Asymptotic Performance of Wireless Communications with Generalized Selection Combining

Yao Ma
Dept. of ECpE
Iowa State Univ.
Email: mayao@iastate.edu

Zhengdao Wang
Dept. of ECpE
Iowa State Univ.
Email: zhengdao@iastate.edu

S. Pasupathy
Dept. of ECE
Univ. of Toronto
Email: pas@comm.utoronto.ca.

***Abstract*— In this paper, we study the asymptotic performance of generalized selection combining (GSC) over different fading channels for large average signal-to-noise ratio (ASNR). Employing a moment generating function (MGF) method and the polynomial approximation of the fading channel probability density function (pdf), we derive general asymptotic MGF expressions for the GSC output SNR over generalized independent fading channels. The diversity and coding gains for GSC over different fading channels are derived. Our analytical results reveal that the diversity gain of GSC is equivalent to that of maximum ratio combining (MRC), for different modulations and generalized fading channels. We also show that the difference in the coding gains for different modulations is manifested in terms of a modulation factor defined in this paper, and thus the performance gaps between different modulations can be analytically predicted. Asymptotic performance gap between GSC and MRC is also studied in terms of the coding gain. Numerical examples are presented to illustrate the application of the new results.**

I. Introduction

Generalized selection combining (GSC) [1]–[9] is a practical and useful diversity combining scheme for wireless communications. In GSC, N $(1 \leq N \leq L)$ branches with the largest instantaneous SNRs are selected from a total of L branches and combined, and we refer to it as GSC (N, L) in this paper, and in the literature it may also be called hybrid selection/maximum-ratio combining (HS/MRC) [3], [9]. Note that GSC (L, L) and GSC $(1, L)$ correspond to the maximum ratio combining (MRC) and the conventional selection combining (CSC), respectively. Performance evaluation of GSC has attracted a lot of research interest recently for different applications, for example, for wideband CDMA [10] and ultra-wide bandwidth (UWB) communications [11], etc.

However, some problems about performance evaluation of GSC remain to be solved. For example, for GSC with different modulations or in different fading channels, how to analytically predict the differences in performance? Also, although it is known that performance of GSC (N, L) is upper-bounded by that of MRC [or GSC (L, L)], and lower-bounded by that of CSC [or GSC $(1, L)$], there is still lack of a unified analysis on the performance gaps among them for different modulations and in different fading environments.

In this paper, we try to approach these issues for an asymptotic scenario of high average signal-to-noise ratio (ASNR). Based on the moment generating function (MGF) result of the GSC output SNR in [7], and the polynomial approximation of the fading channel probability density function (pdf) for the small instantaneous SNR [12], we provide a unified analysis of the GSC performance (error and outage probabilities) in terms of the diversity and coding gains at high ASNR. A general asymptotic MGF expression for GSC SNR is derived, and some closed-form results for special cases are provided. Furthermore, the diversity and coding gains for GSC with a large class of modulations in independent but non-identically distributed (i.n.d.) fading channels are derived. Our results demonstrate that GSC (N, L) achieves the same diversity gain as MRC, for the error and outage probability performance. We also show that the difference in the coding gains for different modulations is manifested in terms of a *modulation factor* defined in this paper, and thus the performance gaps between different modulations can be analytically predicted. The performance loss of GSC w.r.t. MRC, and the performance improvement of GSC w.r.t. CSC are analytically quantized. These results put new insight into the effect of different system and channel parameters on the receiver performance at high ASNRs, and they are useful for analysis and design of GSC receivers over generalized fading channels.

II. Asymptotic MGF of the GSC Output SNR

A. Preliminaries

For a GSC (N, L) receiver, we define the SNR vector from the L receiving channels as $\boldsymbol{\gamma} = [\gamma_{n_1}, \gamma_{n_2}, \ldots, \gamma_{n_L}]^\top$, where T denotes vector transpose, γ_{n_l} represents the instantaneous SNR in the n_lth diversity branch; $n_l \in (1, 2, \ldots, L)$ for $1 \leq l \leq L$, and $(n_1, n_2, \ldots, n_L)$ is a permutation of $(1, 2, \ldots, L)$. We arrange the elements in $\boldsymbol{\gamma}$ in descending order as $\tilde{\boldsymbol{\gamma}} = [\gamma_{(1)}, \gamma_{(2)}, \ldots, \gamma_{(L)}]^\top$, so $\tilde{\boldsymbol{\gamma}}$ is an ordered SNR set. The GSC output SNR is given by $\gamma_s = \sum_{l=1}^{N} \gamma_{(l)}$ and its MGF is defined as $\Phi_{\gamma_s}(s) = E[\exp(-\gamma_s s)]$.

From the result in [7], a general MGF expression for the GSC output SNR on arbitrary i.n.d. fading channels is given

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by

$$\Phi_{\gamma_s}(s) = \sum_{\substack{n_1,\ldots,n_{N-1} \\ n_1<n_2<\ldots<n_{N-1}}} \sum_{n_N} \int_0^\infty e^{-sx} f_{n_N}(x) \cdot \left[\prod_{l=1}^{N-1} \Phi_{n_l}(s,x)\right] \left[\prod_{l'=N+1}^{L} F_{n_{l'}}(x)\right] dx \quad (1)$$

where $f_{n_l}(x)$ and $F_{n_l}(x) = \int_0^x f_{n_l}(y)dy$ are the pdf and the cumulative distribution function (cdf) expressions for the output SNR in the n_lth branch, respectively, for $n_l = 1, \ldots, L$. $\Phi_{n_l}(s,x) = \int_x^\infty f_{n_l}(y)e^{-sy}dy$ is the incomplete MGF expression originally defined in [8], where $f_{n_l}(x)$ is the pdf expression for the SNR γ_{n_l}. Note that $\Phi_{n_l}(s,0) = \Phi_{n_l}(s)$, and $\Phi_{n_l}(0,x) = 1 - F_{n_l}(x)$, where $F_{n_l}(x)$ is the cdf of the SNR γ_{n_l}. In (1), $\sum_{\substack{n_1,\ldots,n_{N-1} \\ n_1<n_2<\ldots<n_{N-1}}}$ denotes the $\binom{L}{N-1}$ possible combinations of selecting the $(N-1)$ largest SNR branches out of the L branches, and in $\sum_{n_N}$ the index n_N is chosen from the remaining $L-N+1$ branches.

However, (1) does not provide the insight as to which parameters determine the performance of GSC at high ASNRs. Also, it is not clear what are the diversity order and coding gain of GSC.

For high ASNRs, if the error probability can be written as

$$P_{\text{Err}} = (G_c\bar{\gamma})^{-G_d} + o(\bar{\gamma}^{-G_d}) \quad (2)$$

where $\bar{\gamma}$ is the input ASNR per branch, and $o(x)$ satisfies the property that $\lim_{x\to 0} \frac{o(x)}{x} = 0$, then we call G_c the coding gain which illustrates an improvement (or degradation) factor to the ASNR per branch, and G_d the diversity gain, which manifests the effective diversity order and determines the asymptotic slope of the error probability curves versus the input ASNR at high ASNRs, in a log-log scale, see e.g. [12].

For outage performance of GSC with a predefined SNR threshold γ_{th} , if the asymptotic outage probability [defined as $P_{\text{OT}}(\gamma_{\text{th}}) = \Pr(\gamma_s < \gamma_{\text{th}})$] can be expressed as

$$P_{\text{OT}}(\gamma_{\text{th}}) = \left(O_c \frac{\bar{\gamma}}{\gamma_{\text{th}}}\right)^{-O_d} + o(\bar{\gamma}^{-O_d}) \quad (3)$$

then we call O_c and O_d the outage coding gain and diversity gain, respectively.

B. General Results

For different fading channel types, the expressions for the polynomial approximation of the pdfs were originally given in [12]. Here for completeness of this work we list them in Table I, although in a slightly different form due to the definition. We express the pdf as $f(\gamma) = a\gamma^t + o(\gamma^t)$ for the high ASNR. Then we can express a as $a = b/\bar{\gamma}^{\tilde{t}}$, where $\bar{\gamma}$ is the ASNR, and b is not a function of $\bar{\gamma}$. We find that for all the fading channel types studied in this paper, $\tilde{t} = t+1$ holds true, that is, we can always express a as $a = b/\bar{\gamma}^{t+1}$.

For large ASNRs, the incomplete MGF can be approximated as $\Phi(s,x) \simeq a\int_x^\infty e^{-su}u^t du$. Furthermore, assuming s is a positive real scalar, a condition which holds true for the analysis we use later, we can write $\Phi(s,x) \simeq a\Gamma(t+1,sx)/s^{t+1}$, where $\Gamma(t,x) = \int_x^\infty e^{-u}u^{t-1}du$ is the complementary incomplete Gamma function. Also, the cdf $F(x)$ is approximated as $F(x) \simeq ax^{t+1}/(t+1)$.

By substituting the result above to the general MGF expression (1), we obtain that

$$\Phi_{\text{GSC}}(s) \simeq \left[\prod_{l=1}^{L} a_l\right] \sum_{\substack{n_1,\ldots,n_{N-1} \\ n_1<n_2<\ldots<n_{N-1}}} \sum_{n_N} \int_0^\infty e^{-sx} \cdot \left[\prod_{l=1}^{N-1} \frac{\Gamma(t_{n_l}+1,sx)}{s^{t_{n_l}+1}}\right] \frac{x^{\sum_{l'=N}^{L}(t_{n_{l'}}+1)-1}}{\prod_{l'=N+1}^{L}(t_{n_{l'}}+1)} dx. \quad (4)$$

By using a change of variable that $y = sx$, we can obtain a general asymptotic MGF expression for GSC in generalized fading channels as

$$\Phi_{\text{GSC}}(s) \simeq \frac{\prod_{l=1}^{L} a_l}{s^{\sum_{l=1}^{L}(t_l+1)}} \sum_{\substack{n_1,\ldots,n_{N-1} \\ n_1<n_2<\ldots<n_{N-1}}} \sum_{n_N} \int_0^\infty e^{-y} \cdot \prod_{l=1}^{N-1} \Gamma(t_{n_l}+1,y) \frac{y^{\sum_{l'=N}^{L}(t_{n_{l'}}+1)-1}}{\left(\prod_{l'=N+1}^{L}(t_{n_{l'}}+1)\right)} dy. \quad (5)$$

Since $\prod_{l=1}^{L} a_l$ is the only term in (5) that is related to the ASNR, by $a_l = b_l/\bar{\gamma}_l^{t_l+1}$, where $\bar{\gamma}_l$ is the ASNR in the lth branch, we conclude that the diversity gain is determined by parameter a_l, which is then manifested as a function of t_l (for $l = 1, \ldots, L$).

To proceed further, let $g_l = \bar{\gamma}_l/\bar{\gamma}$, where $\bar{\gamma}$ is the total input ASNR divided by L. Thus $\sum_{k=1}^{L} g_k = L$. Also, we define two vectors $\mathbf{b} = [b_1, \ldots, b_L]^\top$ and $\mathbf{t} = [t_1, \ldots, t_L]^\top$. Now the asymptotic MGF can be given in an explicit expression of the ASNR $\bar{\gamma}$ as

$$\Phi_{\text{GSC}}(s) \simeq F(\mathbf{b},\mathbf{t})(s\bar{\gamma})^{-\sum_{l=1}^{L}(t_l+1)} \quad (6)$$

where

$$F(\mathbf{b},\mathbf{t}) = \left(\prod_{l=1}^{L} b_l g_l^{-(t_l+1)}\right) \sum_{\substack{n_1,\ldots,n_{N-1} \\ n_1<n_2<\ldots<n_{N-1}}} \sum_{n_N} \int_0^\infty e^{-x} \cdot \prod_{l=1}^{N-1} \Gamma(t_{n_l}+1,x) \frac{x^{\sum_{l'=N}^{L}(t_{n_{l'}}+1)-1}}{\left(\prod_{l'=N+1}^{L}(t_{n_{l'}}+1)\right)} dx. \quad (7)$$

TABLE I

EXPRESSIONS FOR THE PDF AND OTHER PARAMETERS FOR THE SNR IN A DIVERSITY BRANCH OVER RAYLEIGH, RICIAN, NAKAGAMI-q AND NAKAGAMI-m FADING CHANNELS.

Fading Types	PDF expression $f(\gamma)$	$f(\gamma) \simeq a\gamma^t$, t, a, and b
Rayleigh	$\frac{1}{\bar{\gamma}}\exp(\frac{-\gamma}{\bar{\gamma}})$	$t=0$, $a=1/\bar{\gamma}$, $b=1$.
Nakagami-q	$\frac{1+q^2}{2q\bar{\gamma}}\exp\left(-\frac{(1+q^2)^2\gamma}{4q^2\bar{\gamma}}\right)$ $\cdot I_0\left(\frac{(1-q^2)\gamma}{4q^2\bar{\gamma}}\right)$	$t=0$, $a=\frac{1+q^2}{2q\bar{\gamma}}$, $b=\frac{1+q^2}{2q}$
Rician	$\frac{1+K}{\bar{\gamma}}\exp\left(-K-\frac{(1+K)\gamma}{\bar{\gamma}}\right)$ $\cdot I_0\left(2\sqrt{K(1+K)\gamma/\bar{\gamma}}\right)$	$t=0$, $a=(1+K)e^{-K}/\bar{\gamma}$, $b=(1+K)e^{-K}$
Nakagami-m	$(\frac{m}{\bar{\gamma}})^m\frac{\gamma^{m-1}}{\Gamma(m)}\exp(-\frac{m}{\bar{\gamma}}\gamma)$	$t=m-1$, $a=\frac{m^m}{\Gamma(m)\bar{\gamma}^m}$, $b=\frac{m^m}{\Gamma(m)}$.

Using the fact that the MGF for the SNR at the lth branch (for $l = 1, \ldots, L$) can be approximated as

$$\begin{aligned}\Phi_l(s) &\simeq a_l \int_0^\infty e^{-su} u^{t_l} du \\ &= a_l \frac{\Gamma(t_l+1)}{s^{t_l+1}} = \frac{b_l g_l^{-(t_l+1)} \Gamma(t_l+1)}{(s\bar{\gamma})^{(t_l+1)}} \quad (8)\end{aligned}$$

we obtain the MGF for GSC (L, L) (or MRC) for generalized fading channels as

$$\Phi_{\text{MRC}}(s) \simeq \frac{\prod_{l=1}^{L} b_l g_l^{-(t_l+1)} \Gamma(t_l+1)}{(s\bar{\gamma})^{\sum_{l=1}^{L}(t_l+1)}}. \quad (9)$$

C. Results for Special Cases

1) I.N.D. Channels: Let us consider a general case where the signal SNRs at the different branches may have non-identical statistics, or even follow different families of distributions, including Rayleigh, Rician, and/or Nakagami-q distributions (but excluding Nakagami-m distribution with $m \neq 1$).

For these channels, since $t_l = 0$ for $l = 1, \ldots, L$, we can write $\Gamma(t_{n_l}+1, x) = \Gamma(1, x) = e^{-x}$. Thus (6) can be simplified to

$$\begin{aligned}\Phi_{\text{GSC}}(s) &\simeq \frac{\prod_{l=1}^{L} b_l g_l^{-(t_l+1)}}{(\bar{\gamma}s)^{\sum_{l=1}^{L}(t_l+1)}} N \binom{L}{N} \int_0^\infty e^{-Nx} x^{L-N} dx \\ &= \frac{\prod_{l=1}^{L} b_l g_l^{-(t_l+1)}}{(\bar{\gamma}s)^{L}} \frac{L!}{N! N^{L-N}}. \quad (10)\end{aligned}$$

2) I.I.D. Channels: we will consider two cases for i.i.d. channels, as discussed below.

1) In the first case, t_l are positive integers for $l = 1, \ldots, L$. This may, for example, correspond to i.i.d. branches with Nakagami-m distribution with integer fading figure m.
   When t is an positive integer, we have [13] $\Gamma(t+1, x) = \Gamma(t+1) e^{-x} \sum_{k=0}^{t} x^k / k!$, and
   $$[\Gamma(t+1, x)]^{N-1} = [\Gamma(t+1)]^{N-1} e^{-(N-1)x} \cdot \sum_{n_0, \ldots, n_t} \binom{N-1}{n_0, \ldots, n_t} \frac{x^{\sum_{k=1}^{t} k n_k}}{\prod_{k=1}^{t} (k!)^{n_k}} \quad (11)$$
   where $\binom{N-1}{n_0, \ldots, n_t} = \frac{(N-1)!}{n_0! n_1! \cdots n_t!}$ is the multi-nomial coefficient. By applying (11) to (6), we get
   $$\Phi_{\text{GSC}}(s) \simeq \frac{b^L}{(\bar{\gamma}s)^{L(t+1)}} N \binom{L}{N} \frac{[\Gamma(t+1)]^{N-1}}{(t+1)^{L-N}} \sum_{n_0, \ldots, n_t} \binom{N-1}{n_0, \ldots, n_t} \frac{\Gamma(B)}{N^B \prod_{k=1}^{t} (k!)^{n_k}} \quad (12)$$
   where $B = \sum_{k=1}^{t} k n_k + (t+1)(L-N+1)$.
2) In the second case, $t_l = 0$, for $l = 1, \ldots, L$, and the ASNRs at all the branches are identical.
   By setting $t = 0$ in (12), or by setting $b_l = b$ for $l = 1, \ldots, L$ in (10), we obtain
   $$\Phi_{\text{GSC}}(s) \simeq \frac{b^L}{(\bar{\gamma}s)^L} \frac{L!}{N! N^{L-N}}. \quad (13)$$

III. Performance Analysis

We illustrate the application of our asymptotic MGF result to performance evaluation for many modulation schemes with GSC diversity. We will derive the asymptotic coding gain and diversity gain for the error probability and outage probability of GSC in generalized fading channels, as given next.

A. Average Error Probability

The average symbol and bit error probabilities (SEP and BEP) for a large class of modulation are studied.

1) BPSK: By using the integral form of the BEP expression for BPSK [14], we get

$$P_{\text{BPSK}}(\bar{\gamma}) \simeq \frac{1}{\pi} \int_0^{\pi/2} \Phi_{\text{GSC}}\left(\frac{1}{\sin^2\theta}\right) d\theta \quad (14)$$

where $\Phi_{\text{GSC}}\left(\frac{1}{\sin^2\theta}\right)$ is obtained from (6) by replacing variable s with $\frac{1}{\sin^2\theta}$. Since $s^{-\sum_{l=1}^{L}(t_l+1)}$ is the only factor in (6) that is involved in the integral (14), we can extract the factor in (14) which is directly related with the BPSK modulation format as

$$\begin{aligned}\tilde{G}_{M,\text{BPSK}} &= \frac{1}{\pi} \int_0^{\pi/2} (\sin\theta)^{(2\sum_{l=1}^{L}(t_l+1))} d\theta \quad (15) \\ &= \frac{1}{2\sqrt{\pi}} \frac{\Gamma(\sum_{l=1}^{L}(t_l+1)+0.5)}{\Gamma(\sum_{l=1}^{L}(t_l+1)+1)}. \quad (16)\end{aligned}$$

We define $\tilde{G}_{M,\text{BPSK}}$ as the *modulation factor* for BPSK.

The BEP of BPSK can be rewritten as

$$P_{\text{BPSK}}(\bar{\gamma}) = F(\mathbf{b}, \mathbf{t}) \tilde{G}_{M,\text{BPSK}} \bar{\gamma}^{-\sum_{l=1}^{L}(t_l+1)}.$$

By comparing $P_{\text{BPSK}}(\bar{\gamma})$ with (2), we get the diversity gain for GSC (N, L) as $G_d = \sum_{l=1}^{L}(t_l+1)$, and the coding gain as

$$G_{c,\text{BPSK}} = \left(F(\mathbf{b}, \mathbf{t}) \frac{1}{2\sqrt{\pi}} \frac{\Gamma(G_d+0.5)}{\Gamma(G_d+1)}\right)^{-1/G_d}. \quad (17)$$

When $N = L$, the BEP for MRC can be computed by using (9) as

$$P_{\text{BPSK}}(\bar{\gamma}) = \frac{\left[\prod_{l=1}^{L} b_l g_l^{-(t_l+1)} \Gamma(t_l+1)\right] \tilde{G}_{M,\text{BPSK}}}{\bar{\gamma}^{\sum_{l=1}^{L}(t_l+1)}}.$$

Thus the diversity gain of MRC is also given by $G_d = \sum_{l=1}^{L}(t_l+1)$, but the coding gain is given by $G_c = \left([\prod_{l=1}^{L} b_l g_l^{-(t_l+1)} \Gamma(t_l+1)] \tilde{G}_{M,\text{BPSK}}\right)^{-1/G_d}$.

This result shows that the GSC achieves the same diversity gain as the MRC on generalized fading channels, but the coding gains differ by a constant depending on N, L and the channel parameter $\{t_l\}_{l=1}^{L}$.

We also note that for BPSK and all the other modulations studied later in this paper, the diversity gain of GSC (N, L) is always given by $G_d = \sum_{l=1}^{L}(t_l+1)$, and the coding gain is related with the diversity gain G_d, the modulation factor $\tilde{G}_M$, and the channel parameters $\mathbf{b}, \mathbf{t}$ by the formula that

$$G_c = \left(F(\mathbf{b}, \mathbf{t}) \tilde{G}_M\right)^{-1/G_d} \quad (18)$$

Due to the space limitations, we present only the modulation factors for some popular modulations below, omitting details.

2) MPSK: The modulation factor is given as

$$\tilde{G}_{M,\text{MPSK}} = \frac{1}{\pi}\int_0^{(M-1)\pi/M}\left(\frac{\sin\theta}{\sin(\pi/M)}\right)^{2G_d} d\theta = \frac{B_{(\sin[(M-1)\pi/M])^2}(G_d+0.5, 0.5)}{2\pi[\sin(\pi/M)]^{2G_d}}.$$

where $B_b(t_1,t_2) = \int_0^b x^{t_1-1}(1-x)^{t_2-1}dx$ is the incomplete beta function [13].

3) M-ary Quadrature Amplitude Modulation (M-QAM):

$$\tilde{G}_{M,\text{MQAM}} = \frac{2}{\pi g_{\text{qam}}^{G_d}}\left(1-\frac{1}{\sqrt{M}}\right) \cdot \left[B(G_d+0.5,0.5) - \left(1-\frac{1}{\sqrt{M}}\right)B_{0.5}(G_d+0.5,0.5)\right]$$

4) BDPSK and BFSK: First, from the inverse Laplace transform of (6), we get the pdf of the GSC output SNR as

$$f_{\text{GSC}}(\gamma) \simeq F(\mathbf{b},\mathbf{t})(\bar{\gamma})^{-G_d}\gamma^{G_d-1}/\Gamma(G_d). \tag{19}$$

By using (19) and integrate the conditional BEP over the fading channel pdf, we have

$$\tilde{G}_M^{(N)} = \frac{1}{2^{2N-1}g^{G_d}}\sum_{k=0}^{N-1}\frac{c_k\Gamma(k+G_d)}{k!\,\Gamma(G_d)}. \tag{20}$$

where $c_k = \sum_{n=0}^{N-1-k}\binom{2N-1}{n}$, $g=1$ for BDPSK and $g=0.5$ for BFSK.

To summarize, the results above demonstrate that both coherent and non-coherent GSCs have the same diversity gain (same as MRC or post-detection EGC), and the differences in the coding gains for different modulations are manifested in the modulation factor $\tilde{G}_M$ only, which can be analytically evaluated by using the results given above.

B. Outage Probability

The outage probability is defined as $P_{\text{OT}}(\gamma_{\text{th}}) = \int_0^{\gamma_{\text{th}}} f_{\gamma_s}(\gamma_s)d\gamma_s$, where $f_{\gamma_s}(\gamma_s)$ is the pdf of the GSC output SNR, and γ_{th} is the specified SNR threshold.

We can use (19) and obtain an asymptotic outage probability as

$$P_{\text{OT,GSC}}(\gamma_{\text{th}}) \simeq \frac{F(\mathbf{b},\mathbf{t})}{\Gamma(G_d+1)}(\bar{\gamma}/\gamma_{\text{th}})^{-G_d}. \tag{21}$$

The result shows that for GSC the outage diversity gain is the same as the error probability diversity gain, that is $O_d = G_d$. Also, the outage coding gain is given by $O_c = \left(\frac{F(\mathbf{b},\mathbf{t})}{\Gamma(G_d+1)}\right)^{-1/G_d}$.

IV. Loss of Coding Gain of Coherent GSC

The performance of coherent GSC (N,L) is upper-bounded by that of GSC (L,L) (or MRC), and in this subsection we study the performance gap in terms of the coding gain. The loss of the coding gain of GSC with respect to that of MRC can be defined as $\beta_{(N,L)} = \frac{G_{c,\text{MRC}}}{G_{c,\text{GSC}(N,L)}}$. The general result is given by

$$\beta_{(N,L)} = \left(\frac{\tilde{F}(\mathbf{t})}{\prod_{k=1}^{L}\Gamma(t_k+1)}\right)^{1/G_d} \tag{22}$$

where G_d is the diversity gain, and $\tilde{F}(\mathbf{t})$ is given by

$$\tilde{F}(\mathbf{t}) = \sum_{\substack{n_1,\ldots,n_{N-1}\\ n_1<n_2<\ldots<n_{N-1}}}\sum_{n_N}\int_0^\infty e^{-x} \cdot \prod_{l=1}^{N-1}\Gamma(t_{n_l}+1,x)\frac{x^{\sum_{l'=N}^{L}(t_{n_{l'}}+1)-1}}{\left(\prod_{l'=N+1}^{L}(t_{n_{l'}}+1)\right)}dx.$$

For the case of $t_1 = \ldots = t_L = t = 0$, but the ASNRs are not necessarily identical, (22) can be reduced to

$$\beta_{(N,L)} = \left(\frac{L!}{N!N^{L-N}}\right)^{1/L}. \tag{23}$$

We note that in [9] it was proved that the ASNR penalty of GSC w.r.t. the MRC for MPSK modulation at high ASNR in the i.i.d Rayleigh fading channel is given by $\left(\frac{L!}{N!N^{L-N}}\right)^{1/L}$. In comparison, our result in (23) is more general because we proved that this holds true for a large class of modulations at high ASNR and in different i.n.d channels where $t=0$ (including mixed Rayleigh, Rician and Nakagami-q fading distributions).

1) For a small N and $N>1$, we study the improvement factor of the coding gain w.r.t. that of CSC, which is defined as $\alpha_{(N,L)} = \frac{G_{c,\text{GSC}(N,L)}}{G_{c,\text{GSC}(1,L)}} = \frac{\beta_{(1,L)}}{\beta_{(N,L)}}$. It is not difficult to show that
$$\alpha_{(N,L)} = N\left(\frac{(N-1)!}{N^{N-1}}\right)^{1/L},$$
and $N^{1-\frac{N-1}{L}} \le \alpha_{(N,L)} \le N$.
2) When $N = L/2$, the coding gain loss of GSC $(L/2,L)$ is given by (23) by replacing N with $L/2$. By apply the Stirling's formula we can get
$$\beta_{(L/2,L)} \simeq 2^{\frac{1}{2L}}2/\sqrt{e}. \tag{24}$$
It can be show that $\beta_{(L/2,L)}$ decreases monotonically as L increases and $10\log_{10}\beta_{(L/2,L)} < 1$ dB when $L \ge 10$.

V. Numerical Results

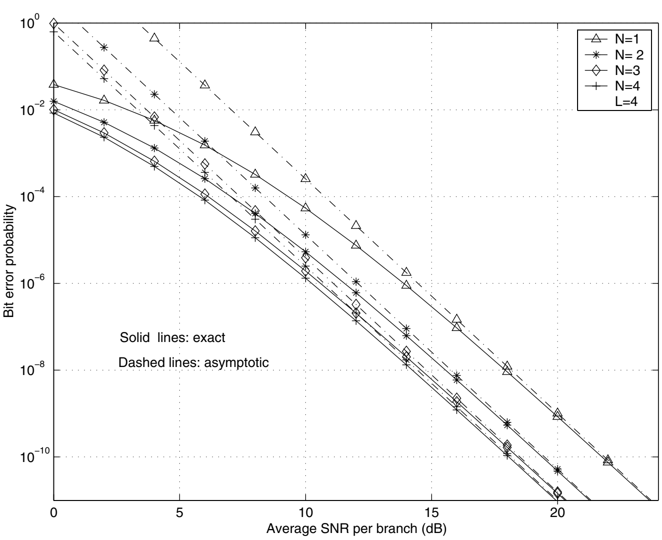

Fig. 1. Exact (in solid lines) and asymptotic (in dashed lines) BEPs of GSC $(N,4)$ on an i.n.d Nakagami fading channel.

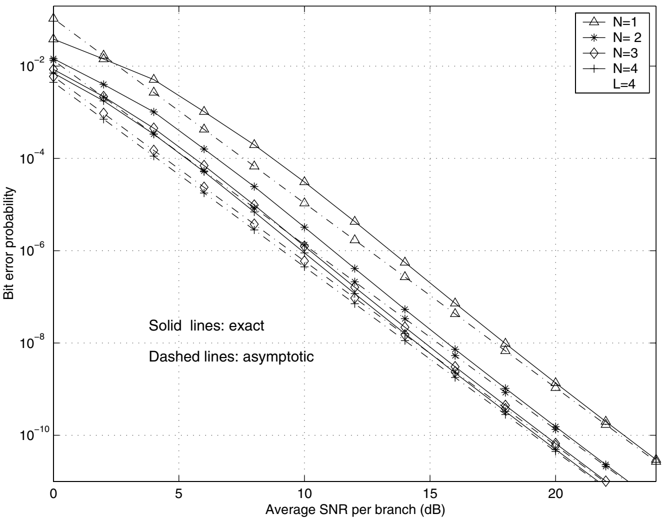

Fig. 2. Exact (in solid lines) and asymptotic (in dashed lines) BEPs of GSC $(N, 4)$ on an i.n.d Rician fading channel.

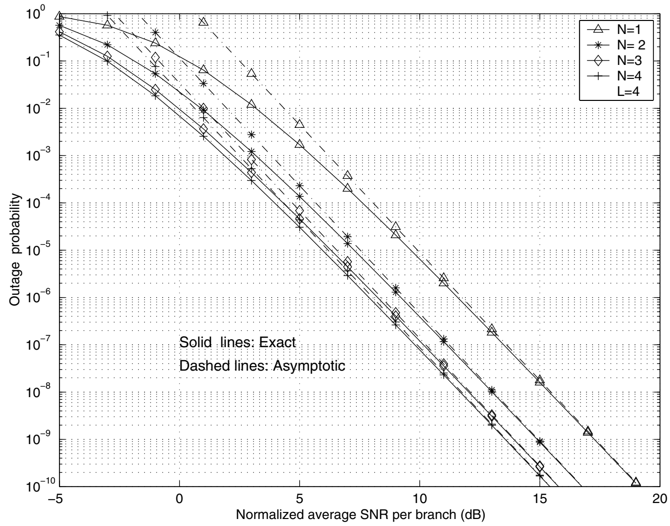

Fig. 3. Exact (in solid lines) and asymptotic (in dashed lines) outage probabilities of GSC $(N, 4)$ diversity on an i.n.d Nakagami fading channel.

We first present exact and asymptotic BEP results for GSC $(N, 4)$ with BPSK modulation on i.n.d Nakagami and Rician fading channels in Figures 1 and 2, respectively. For both Rician and Nakagami fading channels, we assume the ASNRs at different receiving branches follow an exponentially decaying power delay profile (PDP), where the ASNRs decrease by 1.5 dB between the adjacent paths successively. The x-axis is defined as the total input ASNR divided by L. For the Nakagami channel, we assume $m = [1.8, 1.5, 1.2, 0.9]$ from the strongest to the weakest branches; while for the Rician channel, we assume $K = [5, 3, 2, 1]$ dB, respectively. The results in Figures 1 and 2 show that the asymptotic BEP curves become accurate at high ASNR for all N and both types of fading channels. In comparison, the BEP curves for Nakagami channels exhibits a higher diversity gain ($G_d = 5.4$) than those for the Rician channels ($G_d = 4$). On the other hand, the asymptotic BEP curves show that GSC in the studied Rician fading channel has a higher coding gain than in the Nakagami fading channel.

In Fig. 3, we presented the exact and asymptotic outage probability result versus the normalized ASNR per branch (defined as $\bar{\gamma}/\gamma_{\text{th}}$) for GSC $(N, 4)$ in an i.n.d Nakagami fading channel. The parameter setting is assumed to be identical to that for Fig. 1. The result shows that the asymptotic outage probability curves fit the exact ones well when the input ASNR exceeds 15 dB. Comparison between Figs. 1 and 3 reveals that the error probability and outage probability diversity gains for GSC are identical, and the coding gains differ by a constant only.

VI. CONCLUSIONS

In this paper, we have analyzed the asymptotic error and outage probabilities of GSC (N, L), in terms of the diversity and coding gains, over generalized fading channels. Our results demonstrate that the diversity gain of GSC (N, L) is equivalent to that of GSC (L, L) (MRC), and the coding gain gap between them can be analytically predicted. In a future study, our results will be used to evaluate the performance of GSC in space-time communication and UWB communication.

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