13 shows rotor displacements for different speeds measured by one of the PCB-based eddy-current sensors with a sampling frequency of 33 kHz. The sensor measures radial rotor displacements in an axial distance of 6.4 mm from the rotor tip. Given the high rotational speed, an external 1-D ultrahigh-speed optical displacement sensor (Keyence LK-H022) with a sampling frequency of 100 kHz is used to track the rotor tip displacement at 500 kr/min in Fig. 14. In contrast to the eddy-current sensor, the optical sensor is positioned to measure the radial rotor displacements at the rotor tip. An \( \Omega \)-synchronous displacement of about \( \pm 17 \mu m \) can be observed. Due to the frequency and the fact that larger displacements are measured at the rotor tips, it is assumed that the displacements are caused by the unbalance of the rotor. The rotor could not be balanced during manufacturing as it is intended for high speeds.

E. Power Loss Measurements

Due to the high speed and the low torque of the presented machine, a loss measurement based on a rotary torque transducer is not applicable [1]. Therefore, the no-load losses are measured by means of a deceleration test. The rotor is accelerated to 505 kr/min, after which the motor inverter is switched off, letting the losses decelerate the rotor [1]. The recorded speed versus time is shown in Fig. 16. The breaking losses, consisting
of eddy-current, iron, and air-friction losses, are calculated using the known moment of inertia of the rotor. The measurement is performed in vacuum and repeated in ambient air-pressure conditions. The ambient air test setup is shown in Fig. 4 and the vacuum test setup in Fig. 5. With the measured losses of both experiments, the air-friction losses can be separated from the rest. Using the presented measurement method, the eddy-current losses and iron losses cannot be separated from each other. The total magnetic bearing power is measured, during the deceleration, by measuring the bearing inverter dc-link input power. Thus, the measured total bearing power consists of switching losses and conduction losses in the bearing inverter and winding system.

Figs. 17 and 18 show the measured power loss in vacuum and in ambient air-pressure conditions, respectively. Both plots feature a distinctive bearing power drop at 60 kr/min. This loss reduction is achieved by enabling a rotor position notch filter, as described in [26], for rotational frequencies above the cutoff frequency of the mechanical bearing system, i.e., for speeds above 60 kr/min. The purpose of the notch filter is to reduce the power loss in the bearing windings. In vacuum, the bearing power features practically no speed dependence over 60 kr/min. In ambient air-pressure conditions, the bearing power increases for speeds above 300 kr/min because the parasitic aerodynamic forces acting on the rotor have to be compensated by the magnetic bearing.

The measured sum of the eddy-current and iron losses is 13.7 W at 500 kr/min. Based on field simulations, it can be assumed that approximately 80% of these losses are due to eddy currents in the nonlaminated stator aluminum. The eddy currents are caused by the stray field of the permanent magnets in the aluminum parts of the stator that are used as the thermal connection of the bearing windings with the rest of the stator. Proximity losses in the windings of 0.9 W and iron losses of 1.8 W are expected from calculations. The proximity losses are calculated using the Ferreira method presented in [27].

VI. CONCLUSION

The slotless self-bearing motor prototype presented in this paper overcomes several limitations of previously presented high-speed magnetic bearings. The ironless construction of the rotor and the slotless design of the stator enable ultrahigh-speed rotation and very high bearing actuator bandwidths. A mechanical and electrical bearing model is presented and verified by multidimensional force measurements. It is shown that the bearing actuators feature linear characteristics. In order to control the bearing currents, an inverter system is designed...
using a switching frequency of 1 MHz, and its controller performance is shown. Closed-loop system measurements up to speeds of 505 kr/min verify the functionality of the overall system. To the authors’ knowledge, this is the highest speed achieved by magnetically levitated electrical drive systems so far.

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REFERENCES


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