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The fundamental meaning of Kolmogorov's ideas for the contemporary development of studies in turbulence is widely known. In the 42 years since his classical papers ([1], [2]) on the theory of locally isotropic turbulence there has arisen a vast literature on this problem, and numerous measurements of the characteristics of turbulence have been performed in natural media (the atmosphere, the ocean) as well as in large wind tunnels; the measurements have completely confirmed the predictions of the theory put forth by Kolmogorov.(1)

This relates first of all to the "two-thirds law", according to which the mean square difference of the velocities at two points of a developed turbulent flow is proportional to the distance between the points of observation to the power 2/3 in the region of intermediate scales (which are greater than the internal scale determined by the viscosity and are less than the external scale). The modern development of measurement techniques has enabled us to penetrate into the subtle structure of turbulence, where the influence of the viscosity of the medium [illegible] Kolmogorov gave a theoretical estimate of the corresponding scale as far back as 1941. For the boundary-layer conditions in the atmosphere this "internal scale" is several millimeters, as has also been confirmed experimentally [5].

Kolmogorov was always interested also in fluid mechanics problems connected with the origin of turbulent disturbances—hydrodynamic instability of flows of a fluid with small viscosity.

The corresponding mathematical theory is extremely complicated, and in order to come to a better understanding of this phenomenon Kolmogorov suggested as far back as 1959 in his seminar [6] that an investigation should be made of the simplest model—the two-dimensional motion of a viscous fluid due to the action of a periodic (in one of the coordinates) external force field. An elegant solution of the problem of stability of such a flow (a Kolmogorov flow) was given shortly thereafter by Meshalkin and Sinai [7]. The corresponding model was then conceived of only as a convenient

(1)There is a very thorough presentation of the contemporary state of the statistical theory of turbulence in the two-volume monograph [3], [4] of Monin and Yaglom.

object for theoretical investigations, and hardly anyone thought of the possibility that it might be realized physically under laboratory conditions. This was done later in 1979 by using a "magnetohydrodynamic drive" resulting in the creation of an external Ampère force field

$$\vec{f} = [\vec{H}, \vec{j}]$$

that is periodic in space in a weakly conducting fluid (an electrolyte), where $\vec{H}$ is the external magnetic field strength, with z-component periodic in the coordinate y, and $\vec{j}$ is the constant density of the current between two electrodes placed in the fluid layer under investigation. The quantity $\vec{j}$ can be varied as the experimenter wishes by changing the voltage on the electrodes, and a Kolmogorov flow with various Reynolds numbers can thereby be created. The first experiments of this kind were carried out in a 24 x 12 cm^2 cuvette with spatial period (along the short axis) 4.4 cm, and are described in [8]. A discussion of the results of these experiments along with a brief presentation of the theoretical investigations mentioned above is also contained in the monograph [9].

Dolzhanskii ([8], [9]) showed that for a more precise description of the motion of thin layers in a fluid that are caused by a periodic external force field we should use a modified Kolmogorov model that also takes into account the "external" friction (the friction on the bottom of the cuvette). The corresponding equations of two-dimensional hydrodynamics take the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\rho^{-1}\frac{\partial p}{\partial x} + \nu\,\Delta u - \lambda u + \gamma \sin py,$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\rho^{-1}\frac{\partial p}{\partial y} + \nu\,\Delta v - \lambda u,$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

where u and v are the velocity components, p is the pressure, ρ is the density of the fluid, ν is the kinematic viscosity, γ is the intensity of the external action (γ can be expressed in terms of H and j), Δ is the two-dimensional Laplace operator, and λ is the coefficient of "external" friction, which can be determined approximately by the formula

$$\lambda = \frac{2\nu}{h^2},$$

with h the depth of the fluid layer.

Taking into account the external friction turns out to be very essential for a correct estimate of the critical Reynolds number, which is calculated to be of order 10^3 (instead of the $\sqrt{2}$ according to Meshalkin and Sinai), and this agrees completely with experiment. After passing through the critical value of the Reynolds number in a periodic shear flow, a system of steady-state (in a weakly supercritical situation) vortices forming a kind of parquet takes shape. In the absence of external friction the most unstable disturbances, according to the theory of Meshalkin and Sinai, turn out to be "long-wave" vortices whose size is restricted only by the external size of the system. In reality, an effect of the external friction is that there is an optimal longitudinal wave number comparable with the transversal period of the external force for which the initial disturbances grow most rapidly. The question of the form of the vorticial disturbances arising on the background of a two-dimensional periodic flow in the weakly supercritical case can be investigated to a first approximation on the basis of the linear theory, for selection of the most active modes. The subsequent application of the approximate nonlinear theory, which takes into account the interaction of the developing disturbances with the basic flow, enables us to estimate the steady-state values of the amplitudes. In connection with a Kolmogorov flow this approach was worked out by Klyatskin [10], who first constructed a "map" of disturbances of finite amplitude that corresponds to the weakly supercritical case. Use of the Galerkin method for approximating the hydrodynamic equations in the Kolmogorov model and subsequent solution of the finite algebraic systems on a computer enabled the authors of [11] to construct a completely realistic picture of the "vortex parquet" arising in a Kolmogorov flow for significantly supercritical cases, and the picture was very similar to that obtained in the experiment described above with a magnetohydrodynamic drive (which was carried out shortly after publication of [11]). A comparison of the results of theoretical calculations with the picture of the disturbances obtained in the experiment can be found in the monograph [9].

The simulation of a Kolmogorov flow in a confined rectangular cuvette has an essential deficiency

flow close to the theoretical model can be observed only in the central part of the cuvette, and certain unstable modes may be lost. In this connection it was suggested by the author that analogous experiments be performed with an axially symmetric arrangement in an annular vessel with conducting walls of circular form playing the role of electrodes, and with a magnetic field, depending only on the radius, created by a magnetic system located under the circular cuvette.(1) In the arrangement described the dependence of the azimuth dispersing force on the radius was close to sinusoidal (more precisely, to the Bessel function of zero order), with one full wave between the electrodes. Aluminium powder was introduced in the electrolyte solution in order to observe the flow. Under weak excitation a slow motion of the particles was observed along the circles, with the largest velocity gradient on the nodal line in the middle of the annulus. According to the results of a stability analysis with the help of the linearized equations, for a

(1)The simulation of hydrodynamic processes in annular channels is of interest for geophysical applications in connection with the study of the general circulation of the atmosphere. Details of the experiment are presented in [9].

certain definite excitation (the "dispersal" Reynolds number) there arises a disturbance whose symmetry index can be computed theoretically and depends on the geometry of the flow (the ratio of the radii of the annular region). Figure 1 shows the observed picture of the steady-state disturbances in a shear flow in an annular channel that were observed under supercritical conditions ($Re/Re_{cr} \approx 2.8$) for a geometry corresponding to the index 3.

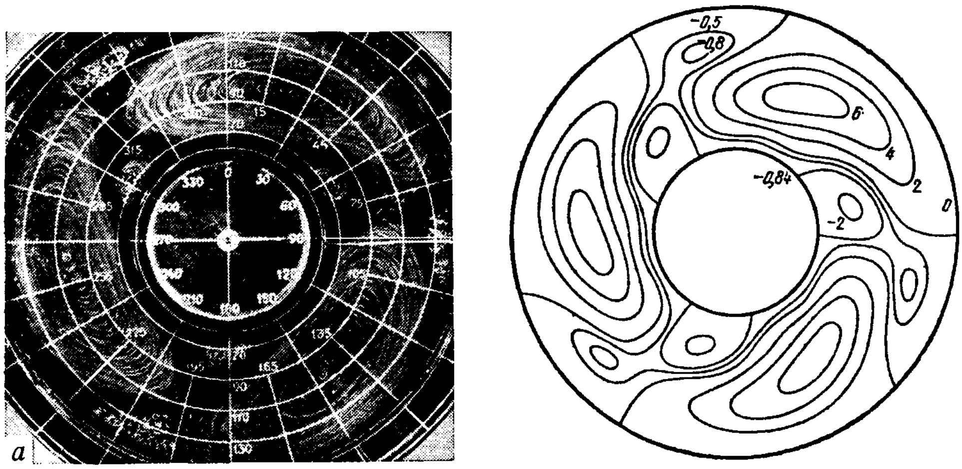

Fig. 1. A stationary disturbance in a shear flow in an annular channel: a) experiment; b) theoretical computation.

For comparison we also give a theoretically calculated (by Ponomarev, using the Galerkin method) picture of the motion (isolines of the flow function) for $Re/Re_{cr} = 3$. The products of Bessel functions of the radius r and trigonometric functions of the azimuth φ were used as basis (reference) functions in the Galerkin method [12].

If the geometry is changed by narrowing the channel while keeping the mean radius, then the critical Reynolds number increases, and finer disturbances corresponding to a larger symmetry index are generated.

In a number of papers on the theory of hydrodynamic stability of two-dimensional flows (see, for example, [13]) it is noted that instabilities connected with limit cycles can appear, which corresponds to excitation of self-oscillations. Such self-oscillations have actually been observed for an elementary system consisting of only four vortices arising as a result of convection [14] or under the influence of magnetohydrodynamic forces [15]; there is a theoretical analysis of a corresponding model in Pleshanova's paper [16].

Experiments recently carried out at the Institute of Atmospheric Physics of the Academy of Sciences of the USSR indicated the existence of a self-oscillating mode for a Kolmogorov flow. The authors Batchaev and Dovzhenko of the experiment in [17] employed a vertical cuvette (Fig. 2) in which the particles of a fluid (a solution of copper sulphate) could move under the action of a magnetohydrodynamic force along a closed trajectory around a slab inside a gap of constant thickness. In the vertical direction four full waves are included.

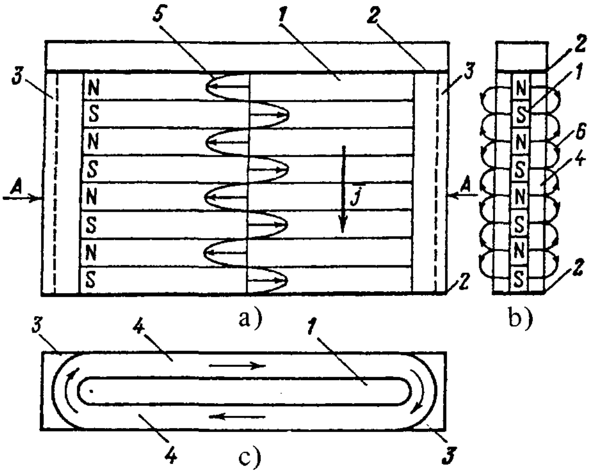

Fig.2. A vertical MHD arrangement for simulating a Kolmogorov flow. Front view, side view, and top view (cut along A-A). 1) a sheet of magnetoelastic rubber of size 245 x 180 x 5 mm^3; 2) copper electrode; 3) side wall rounded on the inside; 4) the channel of flow for the fluid; 5) profile of the MHD force; 6) lines of force of the magnetic field.

It can be assumed that this construction provides a good imitation of a Kolmogorov flow on the surface of a cylinder, and the difficulties connected

pictures of the flow observed under various excitation conditions.

For weak excitation there is a clear picture of ordered jets which call to mind the bands on Jupiter (where zonal atmospheric circulation has also been observed) with the method of visualization used. After the current feeding the system passes through the first critical value (1.1 A in this experiment), steady-state vorticial disturbances arise on the background of zonal circulation. The intensity of these disturbances grows as the excitation increases (the intensity of the average zonal flow varies weakly), and on the next frame the picture of a developed "vortex parquet" is seen. At a certain value of the excitation (the second critical value of the current is 2.6 A) the steady-state vortex mode becomes unstable, and a self-oscillation mode arises in the system of vortices. Cinema frames of the successive phases of the oscillatory mode are shown in Fig. 3. The evolution can be traced by watching the four features of the "vortex parquet" singled out. A full cycle is completed in 11 sec. The length of the period depends on the voltage (current) applied; the period becomes smaller as the excitation increases. For a current less than 3 A the oscillations have a clear "monochromatic" character: the system works like a clock.

For a strong excitation (for a current greater than 3 A) the picture of the oscillations becomes more complicated. The spectrum is enriched by higher harmonics, and a new transition to a mode with continuous spectrum can be

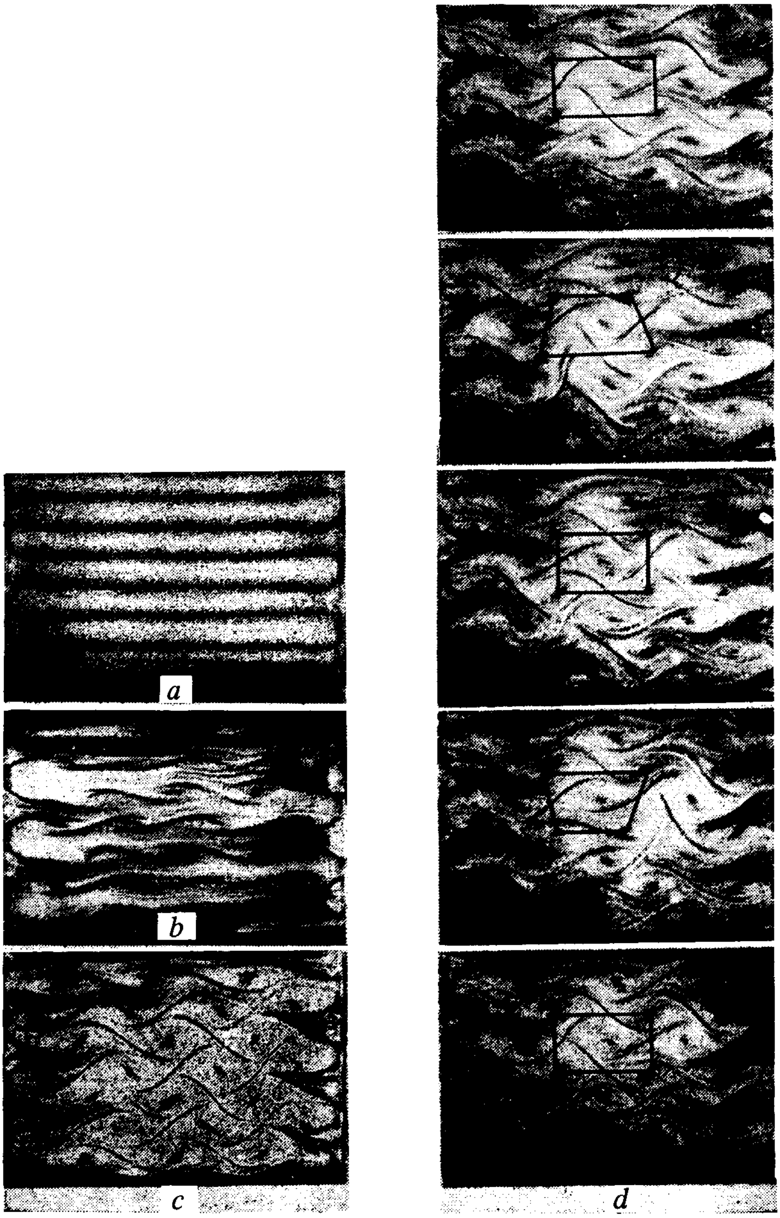

Fig.3. Generation of disturbances in a periodic shear flow: a) a subcritical stationary flow with $I = 0.45$ A (on the right is the velocity profile); b) secondary stationary flow with $I = 1.4$ A; c) secondary stationary flow with $I = 2.5$ A; d) cinema frames of the self-oscillating mode for $I = 2.9$ A and $t = 0, 3, 5, 8$, and 11 sec.

expected; this must correspond to the appearance of a "strange attractor", in modern terminology.[(1)] There was no special investigation of this mode in the experiments described.

Researchers are gradually getting used to the idea that turbulence is a phenomenon that is far less chaotic than was thought 30 or 40 years ago. In the initial stage of development at least, it has much in common with ordered structures studied in crystallography. The vortex parquet arising on the background of a Kolmogorov flow is quite amenable to computation; the difficulties here are not fundamental and can be overcome by using modern computational techniques.

It is also possible to calculate the mean effect of momentum and heat transfer (in the direction transversal to the basic flow) of a steady-state system of elliptic vortices, which is of immediate interest for applications. The comprehensive investigation of Kolmogorov flows (which cannot yet be regarded as complete) has turned out to be extremely useful in this scheme.

All this testifies to the definite success of the dynamical approach to the study of turbulence, although we are still very far from a purely theoretical computation of the transfer characteristics in a developed turbulent flow. The methods presently used in practice for such computations unavoidably contain certain numerical coefficients (the Kármán constant, the coefficient in the Kolmogorov formula, and so on) that are determined on the basis of experimental data; so far it has not been possible to compute them theoretically.

For sufficiently large Reynolds numbers a developed turbulence is characterized by a number of properties (continuity of the time spectrum, ergodicity) that do not appear in the weakly supercritical case, though the effect of a sharp increase in the transport properties of the flow (momentum and heat transfer) takes place already at this stage.

In formulating the main propositions of the local structure theory of a developed turbulent flow [1], Kolmogorov started with a cascade model containing a sufficiently large number of cascades ("levels") characterized by a sequence of scales: from the large-scale, comparable to the diametrical size of the system, to the very small-scale, of the order of the internal scale

$$\lambda_k = \sqrt[4]{\nu^3/\varepsilon},$$

which is determined by the kinematic viscosity ν and the mean dissipation of energy per unit mass ε.

In purely qualitative form the idea of a cascade mechanism for transforming energy in a turbulent atmosphere was formulated as early as 1922 by the famous English meteorologist Richardson in a quatrain now cited frequently

[(1)]Strange attractors are presently receiving much attention, and the number of papers on this question is increasing rapidly. A number of articles in the collection [18] deal with the concept of a strange attractor in connection with the structure of turbulent flows.

(for example, in the monograph of Monin and Yaglom):

Big whorls have little whorls,
which feed on their velocity;
and little whorls have lesser whorls,
and so on to viscosity.

Study of the loss of stability mechanism for actual hydrodynamic models (including Kolmogorov flows) by using the methods of nonlinear mechanics has promoted a better understanding of the process by which vorticial modes of different scale interact.(1)

It is of interest to construct simplified dynamical models that imitate the whole turbulent cascade whose statistical description was so brilliantly given by Kolmogorov more than 40 years ago.

One of the attempts at constructing a discrete model of turbulence was undertaken by the author on the basis of his concept of systems of hydrodynamic type ([19], [20]). A system of hydrodynamic type (SHT) is defined to be a system of ordinary quadratically nonlinear differential equations that are defined in n-dimensional linear phase space by

$$\dot{v}^i = \frac{1}{2}\Gamma^i_{jk} v^j v^k,$$

admit a positive-definite energy integral $E = (1/2) g_{ik} v^i v^k$ $(dE/dt = 0)$, and preserve phase volume (the Liouville condition: $\partial \dot{v}^i / \partial v^i = 0$). The dynamical tensor Γ^i_{jk}, in addition to the obvious condition of symmetry in the lower indices, satisfies two other restrictions: $\gamma_k = \Gamma^i_{ik} = 0$ ("regularity": preservation of phase volume), and $\Gamma_{i,jk} + \Gamma_{k,ij} + \Gamma_{j,ki} = 0$, a cyclic relation expressing the law of conservation of energy. Lowering of indices is accomplished by means of the tensor g_{ik}, which determines a Euclidean metric in the phase space of the system. For a fixed tensor g the sum of the dynamical tensors of two SHT's gives the tensor corresponding to some system of the same type, since the additional conditions are linear. This "superposition principle" enables us to construct complicated systems from simpler "blocks". It can be proved that the simplest nontrivial SHT is a triplet: a system isomorphic to the classical Euler gyroscope, whose equations of motion are conveniently written in the form

$$\frac{dv_0}{dt} = p\,(v_1 - v_2^2), \quad \frac{dv_1}{dt} = -p v_0 v_1, \quad \frac{dv_2}{dt} = p v_0 v_2.$$

If the phase coordinates v_0, v_1, and v_2 have the dimension of velocities (the energy E per unit mass has the dimension of velocity squared), then the interaction coefficient p has the dimension of a wave number (the reciprocal quantity can be called the "dynamical scale" of the triplet). We remark

(1)A vorticial mode is defined to be a solenoidal vector field satisfying the homogeneous boundary conditions assumed for the velocity field. One of the modes is excited by the external force (field), and this ensures the influx of energy to the system.

that v_0 corresponds to an unstable mode of the system. Systems of hydrodynamic type arise naturally in approximating the equations of hydromechanics for actual hydrodynamic models by the Galerkin method. In the general case we should introduce additional dissipative terms proportional to the viscosity and take into account the effect of the external forces.

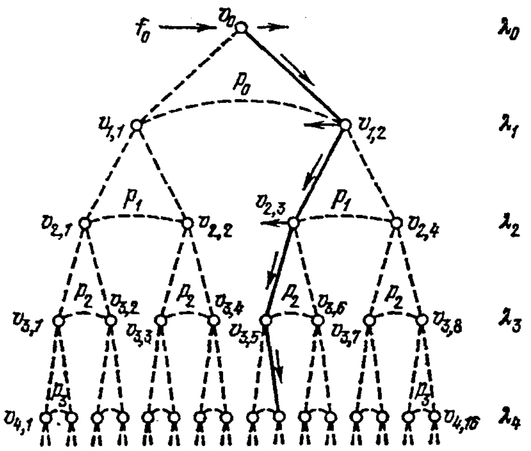

Fig.4. A discrete model of a nonlinear cascade.

Figure 4 gives a schematic representation of the simplest discrete model of a nonlinear cascade; it is constructed from similar triplets of different scales $l_1 = 1/p_i$, linked together in a definite way. The corresponding equations of

$$\dot{v}_0 = p_0(v_{1,1}^2 - v_{1,2}^2) - \lambda_0 v_0 + f_0,$$

$$1\left\{\begin{array}{l}\dot{v}_{1,1} = -p_0 v_0 v_{1,1} + p_1(v_{2,1}^2 - v_{2,2}^2) - \lambda_1 v_{1,1},\\ \dot{v}_{1,2} = p_0 v_0 v_{1,2} + p_1(v_{2,3}^2 - v_{2,4}^2) - \lambda_1 v_{1,2},\end{array}\right.$$

$$2\left\{\begin{array}{l}\dot{v}_{2,1} = -p_1 v_{1,1} v_{2,1} + p_2(v_{3,1}^2 - v_{3,2}^2) - \lambda_2 v_{2,1},\\ \dot{v}_{2,2} = p_1 v_{1,1} v_{2,2} + p_2(v_{3,3}^2 - v_{3,4}^2) - \lambda_2 v_{2,2},\\ \dot{v}_{2,3} = -p_1 v_{1,2} v_{2,3} + p_2(v_{3,5}^2 - v_{3,6}^2) - \lambda_2 v_{2,3},\\ \dot{v}_{2,4} = p_1 v_{1,2} v_{2,4} + p_2(v_{3,7}^2 - v_{3,8}^2) - \lambda_2 v_{2,4},\end{array}\right.$$

. . . .

$$\dot{v}_{i,2s-1} = -p_{i-1} v_{i-1,s} v_{i,2s-1} + p_i(v_{i+1,4s-3}^2 - v_{i+1,4s-2}^2) - \lambda_i v_{i,2s-1},$$

$$\dot{v}_{i,2s} = p_{i-1} v_{i-1,s} v_{i,2s} + p_i(v_{i+1,4s-1}^2 - v_{i+1,4s}^2) - \lambda_i v_{i,2s}$$

$$(1 \leqslant s \leqslant 2^{i-1}),$$

where $\lambda_i = \alpha^2 \nu p_i^2$ are the dissipative coefficients and α is a factor of the order of unity.(1)

In the dynamical variables $v_{i,j}$ the first index numbers the "levels" corresponding to the sequence of p_i and λ_i, and the second index numbers the components of the triplet located on the given level.

The quantity $E_i = (1/2)\sum_j v_{i,j}^2$ corresponds to the energy associated with a disturbance of a given scale $l_i = 1/p_i$.

Since the equations of motion of an ideal fluid admit a group of similarity transformations, it can be assumed (according to the main hypothesis of Kolmogorov on self-similarity of the cascade) that the quantities p_i form a geometric progression. The ratio

$$q = \frac{p_{i+1}}{p_i}$$

is a basic invariant characteristic of the discrete cascade. As proposed by Kolmogorov, it can be called the "coefficient of refinement" of the turbulent disturbances.

In comparison with a continuous energy spectrum, E_i should be regarded as the energy falling on an "octave" (or a definite part of an octave) in the space of frequencies k.

The total energy $E = (1/2)\sum_{i,j} v_{i,j}^2$ satisfies the balance equation

$$\frac{dE}{dt} = W_0 - \Phi,$$

where $W_0 = f_0 v_0$ is the power input and $\Phi = \sum_{i,j} \lambda_i v_{i,j}^2$ is the energy dissipation.

When the external force is "switched on" very slowly, the excitation process can be represented as follows. First, v_0 increases, with the sign of v_0 the same as that of f_0. Then, when $|f_0|$ exceeds the value $\lambda_0 \lambda_1 / p_0$ for which the ground state loses stability, one of the components of the first level is excited: $v_{1,1}$ (if $v_0 < 0$) or $v_{1,2}$ (if $v_0 > 0$). The sign of any excited mode (except for v_0) depends entirely on small initial fluctuations (primings), that is, it is random and at the same time determines the directions of subsequent evolution of the excitation process for the chain. One such excited branch is shown in Fig. 4 as a solid line. The development of excitations on a certain level is accompanied by a growth of the corresponding quadratic

(1)In the spectral representation $\lambda = \nu k^2$, where k is the wave number. For the reference (coordinate) vector fields we can introduce some mean (effective) value of the wave number on the basis of the known expression for the dissipation of energy. At the same time, p is determined by solving the problem of hydrodynamic instability with respect to the maximal increment of increase in the disturbances, which is proportional to the amplitude of the "generating mode". From similarity considerations, $k\alpha$ and p (which have the same dimension) are assumed to be approximately proportional. Actually, α can be essentially larger than unity. For simplicity we can formally set $\alpha = 1$, understanding ν to be a certain "effective" viscosity.

term on the preceding level, which simulates the Reynolds stress. It follows from the equations that the component of the $(i+1)$st level is excited as soon as the intensity at the i-th level exceeds the critical value λ_{i+1}/p_i. For a fixed value of f_0 there is a limiting level above which excitations do not develop.

The value of p_n for this level, which corresponds to the reciprocal value of the Kolmogorov scale, along with the values of the squares of the amplitudes, can be computed by considering the steady-state solutions of the equations of motion for a simple excited chain when there is only one excited mode with amplitude v_i at each level.

Without loss of generality all the v_i are assumed to be positive. The equations of motion of such a chain can be written in the form

$$
\begin{aligned}
\dot{v}_0 &= -p_0 v_1^2 - \lambda_0 v_0 + f_0, \quad f_0 > 0,\\
\dot{v}_1 &= p_0 v_0 v_1 - p_1 v_1^2 - \lambda_1 v_1,\\
&\dots\dots\dots\dots\dots\dots\\
\dot{v}_{i-1} &= p_{i-2} v_{i-2} v_{i-1} - p_{i-1} v_i^2 - \lambda_{i-1} v_{i-1},\\
\dot{v}_i &= p_{i-1} v_{i-1} v_i - p_i v_{i+1}^2 - \lambda_i v_i,\\
&\dots\dots\dots\dots\dots\dots\\
\dot{v}_n &= p_{n-1} v_{n-1} v_n - \lambda_n v_n,\\
v_{n+1} &= O(\varepsilon) \simeq 0
\end{aligned}
$$

with the supplementary condition that the chain breaks off:

Among the steady-state solutions of this system is the amplitude distribution corresponding to the "two-thirds law":

$$v_i^0 = q^{2/9} W^{1/3} p_i^{-1/3} \quad \text{for } i < n.$$

Numerical experiments with the simplest cascade models have shown, however, that steady states of the simple chain type cannot exist for a long time due to the instability of such chains. Glukhovskii ([21], [22]) made a special investigation of this question and showed that steady states of the simple chain type can be unstable even for $n \geqslant 3$. In [22] an interesting connection is established between the phenomenon of instability of chains and the fragmentation coefficient q. For small values of q ($q < q_0 = 2\sqrt{2}$) all sufficiently long chains are unstable. For $q \geqslant q_0$ long stable chains can exist, but they have very narrow stability ranges (with respect to the parameter f_0). Numerical experiments show that destruction of a long chain takes place when, as a result of a "build-up" in one of the links, a "flip" takes place: the corresponding mode $v_{i,j}$ changes sign. Then the higher modes connected with it ($i' > i$) are deprived of an influx of energy and rapidly die out, while the process now develops in a new direction, beginning with $v_{i,j}$.

Therefore, because of the phenomenon of instability of long chains the model described above nevertheless reflects (albeit roughly) the phenomenon of intermittency of very small-scale turbulence, as Kolmogorov pointed out in his report at the 1961 symposium in Marseilles [23]. The report noted that local fluctuations of the dissipation (the intermittency) can lead to a certain deviation of the energy spectrum from the "two-thirds law". The author's article [24] also deals with this question. A number of investigations appearing after 1961 showed that the phenomenon of intermittency is very essential in the study of the higher moments of the difference in the velocities [25]. As for the energy spectrum, it can be well approximated by the "two-thirds law" with an accuracy of order 5% (the law is $-5/3$ for the spectral density). This is indicated by direct measurements of spectra for turbulence in the atmosphere and the ocean, as well as results of numerical experiments with simplified cascade models.

The problem of designing statistical methods for the description and dynamic simulation of turbulence is now attracting a great amount of attention from investigators of diverse types, including mathematicians, theoretical physicists, specialists in plasma physics, hydromechanics, and geophysics, and also astrophysicists.

The personal contribution of Kolmogorov to the study of turbulence and his ideas relating to the general theory of dynamical systems are fundamental reference points in the development of investigations of the most complex phenomenon in nature, namely, turbulence, in connection with diverse areas of knowledge.

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