CONSTANT WALL TEMPERATURE FORCED CONVECTION IN PIPES WITH THE SIMPLIFIED PHAN-THIEN—TANNER FLUID

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Abstract. *The fully-developed thermal and hydrodynamic steady laminar pipe flow of the SPTT fluid is here investigated for the constant wall temperature boundary condition, assuming constant properties and negligible axial conduction. Two limiting conditions were identified: the solution pertaining to the equilibrium between axial convection and radial conduction of thermal energy, and the solution of the equilibrium of radial conduction of energy and energy production by viscous dissipation. Whereas for the first problem the solution was obtained by a successive approximation method, yielding results within 0.3% of the exact solution, for the second problem the final result is exact.*

Keywords: *viscoelastic, viscous dissipation, PTT fluid*

1. Introduction

The simplified Phan-Thien—Tanner (SPTT) constitutive equation, given in Eq. (1), is a reduced version of the full PTT model of Phan-Thien and Tanner (1977), which was derived from considerations of network theory. The simplification involved is to consider only affine motions of the polymer molecules relative to the continuum.

$$Y(\mathrm{tr}\,\tau, T)\tau + \lambda \overset{\nabla}{\tau} = 2\eta \mathbf{D} \tag{1}$$

In Eq. (1) $\overset{\nabla}{\tau}$ stands for Oldroyd's upper convected derivative of the stress tensor τ, as defined by Eq. (2), λ is a relaxation time, η is a viscosity coefficient and $\mathbf{D}$ is the rate of strain tensor.

$$\overset{\nabla}{\tau} = \frac{D\tau}{Dt} - \tau.\nabla\mathbf{u} - \nabla\mathbf{u}^{\mathrm{T}}.\tau \tag{2}$$

The stress-coefficient function Y is related to the rate of destruction of junctions in the molecular network and can be decoupled as

$$Y(\mathrm{tr}\,\tau, T) = \phi(T)\, f(\mathrm{tr}\,\tau) \tag{3}$$

where T is the temperature and tr τ is the trace of the stress tensor τ. Following Phan-Thien (1978) the function $\phi(T)$ is arbitrarily set to unity at the reference temperature in isothermal problems and here we generalise this assumption. Furthermore, although the stress-coefficient has an exponential form we adopt its linearization because the pipe flow is a weak flow according to Tanner's (1985) classification. So,

$$f(\mathrm{tr}\,\tau) = 1 + \frac{\varepsilon\lambda}{\eta}\mathrm{tr}\,\tau \tag{4}$$

In Eq. (4) ε is a parameter related to the elongational behaviour of the model. It imposes an upper limit to the elongational viscosity that is proportional to the inverse of ε (p. 227 in Huilgol and Phan-Thien, 1997), and the upper-convected Maxwell model, which has an unbounded elongational viscosity in simple extensional flow, is recovered when $\varepsilon = 0$.

Polymer melts flow at high temperature in polymer processing and are subject to various heat transfer processes. Polymer melt flows are characterised by very low Reynolds numbers (of the order of 10^{-3} to 10^{-5}) and high Prandtl numbers (10^6 to 10^8), thus leading to a very fast hydrodynamic flow development but rather slow thermal development. This combination yields a Peclet number ($Pe \equiv PrRe$) of the order of 3,000 and it is known that full thermal development for Newtonian fluids, under conditions of hydrodynamic fully-developed flow, requires normalised lengths (length/ transverse size) of $0.05Pe$ (Eckert and Drake, 1972). In extrusion heads the length of the parallel zone is well shorter than this critical length, but it is located at the end of a long, slightly tapered duct in the downstream half of which the flow is likely to be close to thermal and hydrodynamic fully-developed.

The analytical hydrodynamic solution of the pipe flow of an SPTT fluid was derived by Oliveira and Pinho (1999) who subsequently performed the analysis of the corresponding heat transfer problem for imposed constant wall heat flux (Pinho and Oliveira, 2000), taking into account the viscous dissipation. The present work extends this latter analysis to the two equally relevant cases of thermally and hydrodynamically fully-developed steady pipe flow with an imposed constant wall temperature: (1) that of fully developed thermal conditions in the presence of viscous dissipation, and (2) the asymptotic problem in a pipe in the absence of viscous dissipation.

In the next section the problem is formulated, the hydrodynamic solution is presented and the analytical procedure required to solve the heat transfer problem, with constant wall temperature, is outlined. In Section 3, the solutions obtained under the two asymptotic assumptions are presented and discussed.

2. Formulation of the problem

2.1. Fluid model and assumptions

In this steady, laminar, fully-developed pipe flow the fluid properties are taken as constants independent of temperature and the boundary condition is an imposed constant temperature T_w at the pipe wall.

We further make the simplifying assumption of isotropic thermal and thermodynamic properties, although it is known that they are anisotropic due to their intimate relation to molecular structure. It is further assumed that Fourier's law of heat conduction is valid and that the internal energy and thermal conductivity do not depend explicitly on the velocity gradient or other kinematic quantities. This is in agreement with previous heat transfer work relevant to polymer melts cited by Agassant et al (1991) and Bird et al (1987), who showed that consideration of property isotropy did not seriously affected the results.

Under these conditions the hydrodynamic and thermal problems become fully decoupled.

2.2. Hydrodynamic solution

The radial velocity profile for the pipe flow of an SPTT fluid is given in Oliveira and Pinho (1999) as

$$u^* \equiv \frac{u}{U} = 2\frac{U_N}{U}\left[1-r^{*2}\right]\left\{1+16\varepsilon We^2\left(\frac{U_N}{U}\right)^2\left[1+r^{*2}\right]\right\} \tag{5}$$

where stars designate nondimensional quantities and lengths were scaled by the pipe radius $\left(r^* = r/R\right)$. The nondimensional group $We = \lambda U/R$ is the Weissenberg number, a measure of the level of elasticity in the fluid, which is based on the cross-sectional average velocity U for the SPTT fluid. U_N is the average velocity for a Newtonian fluid flowing under the same pressure gradient dp/dx, $U_N \equiv -(dp/dx)R^2/8\eta$ and the ratio of both bulk velocities was shown to be given by:

$$\frac{U_N}{U} = \frac{432^{1/6}\left(\delta^{2/3}-2^{2/3}\right)}{6b^{1/2}\delta^{1/3}} \text{ with } \delta = \left(3^3 b+4\right)^{1/2}+3^{3/2}b^{1/2} \text{ and } b = \frac{64}{3}\varepsilon We^2 \tag{6}$$

The radial profiles of the shear stress τ_{xr} and the corresponding shear rate du/dr are required for the viscous dissipation term of the energy equation:

$$\tau^*_{xr} \equiv \frac{\tau_{xr}}{4\eta\, U/R} = -\frac{U_N}{U}r^* \quad ; \quad \frac{du/dr}{4U/R} = -\frac{U_N}{U}r^*\left[1+32\varepsilon We^2\left(\frac{U_N}{U}\right)^2 r^{*2}\right] \tag{7}$$

It will be convenient for the foregoing analysis to define a modified nondimensional group as $a \equiv 16\varepsilon We^2\left(U_N/U\right)^2$ which gives a measure of both the extensional and the elastic properties of the fluid. It should be recalled here from the hydrodynamic solution (Oliveira and Pinho, 1999) that normal stresses will depend directly on We and inversely on some power of εWe^2, whereas the shear stress will depend exclusively on εWe^2. The dependence on We is solely a normal stress effect but the dependence on εWe^2 combines a normal stress effect with the elongational parameter, and this combination imparts a shear-thinning behaviour to the viscosity function. It is in this context that the expression shear-thinning is used henceforth in the paper when referring to the effect of εWe^2.

2.3. Heat transfer procedure

The equation to be solved is the energy transport equation for the axisymmetric flow with viscous dissipation, but without internal heat sources and negligible axial conduction. The viscous dissipation involves only the shear stress and shear rate and in nondimensional form, the eerngy equation is written as

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right) - 4Br\tau^*_{xr}\frac{\partial u^*}{\partial r^*} = \frac{Pe}{2}u^*\frac{\partial T^*}{\partial x^*} \tag{8}$$

Temperatures were scaled as

$$T^* \equiv \frac{T_w - T}{T_w - \overline{T}_i} \tag{9}$$

where $\overline{T}_i$ represents the bulk temperature at the inlet.

The relevant boundary conditions are symmetry at the axis and an imposed constant temperature at the wall. These are writtem in Eq. (10) in nondimensional form

$$\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=0} = 0 \quad ; \quad T^*_{r^*=1} = 0 \tag{10}$$

Equation (8) expresses the so-called Graetz problem extended to account for viscous dissipation (Shan and London, 1978) and it contains the Peclet number Pe defined by $Pe \equiv Pr.Re = \rho c U 2R/k$ and the Brinkman number Br (following the original definition of Dryden, see Shah and London, 1978) by $Br \equiv \eta U^2 / k\left(T_w - \overline{T}_i\right)$

For Newtonian fluids the solution of Eq. (8) requires transformation of variables followed by such techniques as separation of variables or Lévêque analysis, and was obtained by Brinkman (1951) and Ou and Cheng (1974), amongst others (see also Brown, 1960). For the SPTT fluid, the solution of Eq. (8) is rather more complex, because of the dependence on the Weissenberg number, and is not attempted here. We concentrate instead on obtaining two asymptotic solutions.

For such asymptotic solutions, it is convenient to normalise differently the axial convective term and for that purpose we introduce a nondimensional temperature θ as

$$\theta = \frac{T_w - T}{T_w - \overline{T}} \tag{11}$$

where $\overline{T}$ is the bulk temperature, so that the normalised axial temperature gradient becomes:

$$\frac{\partial T^*}{\partial x^*} = T^*\frac{\partial \theta}{\partial x^*} + \theta\frac{d\overline{T}^*}{dx^*} \tag{12}$$

One possible advantage of using θ is that $\partial\theta/\partial x = 0$ in fully-developed thermal flow situations, Eckert and Drake (1972). So, introducing this simplification into the different non-dimensionalisation of the convective term of Eq. (13), and using again the starred quantities for the remaining normalisations, an alternative dimensionless energy equation is

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right) = \frac{Pe}{2}u^*\theta\frac{d\overline{T}^*}{dx^*} + 4Br\tau^*_{xr}\frac{du^*}{dr^*} \tag{13}$$

One of the important quantities to be obtained is the heat transfer coefficient under the form of a Nusselt number. This solution functional will also be useful to verify convergence of the iterative procedure to be explained below.

The Nusselt number is defined using nondimensional quantities as

$$Nu = \frac{-2\left.\dfrac{\partial T^*}{\partial r^*}\right|_{r=1}}{\overline{T}^*} \tag{14}$$

We explain now the mixed analytical/numerical procedure utilised to solve Eq. (14) which requires application of a successive approximation method following p. 96 of Kays and Crawford (1980). It starts, as a first approximation, with the θ profile for constant wall heat flux (given by Eq. (32) in Pinho and Oliveira, 2000), which is substituted on the right-hand-side of Eq. (13) together with the normalised profiles of velocity and shear stress. Since θ, u^* and τ^*_{xr} are polynomials in r^*, Eq. (13) is readily integrated for T^* to obtain a corrected temperature profile for constant wall temperature. From this corrected T^* profile, new bulk and θ temperatures are calculated using their definitions and a new expression for the Nusselt number is also evaluated. This newly corrected θ profile is then used to start the next iteration. The procedure is repeated in order to obtain systematically improved values of all quantities and of the Nusselt number which converges to a certain value. The procedure is stopped when the Nusselt number variation from successive approximations falls below a prescribed tolerance.

This method is applied to the solution of the developing thermal flow without viscous dissipation ($Br = 0$) with view to obtain an expression for the Nusselt number under fully-developed conditions ($\partial\theta/\partial x = 0$). This constitutes the first asymptotic solution in this paper, henceforth termed the "fully-developed thermal flow with negligible viscous dissipation".

A second asymptotic case is obtained when $d\bar{T}^*/dx^*$ vanishes in Eq. (13), whereby viscous dissipation is balanced by radial conduction only. The solution to this problem is easier and will also be given in the next section under the heading "equilibrium viscous dissipation flow". It is important to realise that, if the successive approximation method were applied to the full equation (with $Br \neq 0$) then the outcome would be this second asymptotic solution which in fact is amenable to an analytical derivation. This is the reason why that approximate method is only applied to the case without dissipation and also the reason for having decided to divide the original problem into two sub-problems.

3. Solutions and discussion

We give in Section 3.1 the approximate solution for the problem of thermally fully-developed flow without viscous dissipation (Eq. 13 with $Br = 0$) and in Section 3.2 we derive the exact solution for the problem of equilibrium between radial conduction and viscous dissipation (Eq. 13 with $dT^*/dx^* = 0$).

3.1. Fully-developed thermal flow with negligible viscous dissipation

As mentioned above, the initial guess for θ was taken as the solution to the problem with a constant wall heat flux (Eq. (32) of Pinho and Oliveira, 2000) for negligible viscous dissipation ($Br = 0$)

$$\theta(r^*) = \frac{\left\{-\frac{1+a}{2} r^{*2} + \frac{1}{8} r^{*4} + \frac{a}{18} r^{*6} + \frac{3}{8} + \frac{8}{18} a\right\}}{\left[\frac{19}{54} a^2 + \frac{17}{30} a + \frac{11}{48}\right] \frac{U_N}{U}} \qquad (15)$$

The successive approximation method described in the previous section was applied until differences from consecutive iteration Nusselt numbers were less than 0.5%. This happened at the end of the third iteration which differed from the second by less than 0.3% as can be assessed by inspection of Fig. 1. This figure plots $1 - Nu_{i+1}/Nu_i$ as a function of εWe^2, where the subscripts designate the iteration number and Nu_q is the initial Nusselt number corresponding to the condition of imposed wall heat flux, from Pinho and Oliveira (2000). It shows the quick convergence, with Nu_2 already within 2% of Nu_1, and that there is basically a decrease of one order of magnitude in the relative difference between consecutive Nusselt numbers. For the Newtonian fluid (a=0), the Nusselt number at the third iteration was well within 0.1% of the value quoted in the literature (3.658 in Kays and Crawford, 1980 or 3.6568 quoted in p. 79 of Shah and London, 1978).

Tracking of the solution by algebraic means up to the third iteration (where an accuracy below 0.1% is estimated) was a tedious operation leading to lengthy equations in terms of powers of r^*. This will not be repeated here, and we shall content ourselves in giving the final "simplified" expressions for the temperature and Nusselt number and analysing their variation from plots.

The normalised temperature profile is a long polynomial in r^* and a given by

$$T^* = \frac{Pe}{2} \frac{d\bar{T}^*}{dx^*} \frac{\left[0.9366 \sum_{i=0}^{4} \delta_i a^i + 1.463 r^{*2} \sum_{i=0}^{4} \left[\alpha_i a^i \times \sum_{j=0}^{11} \beta_{ij} r^{*2j}\right]\right]}{\sum_{i=0}^{4} \gamma_i a^i} \qquad (16)$$

where the coefficients α_i, γ_i and δ_i are presented in Table 1 and β_{ij} in Table 2.

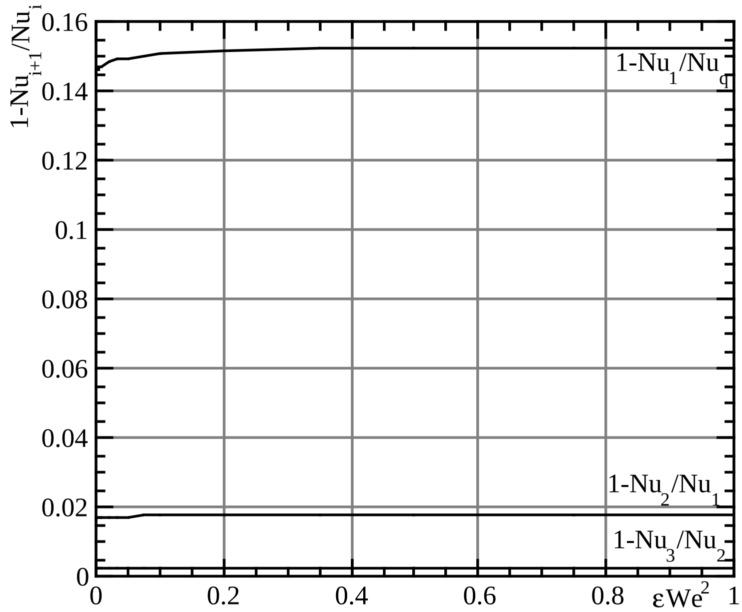

Figure 1- Relative difference between Nusselt numbers at consecutive iterations as a function of εWe^2.

Table 1- Coefficients α_i, γ_i and δ_i for pipe flow

	0	**1**	**2**	**3**	**4**
α_i	0.614	2.779	4.711	3.546	1
γ_i	1	4.7967	8.6410	6.9291	2.0869
δ_i	0.5258	2.4656	4.3405	3.4	1

The Nusselt number was found to be given by

$$Nu = 3.658 \frac{\sum_{i=0}^{4} \gamma_i a^i}{\left[1+\frac{4}{3}a\right]\left[2.4368a^5 + 10.172a^4 + 17.004a^3 + 14.230a^2 + 5.9610a + 1\right]} \tag{17}$$

where the ratio of bulk velocities U/U_N has been be eliminated by means of $U/U_N = 1 + 4a/3$ from Oliveira and Pinho (1999).

The normalised temperature θ, as defined by Eq. (11), is independent of axial location for thermally fully-developed flow, as implied by the definition of this asymptotic condition. It can be obtained from a balance of energy at a cross-section, which shows that

$$\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1} = \frac{Pe}{4}\frac{d\overline{T}^*}{dx^*} \tag{18}$$

hence from Eq. (14) we conclude that

$$\overline{T}^* = -\frac{Pe}{2}\frac{d\overline{T}^*}{dx^*}\frac{1}{Nu} \tag{19}$$

and $\theta = T^*/\overline{T}^*$, with the numerator and denominator given by Eqs. (16) and (19), respectively.

The full lines in Figs. 2-a) and b) show the resulting variation of Nusselt number with εWe^2, which is the relevant viscoelastic number in these flows, in both semi-log and linear scales. The dashed lines in the figures help to show the iterative progression of Nusselt number and complement the information of Fig. 1. They are useful as a check on the correctness of the implementation of the numerical method, and clearly show that more iterations would bring a negligible improvement on the solution.

Table 2- Coefficients β_{ij} for pipe flow.

j \ i	**0**	**1**	**2**	**3**	**4**
0	-1	-1	-1	-1	-1
1	7.021×10^{-1}	6.302×10^{-1}	5.540×10^{-1}	4.730×10^{-1}	3.865×10^{-1}
2	-3.376×10^{-1}	-2.418×10^{-1}	-1.456×10^{-1}	-4.898×10^{-2}	4.817×10^{-2}
3	1.111×10^{-1}	4.659×10^{-2}	-1.096×10^{-2}	-6.102×10^{-2}	-1.029×10^{-1}
4	-2.788×10^{-2}	-4.600×10^{-4}	1.819×10^{-2}	2.802×10^{-2}	2.895×10^{-2}
5	4.865×10^{-3}	-3.383×10^{-3}	-6.130×10^{-3}	-4.104×10^{-3}	1.900×10^{-3}
6	-5.602×10^{-4}	1.108×10^{-3}	6.501×10^{-4}	-1.109×10^{-3}	-3.354×10^{-3}
7	2.814×10^{-5}	-1.861×10^{-4}	1.219×10^{-4}	4.953×10^{-4}	5.679×10^{-4}
8	0	1.186×10^{-5}	-5.305×10^{-5}	-4.994×10^{-5}	8.634×10^{-5}
9	0	0	4.802×10^{-6}	-1.091×10^{-5}	-3.870×10^{-5}
10	0	0	0	1.854×10^{-6}	0
11	0	0	0	0	6.745×10^{-7}

The Nusselt number varies between two asymptotes: at the limit of vanishing εWe^2 it gives the well-known Newtonian value of 3.658 and, as εWe^2 goes to infinity, it tends to 4.178. There is thus a maximum increase of 14% in relation to the Newtonian solution, represented in the figure as a horizontal line. It is very unlikely that flows attain such high values of εWe^2 and Fig. 2-b) presents the same information in linear coordinates and in a limited range, $\varepsilon We^2 \in [0,1]$. At $\varepsilon We^2 = 1$ the Nusselt number is already equal to 4, representing an increase of 9.4% from the Newtonian value. As with the Nusselt number for constant wall heat flux, represented in the figures in long dashes, the Nusselt number increases with εWe^2 due to the more pronounced shear-thinning effect.

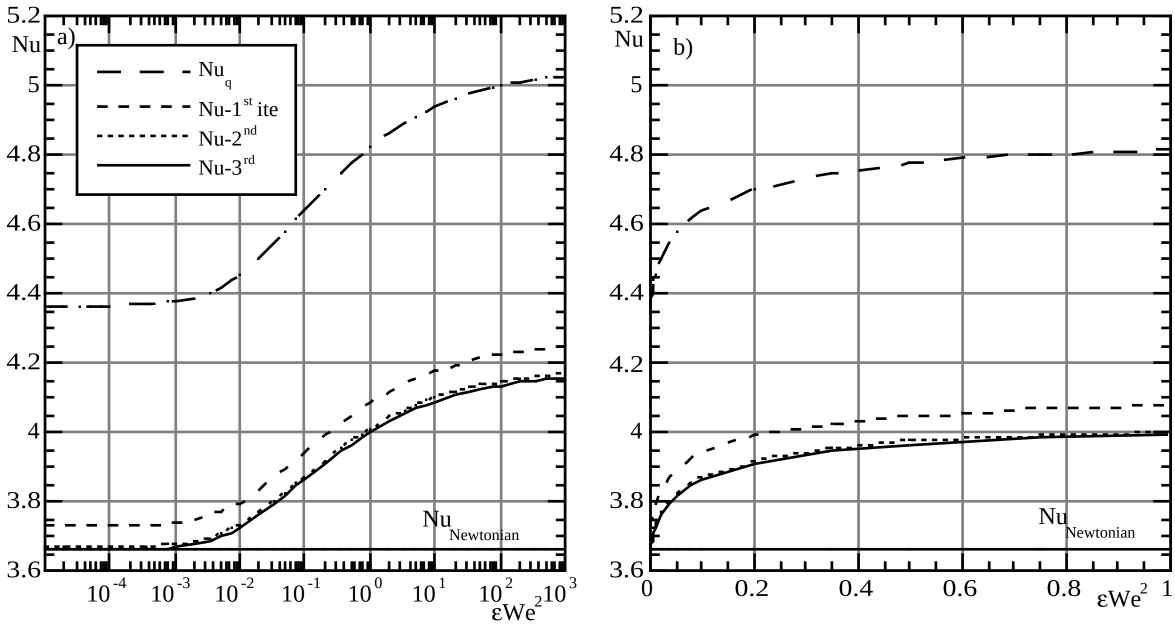

Figure 2- Variation of the Nusselt number with εWe^2. Dashed lines show values at consecutive iterations, starting from the case of imposed heat flux (Nu_q) :a) in semi-log coordinates; b) in linear coordinates.

The variation of θ with radius is given in Fig. 3 for various values of the parameter εWe^2. The variation with εWe^2 is mild since both its numerator and denominator depend similarly on εWe^2, a conclusion similar to that reached in the previous work for the case of imposed wall heat flux (Pinho and Oliveira, 2000).

The practical use of the expressions for temperature (Eqs 16 and 19) is rather limited unless the variation of the bulk temperature with the axial coordinate is known. The successive approximation method used here can not provide such gradient because it only gives the asymptotic radial variation. However, in the fully-developed flow region, where *Nu* has reached its constant asymptotic value, it is possible to carry out an integral energy balance yielding the following axial variation of bulk temperature

$$\overline{T}^* = \overline{T}_I^* \operatorname{Exp}\left[-\frac{2\,Nu}{Pe}\, x^* \right] \qquad (20)$$

from which its axial gradient is obtained

$$\frac{d\overline{T}^*}{dx^*} = -\frac{2\,Nu}{Pe}\,\overline{T}_I^* \times \mathrm{Exp}\left[-\frac{2\,Nu}{Pe}\,x^*\right] \tag{21}$$

If x^* is the axial position relative to a reference location within the fully-developed region, and $\overline{T}_I$ is the bulk temperature at that reference location, then Eqs. (20) and (21) give exact values for the bulk temperature and its gradient; otherwise, if there is no certainty that x^* lies in the fully-developed region, then $\overline{T}_I$ has to be estimated and relations (20) and (21) are only approximate.

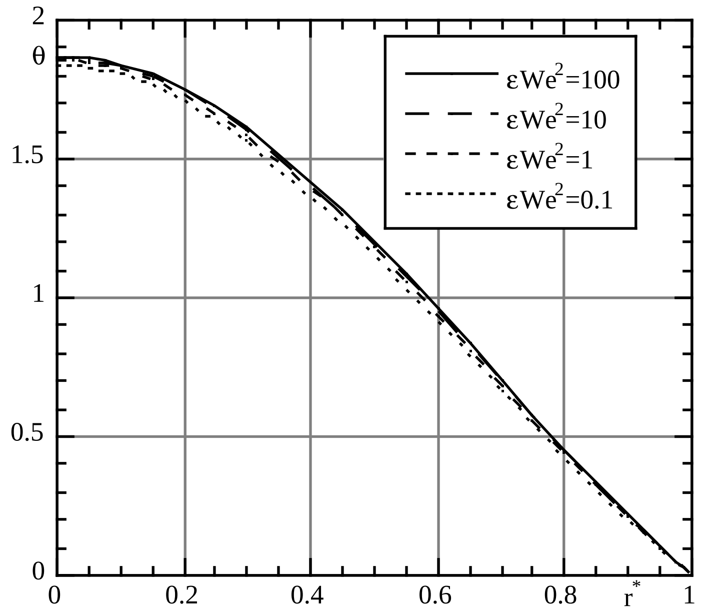

Figure 3- Radial profiles of θ as a function of fluid elasticity.

3.2. Equilibrium viscous dissipation flow

We turn now to the second problem involving a balance between radial conduction and viscous dissipation under fully developed conditions. Substitution of the velocity profile (Eq. 5) and the shear stress and shear rate profiles (Eq 7) into Eq. (13) with $d\overline{T}^*/dx^* = 0$, produces

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right) = 16\,Br\left(\frac{U_N}{U}\right)^2\left[2ar^{*4} + r^{*2}\right] \tag{22}$$

which is of straightforward integration. The first integration gives the temperature gradient at the wall needed to obtain the Nusselt number

$$\left.\frac{dT^*}{dr^*}\right|_{r^*=1} = 4\,Br\left(\frac{U_N}{U}\right)^2\frac{4a+3}{3} = 4\,Br\frac{3}{4a+3} = 4\,Br\frac{U_N}{U} \tag{23}$$

and the second integration and the imposition of the wall temperature boundary condition gives the temperature distribution

$$T^* = Br\left(\frac{U_N}{U}\right)^2\frac{\left[8a\left(r^{*6}-1\right)+9\left(r^{*4}-1\right)\right]}{9} \tag{24}$$

and the dimensionless bulk temperature becomes

$$\overline{T}^* = -Br\left(\frac{U_N}{U}\right)^3\frac{280\,a^2 + 504\,a + 225}{270} \tag{25}$$

Finally, the Nusselt number is given by

$$Nu = \frac{240(4a+3)^2}{280a^2 + 504a + 225} \tag{26}$$

which gives the correct Newtonian value $Nu = 48/5$ for a=0 (p. 80 in Shah and London, 1978). The important conclusion here is that the Nusselt number does not depend on the Brinkman number. The definition of Nu in Eq. (14), involves the ratio of two quantities that in this limiting problem depend linearly on the Brinkman number and consequently Nu becomes independent of Br.

We know from the equivalent constant wall-heat-flux problem that viscous dissipation has a strong impact upon the heat transfer characteristics, and this is confirmed in the present case. For a Newtonian fluid the Nusselt number more than doubles, from 3.658 to 9.6, but this increase is accentuated for the viscoelastic fluid, as shown in the comparative plot of Fig. 4. For εWe^2 =1 the normalised heat transfer coefficient with viscous dissipation is already three times higher than that for negligible dissipation, and this ratio increases to the limiting value of 3.28 as $\varepsilon We^2 \to \infty$ ($Nu \to 13.714$ with dissipation, compared with $Nu \to 4.178$ without dissipation).

Radial temperature profiles for these situations are plotted in Fig. 5. An increase in Brinkman number raises the temperature level linearly in the pipe section. Although the influence of viscous dissipation increases with flow elasticity, as seen in the Nusselt number plot of Fig. 4, the difference between the local and the wall temperatures (recall that $-\bar{T}^* \propto T - T_w$) decreases with εWe^2, because the flow becomes increasingly shear-thinning due to normal stress effects and consequently viscous dissipation becomes progressively more localised in the wall region and less pronounced in the core of the flow. Thus, since the heat is generated closer to the wall, it is evacuated easier without the need to heat the bulk of the flow. Note that the decrease in T^* with viscoelasticity induces a wider core of uniform temperature.

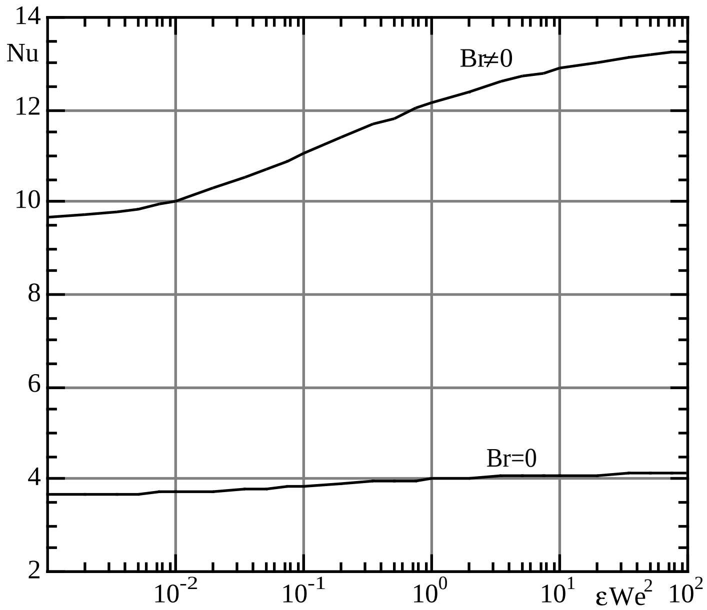

Figure 4-Comparison between the Nusselt numbers for negligible viscous dissipation and the limiting equilibrium viscous dissipation condition, as a function of fluid elasticity (εWe^2).

4. Conclusions

Heat transfer in the steady, laminar pipe flow of the simplified Phan-Thien—Tanner model fluid was investigated for the condition of an imposed constant wall temperature and under fully-developed thermal and hydrodynamic conditions. Two cases were investigated and results were presented for the radial profile of normalised temperature and for the Nusselt number as a function of the relevant nondimensional Brinkman number and the product of ε with the square of the Weissenberg number.

The first fully-developed solution pertained to the equilibrium between axial convection and radial conduction of thermal energy, with negligible viscous dissipation, and here it was observed that an increase in fluid elasticity (as measured by $\sqrt{\varepsilon} We$, to be more precise) raised the normalised heat transfer coefficient by at most 14% due to the increased level of shear-thinning behaviour.

For the second thermally fully-developed problem, where there is equilibrium between radial conduction of energy and heat production by viscous effects, viscous dissipation is responsible for the increase in the Nusselt number which is more pronounced the more elastic the fluid is. Again, that was found to be associated with the more intense shear-thinning fluid behaviour: shear-thinning is enhanced by increasing $\sqrt{\varepsilon} We$, which was seen to raise shear rates close to the wall while simultaneously decreasing them in the core of the flow. Since viscous dissipation is proportional to the shear rate, as shear-thinning is enhanced the internal production of heat increases nearer to the wall where heat is evacuated, and decreases in the core of the duct, thus the thermal resistance decreases and the Nusselt number raises.

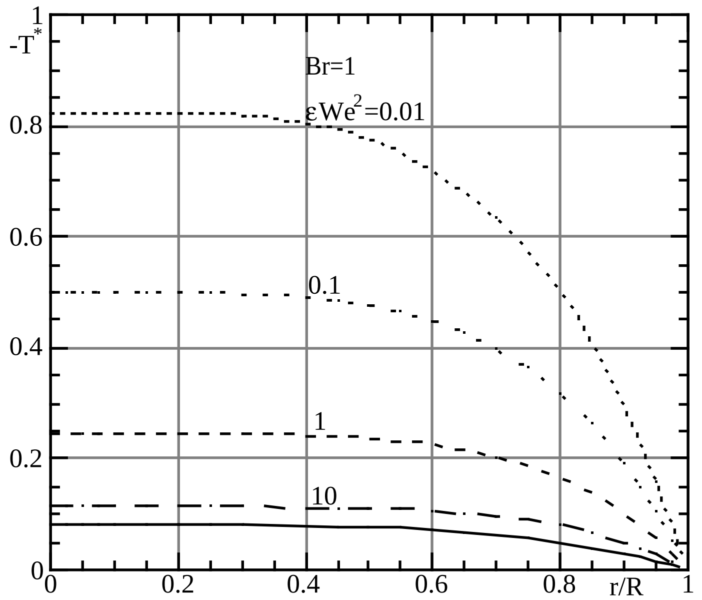

Figure 5- Effect of viscoelasticity and Brinkman number on the radial temperature profiles in the equilibrium viscous dissipation problem. The full line is for $Br = 0.1$ and $\varepsilon We^2 = 0.01$.

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