Discrete Cosine Transform Type-IV-based Multicarrier Modulators in Frequency Offset Channels

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Abstract—Carrier frequency offset (CFO) is one of the main drawbacks of multicarrier modulation based on the discrete Fourier transform (DFT). To solve the above problem, several researchers have proposed the use of discrete trigonometric transforms. In this paper, we analyze the intercarrier interference and the bit-error probability in discrete cosine transform type-IV even (DCT4e)-based multicarrier systems under the presence of carrier frequency offset (CFO). The parameters which are involved in the design are established, and the expressions for both prefix and suffix to be appended into each data symbol to be transmitted are shown. Several computer simulations indicate that the proposed systems outperform the standardized ones based on the discrete Fourier transform (DFT) under the presence of CFO.

Index Terms—Multicarrier modulator (MCM), Discrete Trigonometric Transform (DTT), Discrete Cosine Transform (DCT).

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is a particular form of multicarrier modulation (MCM) [1], and it has been the modulation choice for fixed or nomadic broadband communications (WiFi and WiMAX), digital audio and video broadcasting (DAB and DVB), or power line communications [2]–[6]. In mobile communications, OFDM has also recently been adopted in WiMAX [7] and LTE downlink [8].

Figure 1 shows a general block diagram to implement multicarrier modulation. At the transmitter, data are processed by an $N$-point inverse transform ($T_a^{-1}$) —with $N$ being the number of subchannels or subcarriers—by fast algorithm of implementation. At the receiver, a discrete transform ($T_c$), also implemented by a fast algorithm, is performed. In OFDM, $T_a$ and $T_c$ are carried out using discrete Fourier transforms (DFTs).

DFT-based systems are efficient to fight against multi-path fading channels, but they have some drawbacks, as the peak-to-average power ratio, a poor behaviour of uncoded DFT-based systems in noisy-environments, or the sensitivity to carrier-frequency offsets (CFO). Regarding the last one, frequency-offset interferences are mainly caused in mobile OFDM communications by mismatch or ill-stability of the local oscillators in the transmitter and the receiver, and also by the movement-induced Doppler shift.

Different solutions to correct some of the above drawbacks are based on the use of discrete trigonometric transforms (DTTs) as multicarrier modulators (MCMs) [9]–[15]. In this work, the use of discrete cosine transform type-IV even-based multicarrier modulators (DCT4e-MCMs) is considered. In the above systems, the intercarrier interference coefficients and the expressions of the BEP under additive Gaussian noise in the presence of CFO, are obtained. The performance of the DCT4e-MCM, the DCT2e-MCM introduced in [12], and the DFT-MCM (OFDM), considering the impact of CFO on the bit error rate, is evaluated by computer simulations.

II. DISCRETE COSINE TRANSFORM-IV FOR MCM

When the DCT4e [16] is used for multicarrier data transmission ($T_a^{-1} = DCT4e^{-1}$ and $T_c = DCT4e$ in Fig. 1), the inter-block interference is eliminated adding to each block a left prefix $x_{pre}$, and also a right suffix $x_{suf}$, both of length $\nu$, where $\nu$ is at least the channel order. Hence, we consider the extended block

$$x_e = \begin{bmatrix} x_{pre} \\ x \\ x_{suf} \end{bmatrix}$$

of length $N + 2\nu$. The prefix and suffix must be defined as

$$x_{pre}^T = \begin{bmatrix} x_{\nu-1} & \cdots & x_0 \end{bmatrix},$$

$$x_{suf}^T = \begin{bmatrix} -x_{N-1} & \cdots & -x_{N-\nu} \end{bmatrix}.$$
III. ICI ANALYSIS FOR THE DCT4e-MCM

In this section, we analyze the intercarrier interference (ICI) for DCT4e-MCM systems operating in the presence of CFO over an AWGN channel. For this purpose, we consider the block diagram of Fig. 2, which includes in-phase and quadrature modulators in the presence of CFO ($\Delta f$) and phase error ($\phi$). For one-dimensional modulations, the quadrature modulator is not necessary in DCT-MCM systems, whereas in the DFT-MCM the quadrature modulator would be included due to the complex values after the IDFT block.

The $N$-length discrete-time sequence of the baseband IDCT4e block—including in-phase $x^I_n$ and quadrature $x^Q_n$ components—is given by

$$x^I_n = N^N \sum_{k=0}^{N-1} X_k \cos \left( \frac{\pi(2n+1)(2k+1)}{4N} \right),$$

setting each subcarrier in frequency at $\frac{(2n+1)\Delta f}{2N}$, which are fed into the I and Q modulators, respectively.

With $f(t)$ being the lowpass reconstruction filter included in the D/A converter, the real and imaginary parts of the base-band symbol are obtained as

$$x^I_n = x^I_n + jx^Q_n = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} X_k \cos \left( \frac{\pi(2n+1)(2k+1)}{4N} \right),$$

$$x^I_n = \sum_{n=0}^{N-1} x^I_n f \left( t - \frac{(2n+1)T}{2N} \right),$$

$$x^Q_n = \sum_{n=0}^{N-1} x^Q_n f \left( t - \frac{(2n+1)T}{2N} \right).$$

Next, the passband block transmitted signal is

$$r^I(t) = \sum_{n=0}^{N-1} x^I_n q \left( t - \frac{(2n+1)T}{2N} \right) \cos \left( 2\pi \Delta f t + \phi \right)$$

$$-x^Q_n q \left( t - \frac{(2n+1)T}{2N} \right) \sin \left( 2\pi \Delta f t + \phi \right) + z(t),$$

$$r^Q(t) = \sum_{n=0}^{N-1} x^Q_n q \left( t - \frac{(2n+1)T}{2N} \right) \sin \left( 2\pi \Delta f t + \phi \right)$$

$$+ x^I_n q \left( t - \frac{(2n+1)T}{2N} \right) \cos \left( 2\pi \Delta f t + \phi \right) + z(t),$$

where $z(t) = z(t) + j : q(t)$ is Gaussian noise, and $q(t)$ is the cascade of the transmitter and receiver lowpass filters, that satisfies

$$q \left( \frac{nT}{N} \right) = \begin{cases} 1, & n = 0, \\ 0, & \text{otherwise}. \end{cases}$$

The received in-phase and quadrature signals are sampled at instants $T(2m+1)/2N, m = 0, \ldots, N-1$, getting

$$\tilde{r}_m = (x^I_m + jx^Q_m)e^{j\left( \frac{2\pi \Delta f T(2m+1)}{2N} + \phi \right)} + \tilde{z}_m.$$  \(6\)

After the $T_c$ block transform, the decision variable $Y_k$ in the $k$th subcarrier can be expressed as

$$Y_k = (X^I_k + jX^Q_k)(V^I_{k,k} + jV^Q_{k,k})$$

$$+ \sum_{i=0}^{N-1} (X^I_{i} + jX^Q_{i})(V^I_{i,k} + jV^Q_{i,k}) + \tilde{Z}_k,$$

where $\tilde{Z}_k$ is a term related to the noise and

$$V^I_{\ell,k} = \frac{1}{2N} \left[ \Psi(\ell + \frac{1}{2} + k + \epsilon) + \Psi(\ell - \frac{1}{2} - k - \epsilon) \right]$$

$$+ \Phi(\ell + \frac{1}{2} + k + \epsilon) + \Phi(\ell - \frac{1}{2} - k - \epsilon)],$$

$$V^Q_{\ell,k} = \frac{1}{2N} \left[ \Gamma(\ell + \frac{1}{2} + k + \epsilon) + \Gamma(\ell - \frac{1}{2} - k - \epsilon) \right]$$

$$+ \Delta(\ell + \frac{1}{2} + k + \epsilon) + \Delta(\ell - \frac{1}{2} - k - \epsilon)],$$

$$\Psi(x) = \sin \left( \frac{\pi x}{2} \right) \cos \left( \frac{\pi x}{2} \right) / \sin \left( \frac{\pi x}{2N} \right),$$

$$\Psi(x) = \Phi(-x),$$

$$\Gamma(x) = \sin \left( \frac{\pi x}{2} \right) \sin \left( \frac{\pi x}{2} \right) / \sin \left( \frac{\pi x}{2N} \right),$$

$$\Gamma(x) = \Lambda(-x),$$

$$\Psi(0) = \Phi(0) = N \cos (\phi).$$

Fig. 1. Block diagram of a transforms-based multichannel system over a channel with additive noise.
Fig. 2. Block diagram of a multicarrier modulation system in the presence of CFO and phase error.

\[
\Gamma(0) = \Lambda(0) = N \sin(\phi),
\]
and \(\epsilon = 2T\Delta f\). Considering the phase error equal to zero (i.e. \(\phi = 0\)), it is satisfied that \(\Psi(x)\) is an even function and \(\Gamma(x)\) is an odd function, and therefore (7b) and (7c) can be simplified as follows:

\[
V_{\ell,k}^f = \frac{1}{2N} [\Psi(\ell + k + 1 - \epsilon) + \Psi(\ell - k - \epsilon)] + \Psi(\ell + k + 1 + \epsilon) + \Psi(\ell - k + \epsilon)] , 
\tag{8a}
\]

\[
V_{\ell,k}^Q = \frac{1}{2N} [\Gamma(\ell + k + 1 - \epsilon) + \Gamma(\ell - k - \epsilon)] - \Gamma(\ell + k + 1 + \epsilon) - \Gamma(\ell - k + \epsilon)] . 
\tag{8b}
\]

The exact BEP in the case of BPSK modulation for DCT4e-MCM is also expressed as in [13, eq. (34), p. 2119]

\[
P_{BPSK}^{DCT4e}(k) = \frac{1}{2} - \int_0^{\infty} \sin(\sqrt{\frac{\epsilon \sigma^2}{\sigma^2}}V_{\ell,k}^f) \cdot e^{-\frac{\omega^2}{2\sigma^2}} \times \prod_{\ell \neq k}^{N-1} \cos \left( \sqrt{\frac{\epsilon \sigma^2}{\sigma^2}}V_{\ell,k}^f \right) \cdot d\omega , 
\]

but \(V_{\ell,k}^f\) is given by (8a). It is worthwhile to mention that all the expressions included in [13] for the BEP in the case of using DCT2e-MCM are also valid for DCT4e-MCM, but considering eqs. (8a) and (8b).

IV. EXAMPLE DESIGN

In our first set of experiments, we consider in Fig. 2 128 subcarriers, and before proceeding with the multicarrier modulation, the data at each subcarrier were mapped by BPSK modulation. After mapping them, the parallel data were fed into the proposed DCT4e-MCM system, and also into a standardized OFDM system (DFT-MCM) to compare their performances. The resulting bit error rates obtained after the simulation in a discrete-time memoryless Gaussian channel are shown in Fig. 3 considering different normalized frequency offsets\(^2\) \(\Delta f T = 0.02, 0.1, 0.2\). As it can be seen in this figure, the BER of the proposed systems is similar to the DFT-MCM (OFDM) system for low frequency offset, whereas the performance of the DFT-based MCM system suffers from degradation when the frequency offset increases.

The ICI coefficients at \(k = 64\) and also at their adjacent subcarriers for the DCT4e-MCM are shown in Fig. 4, in the presence of two different CFO: \(\Delta f T = 0.2, 0.05\). To compare the above with those corresponding to DFT-MCM, we used the ICI analysis for DFT-MCM with frequency offset \(\xi = \Delta f T\) presented in [17]:

\[
S_{\ell} = \frac{\sin (\pi(\ell + \xi))}{N \sin \left( \frac{\pi(\ell + \xi)}{N} \right)} \left( \frac{\pi(N-1)(\ell+\xi)}{N} \right) , 
\tag{9}
\]

where the decision variable in the \(k\)th subcarrier is

\[
Y_k = X_k \cdot S_{\ell} + \sum_{\ell \neq k}^{N-1} X_{\ell} \cdot S_{\ell-k} + \tilde{z}_k; \ k = 0, \cdots, N-1. 
\tag{10}
\]

Fig. 4 is useful to see that ICI effects in adjacent subcarriers are greater for DFT-MCM than for DCT4e-MCM and DCT2e-MCM, since ICI coefficients in both DCT4e-MCM and DCT2e-MCM are more concentrated around the main coefficient than in DFT-MCM. In addition, we have obtained and plotted in Fig. 5 the signal-to-interference ratio (SIR) defined as in [13, p. 2117]. In this figure, less degradation in DCT4e-MCM and DCT2e-MCM for \(\Delta f T < 0.25\) can also be seen. Finally, Fig. 6 gives a comparison between two MCM systems using ideal channel (without noise) only in the presence of CFO. From this figure, it is seen that DCT4e-MCM and DCT2e-MCM outperform DFT-MCM for \(\Delta f T < 0.27\).

\(^2\)These values have been chosen according to those included in [13], [18].
of CFO.

DFT-MCM considering 128 subcarriers and BPSK modulation in the presence of CFO. In this work, we present a theoretical study of the ICI and the BEP for DCT4e-MCM systems working on the above conditions. In DCT4e-based MCM systems, a prefix and a suffix must be appended into each data symbol to be transmitted, besides imposing a symmetry condition for the channel impulse response by means of a front-end prefilter. The obtained results contribute in recommending the use of DCT4e-MCM for multicarrier communications.

Fig. 6. BER performance results for DCT4e-MCM, DCT2e-MCM, and DFT-MCM considering 128 subcarriers and BPSK modulation in the presence of CFO.

V. CONCLUSIONS

There are attractive features for using discrete trigonometric transforms for multicarrier communications. One of these features is that DTTs can outperform DFT-MCM in the presence of CFO. In this work, we present a theoretical study of the ICI and the BEP for DCT4e-MCM systems working on the above conditions. In DCT4e-based MCM systems, a prefix and a suffix must be appended into each data symbol to be transmitted, besides imposing a symmetry condition for the channel impulse response by means of a front-end prefilter. The obtained results contribute in recommending the use of DCT4e-MCM for multicarrier communications.

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