

# Impact of Different Mobility Models on Connectivity Probability of a Wireless Ad Hoc Network

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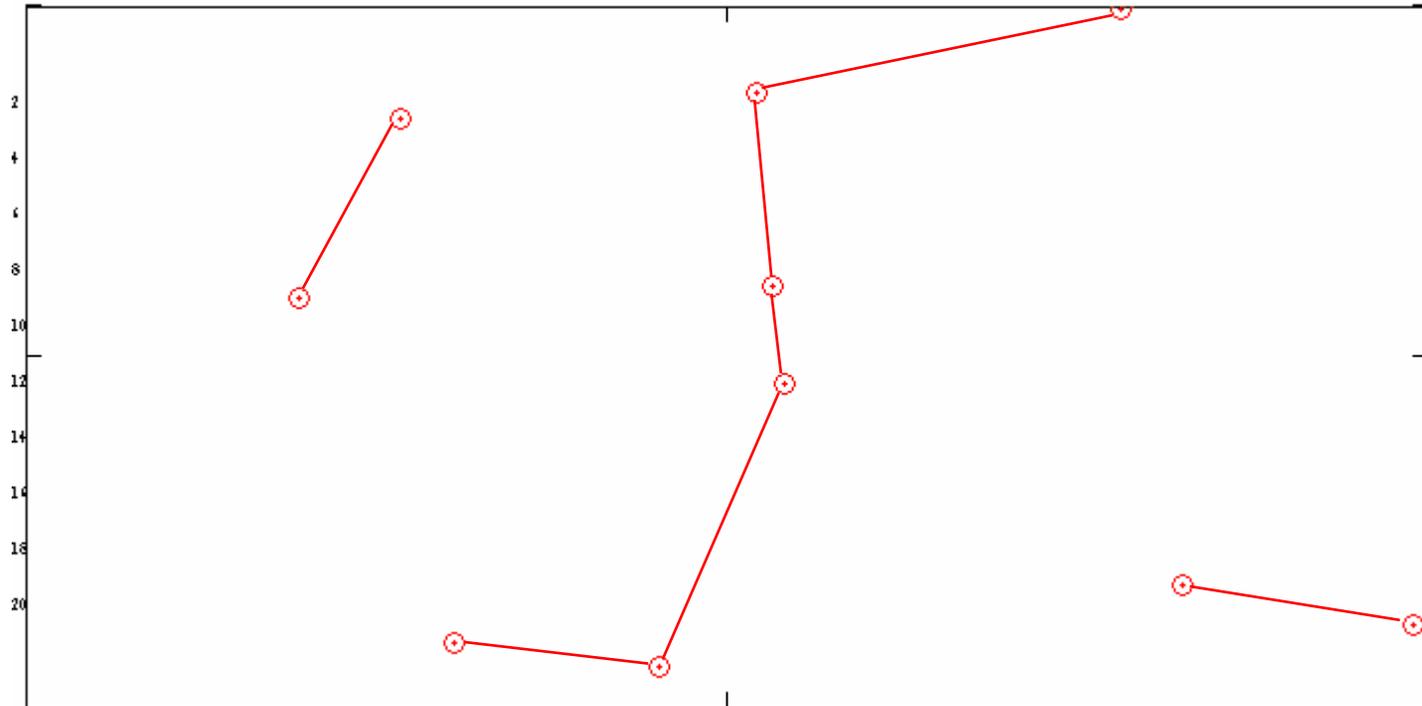
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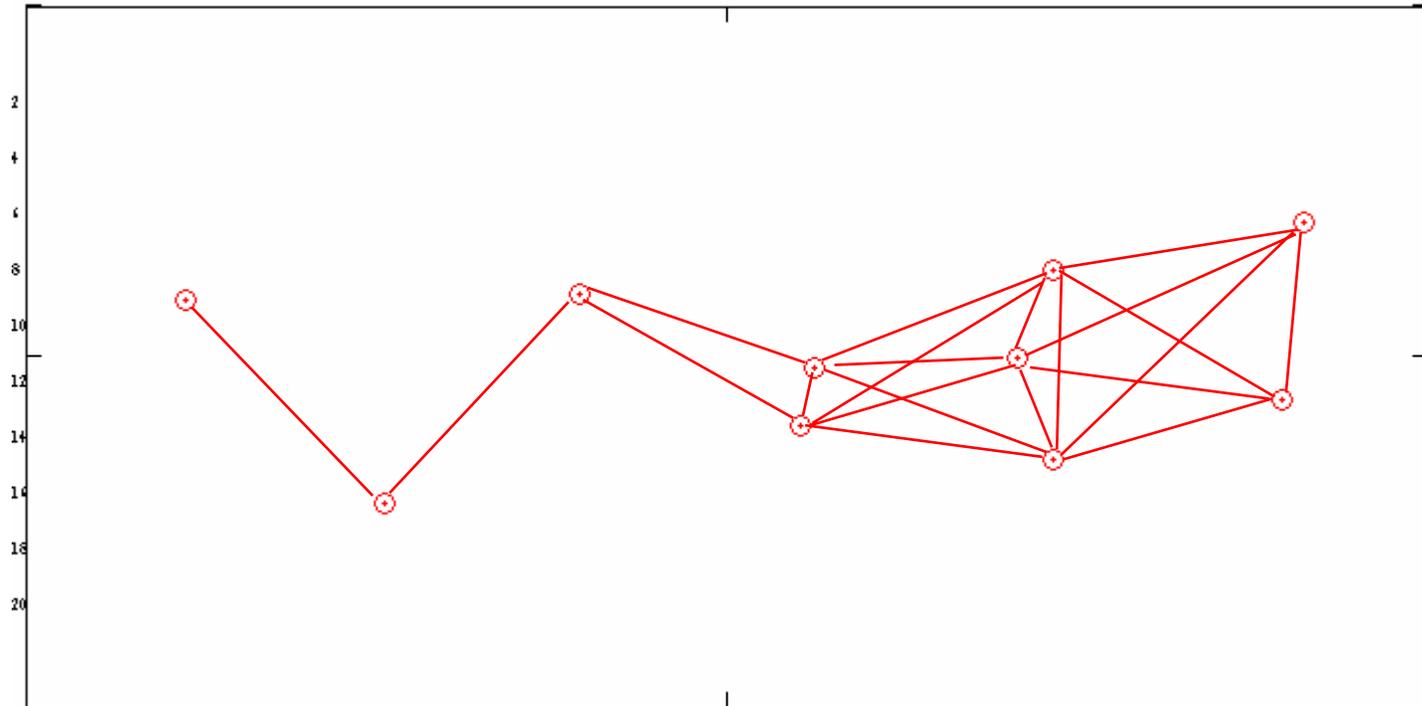
- Introduction to mobility models
- The problem of network connectivity under mobility of nodes
- Definition of *Connectivity Probability*
- Analysis of Random Direction mobility model
- Applicability of the method to other models
- Example: connectivity of a network in an area with obstacles
- Conclusions and future work

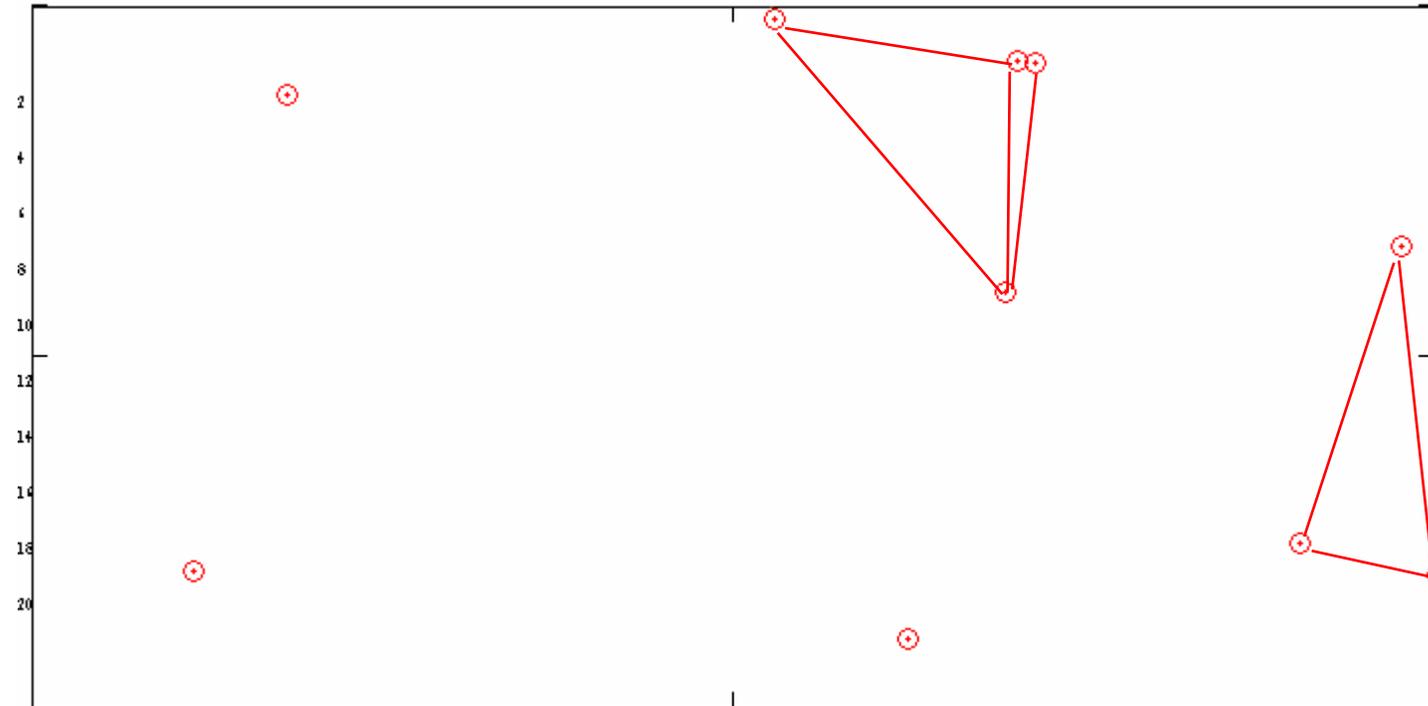
- The movement pattern of users has a significant impact on performance of wireless networks
- Different mobility models have been proposed
  - Random Direction
  - Random Waypoint
  - Virtual World Scenarios
- Important issues for ad hoc networks
  - Connectivity
  - Path duration
  - Stability of routes

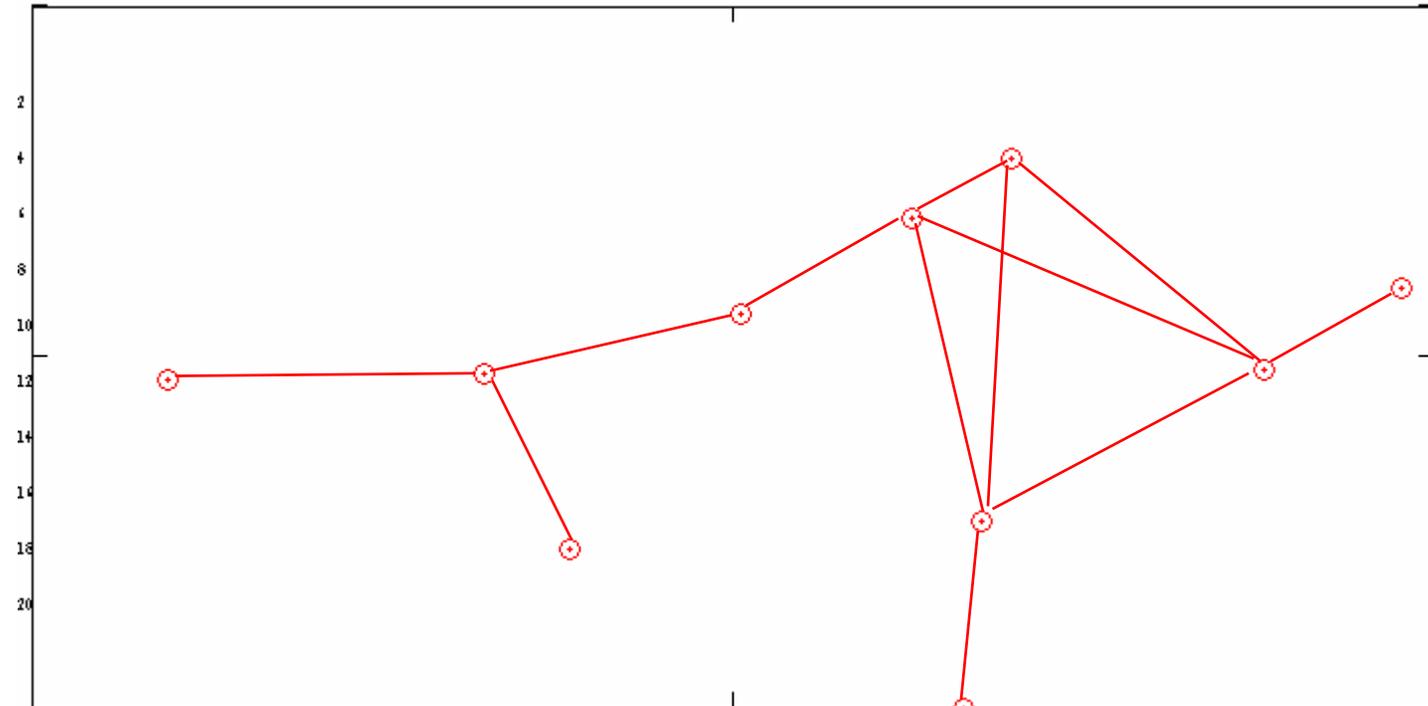
	RWP	RD
slow	0.81	0.8
fast	0.95	0.9





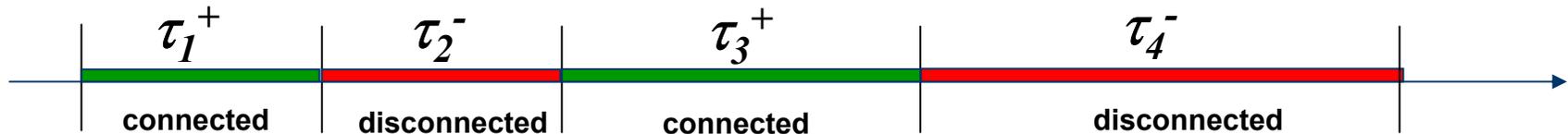






- We introduce a measure of network connectivity in the presence of mobility
  - *Connectivity probability*
- A number that gives an estimate that the whole network is connected in any point in time

- **Definition.** At a given moment in time a network is *connected* if for every pair of nodes there exists a path between them.
- Under motion of nodes a semiaxis of time is divided into intervals  $\tau_1^\pm, \tau_2^\pm, \tau_3^\pm, \dots$



- Denote  $T^+ = \bigcup \tau_k^+, T^- = \bigcup \tau_k^-$
- **Definition.** *Connectivity probability* is defines as a measure of set  $T^+$ , or as it is customary in number theory:

$$P_+ = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \text{mes}(T^+ \cap [0, \tau])$$

if this limit exists

- Since  $(T^- \cap [0, \tau]) \cup (T^+ \cap [0, \tau]) = [0, \tau]$  ,  
then

$$P_+ + P_- = 1$$

- The problem to find connectivity probability is reduced to the problem of existence and calculation of the limit.

- Definition. At a given moment in time a network consisting of  $n$  nodes is *weakly connected* if

$$\max_i \min_{j \neq i} \|r_i - r_j\| \leq r$$

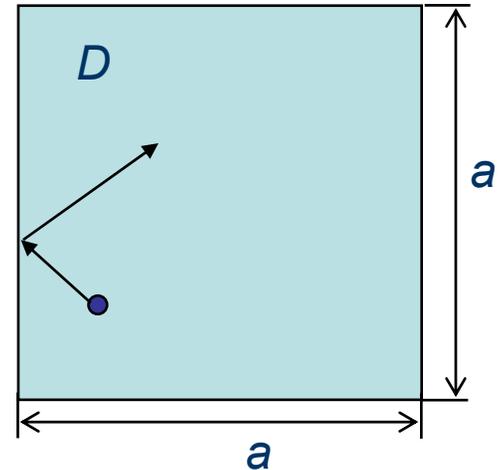
where  $r$  is the communication range and  $\|\cdot\|$  is a metric.

- Number of connections in fully connected network  
 $C = n(n-1)/2$
- Number of connection in weakly connected network  
 $n/2 \leq C \leq n(n-1)/2$

- Node selects a direction and speed to move from a given distribution. The motion is along a straight line. Once a boundary is reached, new direction is chosen towards the inside of the simulation area.
- Why RD model?
  - It is an example of a dynamical system that successfully realizes ergodic properties → analytical approach is feasible based on ideas and methods of ergodic theory

- A particular case of RD model – *billiard* : trajectories form equal angles with the boundary.
- Applying Weyl's ergodic theorem, we show that the limit exists and it is equal to the spatial average:

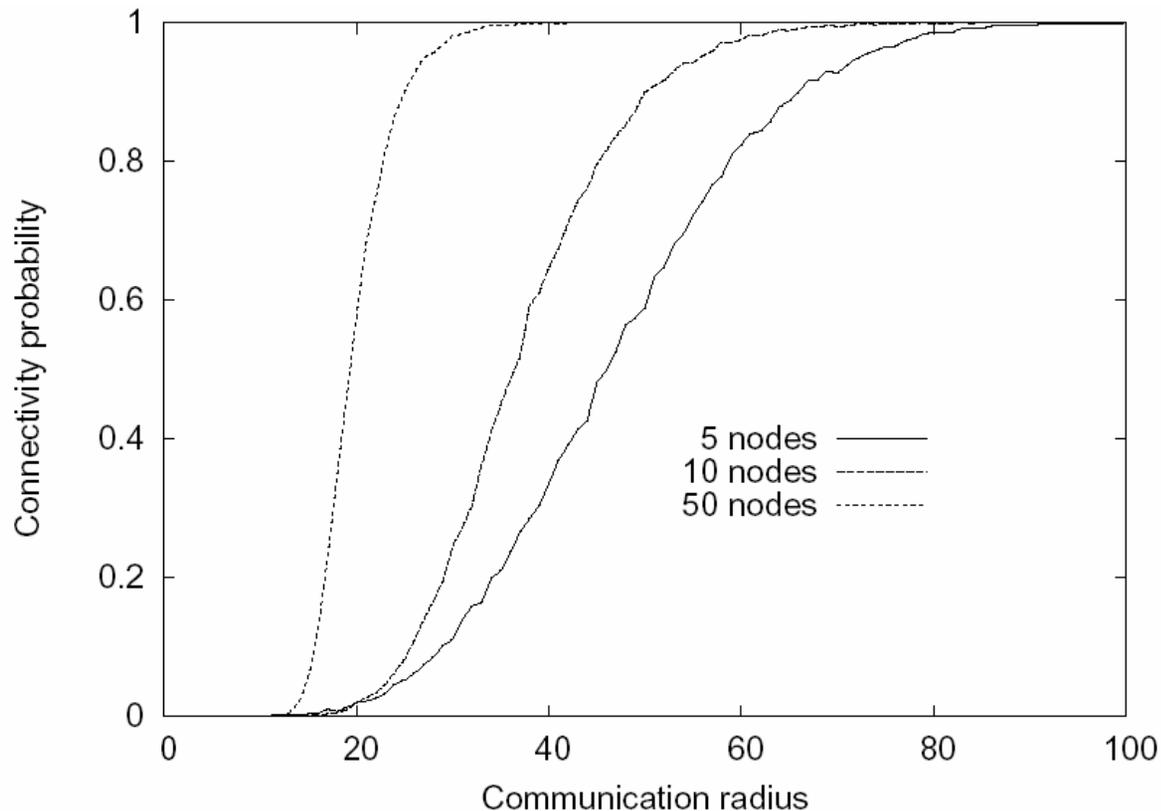
$$P_+ = \frac{\text{mes } S_r}{a^{2n}}$$



where  $a$  is the side of the square,  $n$  is the number of nodes,  $S_r$  is a domain consisting of all possible positions of the nodes such that the network is connected.

- It seems to be difficult to find a close form expression for the measure of  $S_r$  as a function of  $r$ . We propose to use Monte-Carlo simulations to find **mes**  $S_r$ .
- What is the advantage?
  - Using definition for calculation of connectivity probability, we would have to perform in principle infinitely many simulations, one for each point in time.
  - Using the obtained formula, we do not have to model our system in time, - this leads to significant reduction in computational complexity.

- Connectivity probability for RD as a function of communication radius

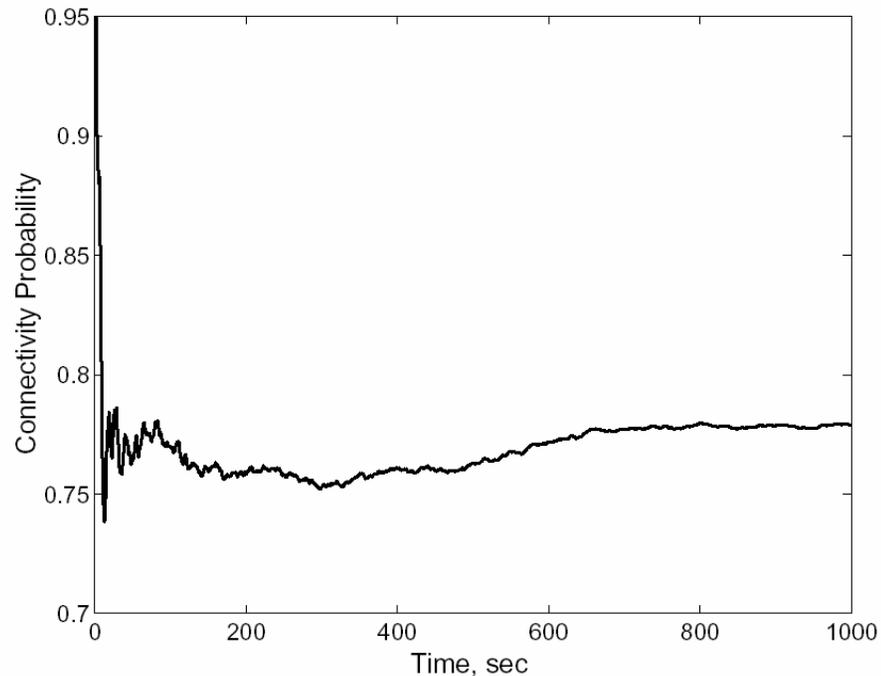


- Simulation area is a square 100x100 m<sup>2</sup>, speed of all nodes is 5 m/s, no pause time

- area 100x100 m
- comm. radius 20m
- 90 nodes
- speed 5 m/s
- Conn. probability = approx. 0.87
- Using formula
  - $N=2000 \rightarrow \text{cpu time}=132 \text{ sec}$
  - $N=10.000 \rightarrow \text{cpu time}=666 \text{ sec}$
- Using definition
  - $t=5.000 \text{ sec} \rightarrow \text{cpu time} = 2431 \text{ sec}$
  - $t=10.000 \text{ sec} \rightarrow \text{cpu time} = 5476 \text{ sec}$

- The same formula is applicable in the case of
  - general RD model
  - When rectangular-shaped obstacles are placed in the simulation area
- If a collection of nodes comprises a dynamical system with some particular properties, then for almost all initial conditions the limit exists. It can happen that for different initial positions and velocities, the limiting values will be different.
- Note that for RD model the limit does not depend on initial conditions, therefore it is defined correctly.

- We propose to introduce averaging over initial node distribution, and the obtained average connectivity probability can be used as an estimate for network connectivity.

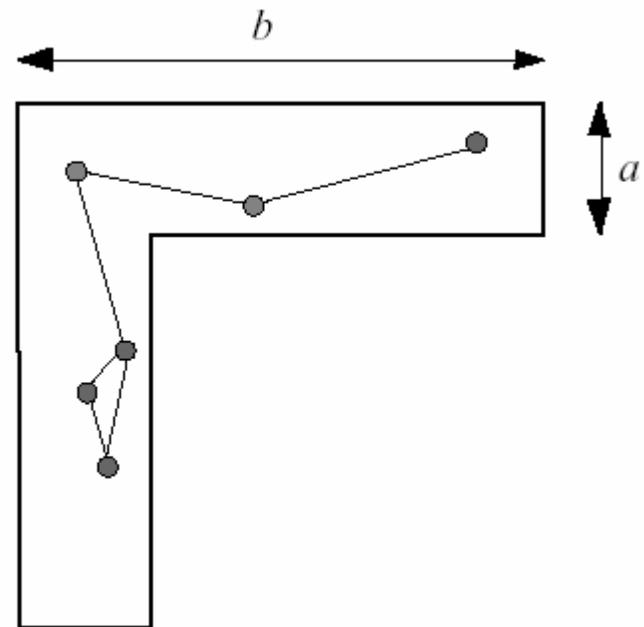


Simulation area is 100x100 m<sup>2</sup>, 90 nodes, speed 5 m/s, communication radius 20 m, 10 simulation runs

- A simulation scenario of a rectangular area with a lot of nodes is considered to be a good approximation for large scale ad hoc networks.
- For small-scale networks the shape of the mobility area becomes important. One way to make simulation scenarios more realistic is to incorporate obstacles simulation area.

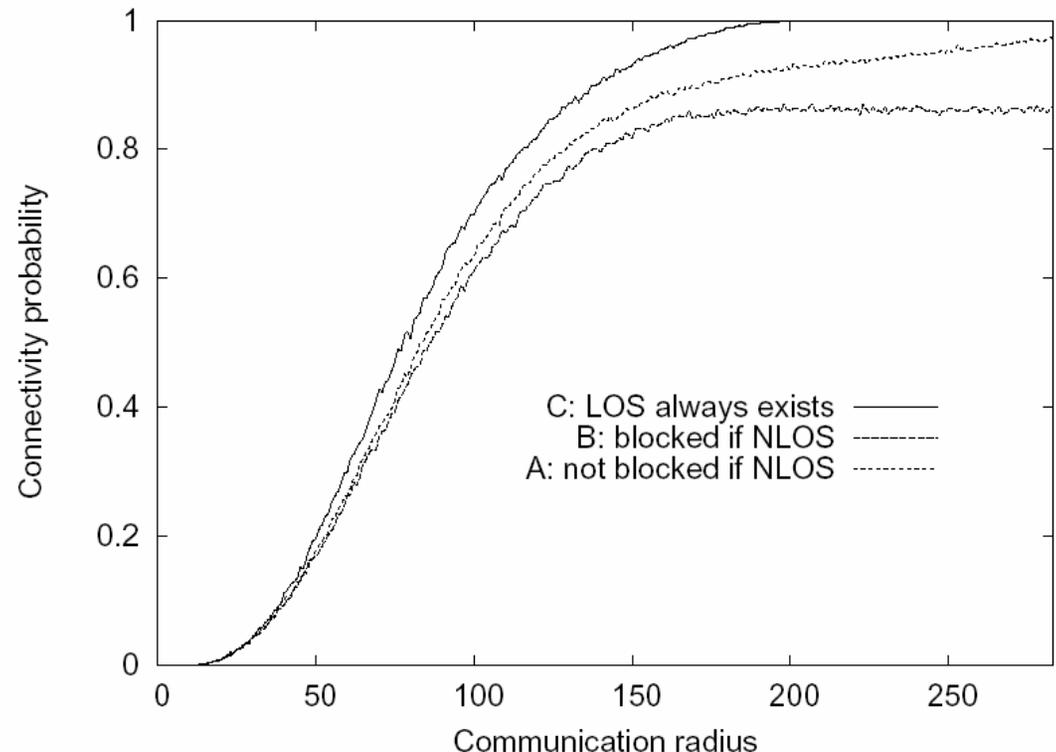
- To illustrate the applicability of the formula in the scenarios with obstacles, we consider the following example:

$a = 25 \text{ m}$ ,  $b = 200 \text{ m}$ , 6 nodes, speed 5 m/s



- Connectivity probability for RD as a function of communication radius

- Line A: we propose use a simplified propagation model: a signal can reach the receiver via non LOS mechanism, but the receiving power is 20 dB smaller compared with the case when a LOS path exists.



- The number 20 dB seems to be realistic for indoor environment, but it can vary a lot depending on materials of walls and ceiling, size of the room etc

- We have presented a new method to describe connectivity of a network in presence of mobility as a measure
- The developed framework was applied to Random Direction model; it's applicability to other models was discussed
- In the future we plan to examine other mobility models, including the one, based on the use of mobility trajectories of the users in a virtual world using a multi-player game such as e.g. Quake II.