APA with Evolving Order and Variable Regularization for Echo Cancellation

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Abstract—Recently APA has become one of most popular algorithms in application of Acoustic Echo Cancellation. Because of the contradictory factors of convergence rate and steady-state misalignment, a new algorithm by the behavior of associating variable regularization and evolving order has been proposed in this paper. Despite of the conventional assumption that the a posteriori error is zero, we take the statistical characteristic of the noise into consideration during the adaptation process. Exact and approximate formulations for the optimal regularization factor are derived. Numerical simulation results show that the proposed algorithm improves the performance of the APA in terms of its faster convergence rate and lower steady-state misalignment compared to existing variable regularization APA and evolving order APA, respectively. Meanwhile it can be seen that near-end speech signal has been restored more effectively.

Index Terms—Variable Regularization; Evolving Order; APA; Acoustic Echo Cancellation

I. INTRODUCTION

The acoustic echo is mainly generated by transferring far-end speech signal to the local to amplify and put out through the speaker, then the signal is picked up by microphone and transmitted to the far-end together with the local voice signal. With the development of communication technology, the communication quality is constantly improved to satisfy the requirement of people’s living standard. Whereas, echo which not only affects our call quality but also makes a normal call impossibly is engendered inevitably through the communication [1]. The basic principle of the echo canceller is that an adaptive filter is utilized to imitate the echo path. Then the impulse response will approach to the actual echo path through adjusting the adaptive filtering algorithm so that we can gain predicted echo signal. In final, echo cancellation will be realized in the way of subtracting the predicted echo signal from voice signal received from the microphone. Adaptive echo cancellation technology is internationally recognized as the most promising technologies, but also the practical application of the most used technologies. At present, there are already a variety of adaptive echo cancellation algorithms and solutions based on high-speed digital signal processing chips in reference [2].

Affine projection algorithm (APA) has many applications in our real life such as wireless channel equalizer, echo cancellation, noise cancellation and speech enhancement and so on in reference [3]. Particularly in echo cancellation, APA has presented a perfect performance. The APA has been advanced optimized and its high performance has been required due to the diversity and complexity of the communications environment, as well as taking into account the characteristics of the adaptive filter.

APA adopts the method of repeating use of the signal sample values. The higher the correlation of input data is, the faster the convergence speed will be. However, there is larger amount of calculation and worse steady-state misalignment than the NLMS algorithm in APA iterative process in the reference [4]. It was shown in reference [5] that a variable regularization factor APA could greatly reduce the steady-state error. An evolving order APA namely E-APA proposed in reference [6] had adjusted the current order based on steady-state mean square error, which could improve the contradiction between the fast convergence rate and the low steady-state misalignment and reduce the calculation greatly. In reference [7], a data selective segment proportionate affine projection algorithm for echo cancellation has been proposed by combining proportionate adaption with the framework of set-membership filtering in order to decrease the computational complexity of the algorithm. The frequency of updates of the filter coefficients has been reduced, where the filter coefficients are updated so that the output estimation error is upper bounded by a predetermined threshold. In reference [8], an improved set-membership affine-projection adaptive-filtering algorithm has been proposed. The proposed paper utilized two error bounds one of which is used to realize faster convergence and the other is used to suppress impulsive-noise interference. Through this way, the misalignment is reduced.

This article derives a new affine projection algorithm in the way of coordinating variable regularization in reference [5] and evolving order in reference [6], because
we have found that variable regularization can reduce the misalignment and the evolving order can speed up the convergence. We also incorporate the statistical characteristic of the noise into the adaptation process.

Moreover we will test and confirm the performance of the algorithm proposed in this paper from three aspects of convergence rate, steady-state misalignment and computational complexity [10]. Convergence rate refers to iteration time or number of iteration of the algorithm from the beginning to the stable stage. Convergence speed has a high relationship with the step size that we have chosen in the affine projection algorithm. Practically, convergence rate can be controlled by the selected step size [11]. The judgment for the iteration time should be based on the hardware environment. The steady-state misalignment provides the degree of deviation between the final mean square error adaptive algorithm and minimum mean square error generated by Wiener filter for an interesting algorithm [12]. We can conclude that step size and the order of the algorithm have a great effect on steady-state misalignment. Computation of an algorithm is the memory size used or occupied by the cost of a full iteration and storage of data and program. The greater the amount of calculation is, the higher the requirement of algorithm hardware environment will be, so the construction costs will be even great [13].

The paper is organized as follows. Section II give a brief introduction of the conventional APA model where will introduce its basic conception. In Section III, the optimization APA is provided including variable regularization, evolving order and its combination. The simulation results will be depicted in Section IV and conclusions are presented in Section V.

II. CONVENTIONAL APA MODEL

Many algorithms will appear to the problem of slow convergence rate when the input signal is colored in the adaptive filter. Data reuse algorithms are considered to be one of methods to improve the convergence rate of the adaptive filter algorithm in case of the relevance for input data [14]. Affine projection algorithm (APA) is one of data reuse algorithms; APA which evolves from the minimum mean square algorithm is also a normalized least mean square algorithm. Actually, APA is derived from least mean square algorithm (LMS) [15]. LMS is a searching algorithm where the calculation of the gradient vector is simplified by adjusting the objective functions appropriately. It is because of simple calculation that LMS algorithm and its associated algorithms have been widely used in reference [16].

We first define the adaptive filter coefficients \( \mathbf{w}(n) = [w_0(n), w_1(n), \ldots, w_{L-1}(n)]^T \) and the input signal vector \( \mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-L+1)]^T \) where the superscript \( T \) denotes the transposition, \( d(n) \) is the desired signal, \( K \) is the order of the APA, \( L \) is the length of unknown FIR system, \( A(n) = [\mathbf{X}(n), \mathbf{X}(n-1), \ldots, \mathbf{X}(n-K+1)]^T \) is \( L \times K \) affine projection matrix, \( \mathbf{w}_\text{opt} \) is the unknown matrix for the filter to be estimated. Suppose \( \mathbf{w}(n) \) is an estimation of \( \mathbf{w}_\text{opt} \), at iteration \( n \), \( n \) is the time index, \( \mathbf{v}(n) \) is the noise signal.

The basic formulas of the affine projection algorithm [17], [18] are as follows:

\[
d(n) = y(n) + v(n) = X^T(n)w_{\text{opt}} + v(n) \quad (1)
\]

The updating formula of classic filter coefficient is:

\[
w(n) = w(n-1) + \mu A(n)(\mathbf{A}^T(n)A(n) + \delta I)^{-1} \mathbf{e}(n) \quad (2)
\]

The error formula is:

\[
\mathbf{e}(n) = d(n) - \mathbf{A}^T(n)w(n-1) \quad (3)
\]

\( I \) is a \( K \times K \) identity matrix. \( \mu \) is the step size. Step size acts to be an important part in iteration. When the step size is big, the algorithm will converge very fast, but steady-state misalignment will increase. When step size is small, the convergence is slow, but the final misalignment is small. Let its value be 1 here. \( \delta \) is the regularization factor which is employed to avoid the inversion of possibly rank-deficient matrix \( \mathbf{A}^T(n)A(n) \). What’s more, it plays a significant role between the convergence rate and the steady-state misalignment of the conventional APA. On the one hand, a big value of step size will be obtained, responding to a small value of regularization factor, at the same time, the speed of the convergence will increase. However, this will lead to high steady-state misalignment. On the other hand, a large value of the regularization factor will result in a small step size, slow convergence rate and of course a low misalignment. In order to solve the contradiction of convergence speed, steady-state misalignment and computation of affine projection algorithm, this paper tries to figure out this problem from the aspect of variable parameters. Consequently, we expect to improve the performance of the APA by using a variable regularization factor.

III. OPTIMIZATION OF APA

In the traditional algorithm, the posteriori error was generally set as zero. It meant that noise was ignored. However, the statistical properties of the noise will be taken into account during the adaptive process in this paper. On the one hand, we want to minimize the steady-state error in order to increase the stability of convergence by adjusting the regularization factor [19]. On the other hand, it is expected to speed up the convergence rate of the algorithm and reduce the computational complexity by the way of regulating projection order. Although the larger projection order is the faster the convergence rate will be, the steady-state error also increases. The proposed algorithm has better performance through the synergistic effect of the variable regularization factor and evolving order [20], [21].

In order to increase the stability and decrease the steady-state error of the algorithm, it uses the variable \( \delta(n) \) to modify the \( \delta \); so formula (2) can be rewritten as

\[
w(n) = w(n-1) + \mu A(n)(\mathbf{A}^T(n)A(n) + \delta(n)I)^{-1} \mathbf{e}(n) \quad (4)
\]

By defining \( S(n) = (\mathbf{A}^T(n)A(n) + \delta(n)I)^{-1} \), then \( w(n) = w(n-1) + A(n)S(n)e(n) \). We can name \( e(n) \) the a priori error and \( \epsilon(n) = d(n) - \mathbf{A}^T(n)w(n) \) the a posteriori error. So the
The relationship of $\varepsilon(n)$ can be expressed as follow:

$$\varepsilon(n) = A^T(n)(w_{opt} - w(n)) + v(n).$$

Ideally supposing that posteriori error $\varepsilon(n) = 0$, a priori error $\varepsilon(n)$ is zero, and the assumption reveals that the reduction of the expectation of the $l_2$ norm of systematic error should be minimum. We can have the formula of $\varepsilon(n)$ through simplification. Actually the noise signal is always present. It is obvious that $\varepsilon(n)$ will be minimum only when $w_{opt} = w(n)$.

So we get $\varepsilon(n) = v(n)$. Therefore, the solution for $\delta(n)$ is obtained such that $E[\xi(n)\xi^T(n)] = E[\nu(n)\nu^T(n)]$. And we can reach to the following equation:

$$E[\|v(n)\|^2] = E[\xi^T(n)(I - A^T(n)A(n)S(n))^T(1 - A^T(n)A(n)S(n))\xi(n)]$$

(5)

The equation will be simplified by means of eigenvalue decomposition. The eigenvalue decomposition of the Gram matrix $A^T(n)A(n)$ is given by

$$A^T(n)A(n) = U(n)\Lambda(n)U^T(n)$$

(6)

where $A(n)$ is the diagonal matrix formed with the Eigenvalues of $A^T(n)A(n)$, and $U(n)$ are eigenvectors of $A^T(n)A(n)$. So the matrix $S(n)$ can be given as

$$S(n) = U(n)\Lambda(n) + \delta(n)I^{-1}U^T(n)$$

(7)

The diagonal matrix whose kth diagonal element is $(\delta(n)/\|\lambda_k(n) + \delta(n)\|)k$ is expressed as

$$Q(n) = (I - \Lambda(n)(\Lambda(n) + \delta(n)I)^{-1})^2$$

(8)

The relationship of noise signal and posteriori error is obtained by putting (6), (7) and (8) into (5)

$$E[\|v(n)\|^2] = E[\xi^T(n)U(n)Q(n)U^T(n)e(n)]$$

(9)

Which mostly relies on the decomposition of Eigenvalues of the matrix $A^T(n)A(n)$ from the formula derivation process. So it is apparently impossible to obtain the variable regularization factor through this method. However, it can be approximated in other ways. It also can be seen that the kth diagonal element of $A^T(n)A(n)$ is the $l_2$ norm of the input signal vector, so an exploratory approximation has been given as

$$\lambda_k(n) \approx L\sigma^2_k(n) \quad 1 \leq k \leq K$$

(10)

$\sigma^2_k(n)$ is the variance of input signal. Substituting this formula into (9) results in

$$\delta(n) = \frac{L\sigma^2_k(n)\sqrt{E[\|v(n)\|^2]}}{\sqrt{E[\|v(n)\|^2] - E[\|e(n)\|^2]}}$$

(11)

From (11), it can be perceived that the value of a priori error $\varepsilon(n)$ is very large at the beginning stage of the adaption, and the value of $E[\|e(n)\|^2]$ is large, so the value of $\delta(n)$ is small corresponding to the fast convergence rate. While during the adaptive process $E[\|v(n)\|^2]$ is gradually play an important role, the value of $\delta(n)$ increases, of course the coefficients of the adaptive filter start to adjust slowly and convergence rate is steady.

In general, $\delta(n) \geq 0$ because of $\|e(n)\| < \|v(n)\|$. When the algorithm begins to converge, the distance between $\|e(n)\|$ and $\|v(n)\|$ will be small and $\delta(n)$ may turn to be negative in reference [11]. So we set $\delta(n) = \delta(n-1)$ to avoid $\delta(n) < 0$.

In this paper it will also use the formulas as follows:

$$E[\|v(n)\|^2] = K\sigma^2_v(n)$$

$$\sigma^2_v(n) = \frac{1}{T}\text{Tr}\left\{A^T(n)A(n)\right\}$$

$$\delta^2_v(n) = \alpha\delta^2_v(n-1) + (1 - \alpha)e^2(n)$$

(12)

The steady-state error will be anabolic with the guarantee of variable regularization factor. What's more, there is a fact that fast convergence is our perpetual and unchanging pursuit. The algorithm in this paper will continue to explore the problem of the convergence rate. The larger the projection order is, the faster the convergence will be, but the greater the steady-state error is, the higher the computational complexity will be in APA in reference [22]. Variable order APA adjusts the current order according to the steady-state mean square error, which not only solves the contradiction between convergence rate and steady-state misalignment effectively, but also can reduce the amount of computation greatly in reference [23].

At the beginning of iteration of the algorithm, the order of the input matrix is large to ensure the convergence rate is high. At the Stable stage, the order decreases which can result in less steady-state misalignment and computational cost [24], [25].

If the order is available in APA, we can set the evolving order as $K_i$ at the time $i$ so, the formula should be rewritten as:

$$\delta_{k_i}(n) = \frac{L\sigma^2_{k_i}(n)\sqrt{E[\|v_{k_i}(n)\|^2]}}{\sqrt{E[\|v_{k_i}(n)\|^2] - E[\|e(n)\|^2]}}$$

(13)

of course the formula has to do a corresponding change when it relates to the order. We will adopt the formula of $K_i$ in [6]:

$$K_i = \begin{cases} \min(K_{i-1} + 1, K_{\text{max}}), \eta_i < e^2(n) \\ K_{i-1}, \eta_i < e^2(n) \leq \theta_i \\ \max(K_{i-1} - 1, 1), e^2(n) \geq \theta_i \end{cases}$$

(14)

where $\eta_i$ is the error limit in the $i$th iteration process and $\theta_i$ is the error floor. The range of $K_i$ presents $1 \leq K_i \leq K_{\text{max}}$. $K_{\text{max}}$ is the maximum projection order, and $K_{\text{max}} \leq L$. During the calculation process of the algorithm, the formula of $\eta_i$ and $\theta_i$ also employs the expression in [6], showing as:
\[
\begin{align*}
\eta_i &= \delta_i^2 \frac{\mu(i-1)K_{i-1} + 2}{2 - \mu(i-1)} \\
\theta_i &= \delta_i^2 \frac{\mu(i-1)(K_{i-1} + 2)}{2 - \mu(i-1)}
\end{align*}
\]  

(15)

\[
\delta 2\nu \text{ is the measured noise variance. Actually we call it as "Evolving order with variable step-size affine projection algorithm (EVSSAPA)". Considering that the step-size in the proposed algorithm is always 1, so } \eta_i \text{ and } \theta_i \text{ can be rewritten as:}
\]

\[
\begin{align*}
\eta_i &= \delta_i^2 (K_{i-1} + 2) \\
\theta_i &= \delta_i^2 (K_{i-1} + 1)
\end{align*}
\]  

(16)

From the analysis of the algorithm of evolving order, it will be found that there is an important relationship between } \eta_i, \theta_i \text{ and steady-state misalignment which is relied on the projection order in the meantime. The order of iteration will decrease one to obtain small steady-state misalignment when the output error is less than } \theta_i. \text{ Moreover the order of iteration will plus one to gain fast convergence rate when the output error is more than } \eta_i \text{ [26].}

As can be seen from the final formula (13), when the state stay in the initial stage of iteration, error is relatively large and regularization factor is high with large order of the algorithm, which results in fast convergence. When the algorithm has converged, the order decreases so that the steady-state misalignment reduces too. Compared to the traditional APA, the order of this algorithm has greater flexibility. At the beginning of the adaption, there is a considerable computational cost in the proposed algorithm just like traditional APA [27], [28], [29]. The computational complexity is } O(K^2 \lambda) \text{ for the traditional affine projection algorithm in one of iterations. The complexity of this algorithm is } O(K^2 \lambda). \text{ But in the adaptation proceeds, the computational cost will be reduced greatly accompanied with the adaption of the order. So the algorithm proposed in the paper shows superiority.}

IV. SIMULATION RESULTS

The voice speech from IEEE-AP database acts as an object of simulation. The effect of echo cancellation and the convergence performance will be compared and analysis with the proposed algorithm and the algorithm in [5], [6]. SNR=25dB, } \alpha=0.998, \mu=1, L=64, \text{ the beginning order is 30, and the misalignment will be evaluated by using:}

\[
MIS = E \left( \frac{\|w_{opt} - w(n)\|^2}{\|w_{opt}\|^2} \right).
\]

In order to make the algorithm more persuasive, we consider some simulations. Before using the real speech we consider the AR and ARMA signals which are obtained by filtering a white zero-mean Gaussian random sequence through a first order system the AR(0.8) and ARMA(2, 2) signals whose formulas are } G_1(z)=1/(1-\rho_1 z^{-1}) \text{ and } G_2(z)=(1+0.5\bar{z}^{-1}+0.81 \bar{z}^{-2})/(1-0.59 \bar{z}^{-1}+0.4\bar{z}^{-2}) \text{ respectively, and } \rho_1=0.80. \text{ The noise } \nu(n) \text{ is white Gaussian.}

Fig. 1 shows the speech signal and the near-end speech signal superimposed the analog echo signal. The simulation in Fig. 2 reveals the channel impulse responses whose input signal is far-end speech signal. And the impulse response order is 64.

![Speech signal and Speech signal coupled with echo signal](image1)

**Figure 1.** Speech signal and Speech signal coupled with echo signal

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**Figure 2.** Simulated echo path by using speech signal

Fig. 3 shows the convergence curve of AR (0.8) for three algorithms. It can be seen that variable regularization APA converges quickly compared to evolving order APA, but the magnitude of convergence of variable regularization APA is large. Moreover the convergence rate of the proposed algorithm is faster than algorithms both in [5] and [6]. The steady-state error of the proposed algorithm are both less than the others apparently. In comparison, Fig. 4 shows the algorithm has a faster convergence speed; the steady-state error of the proposed algorithm is slightly worse than the performance in [6] but significantly better than its behavior in [5]. The two simulations of Fig. 3 and Fig. 4 show the superiority of the proposed algorithm.

As is depicted in Fig. 5, there are three convergence curves of the algorithm in this paper and in reference [5], [6]. According to the convergence rate, in the initial stage of algorithm, relatively, the convergence rate is lower in terms of variable regularization in reference [5]. While compared to the evolving order algorithm in reference [6], the proposed algorithm has a slight advantage and a great benefit over the both. However, in the stable state of
adaption, the magnitude of the convergence curve in reference [5] is higher than the magnitude in reference [6], but both are much lower than the magnitude of the proposed algorithm. Considering the steady-state misalignment, in the entire convergence process, the error in reference [5] tends to be maximum, the error of the proposed algorithm is minimum, and the algorithm is also comparatively stable under the same conditions, which corresponds to a better stability of the system during echo cancellation, the performance and the effect of the echo cancellation is better.

Fig. 6 demonstrates the near-end speech signal obtained after the echo cancellation by the way of the variable regularization factor, evolving order and the proposed algorithm, respectively. It can be concluded from the Fig. 6 compared to Fig. 1 that the variable regularization factor algorithm has poor results; the effect of the proposed algorithm is best which is able to get more realistic near-end speech signal.

V. CONCLUSION

Nowadays along with people's continuously improving requirements on call quality, more and more attention has been paid to the performance of the echo cancellation. Actually APA has a very important position in the echo cancellation. In connection with the status of low convergence and large amount of calculation for the existing APA algorithms, this paper proposes an algorithm combined with variable order and variable regularization factor. The convergence rate has been speeded up by changing the order of the algorithm. And the steady-state misalignment will be reduced by introducing the variable regularization factor during convergence, which makes the algorithm more stable. The method in this paper improves the performance of fast convergence rate and small steady-state error. The simulation results depicted that the algorithm has a good performance in convergence speed, steady-state misalignment and the amount of calculation in echo cancellation, which can better satisfies the needs of the people on the call quality compared with the existing algorithms.

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[16] Changpeng Ji received the B.E. and M.S. degrees, in computer engineering from Liaoning Technical University of China, Fuxin, China, in 1993 and 2002. Professor of Liaoning Technical University, master instructor. The main research direction: computer communication & networks, signal detection and estimation, wireless communication.