An Auction-based Mechanism for Spectrum Leasing in Overlay Cognitive Radio Networks

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Abstract—Spectrum leasing in cognitive radio networks has recently started to draw the attention of the researchers. According to this concept, the primary users own specific spectral resources and they decide how to lease them to the secondary users so as to optimize their performance. In this paper, we consider an overlay cognitive radio network where the secondary users are allowed to transmit simultaneously with the primary user in exchange for devoting a part of their power to relay the primary signal. Focusing on the distributed nature of cognitive radio networks, we propose an auction-based mechanism for the leasing process. The secondary users, who represent the bidders, decide the power that are willing to employ for relaying PU’s data whereas the primary user has the role of the auctioneer deciding the duration of the leasing process as well as the leasing time allocated to each secondary user. Furthermore, three different scenarios are studied, depending on the channel state information available to the secondary users. The performance of the proposed mechanism is investigated through the numerical analysis and the advantages of complete knowledge of channel state information are highlighted.

I. INTRODUCTION

The increasing demand of the scarce spectral resources and the problem of spectrum underutilization have led the researchers to the concept of cognitive radio (CR) [1]. This promising technology considers the simultaneous existence of two different types of users: the licensed users (or primary users, PUs) who have a license for specific frequency bands and the unlicensed users (secondary users, SUs). Depending on the spectrum sharing approach which is employed (interweave/overlay/underlay), the SUs are allowed or not to utilize the licensed spectrum [2]. More specifically, in the overlay model, which is considered in this paper, the SUs may co-exist with a PU but they have to devote a part of their transmit power to relay the PU’s message ameliorating its communication.

Except the classification of CR models based on the spectrum sharing approach, the CRs can be categorized according to the PUs’ awareness degree of the SUs’ existence. In this case, two basic approaches have emerged: the commons model and the property rights model [3]. The first model considers that the PUs are not aware of the presence of SUs whereas, in the property rights model, it is considered that the PUs own the spectral resources and they decide whether to lease them to the SUs or not, in exchange for the adequate compensation. Although the first model has been studied extensively, the property-rights model has recently started to draw the attention of the researchers. In [4], the authors consider a property-rights models where a PU can lease the owned spectrum to a certain number of SUs in exchange for cooperation. More specifically, the PU leases its spectrum to maximize its quality of service whereas the SUs choose whether to cooperate and they compete among themselves for transmission following a distributed power control mechanism. A similar scenario is studied in [5], where the PU sets the spectrum price so as to maximize its utility whereas the SUs choose the power levels for cooperation with the PU so as to obtain the corresponding spectrum access time. In both scenarios, the leasing problem is studied using the framework of Stackelberg games.

In this paper, we consider an overlay CR network and we propose a novel auction-based resource allocation mechanism for the spectrum leasing process, so as to optimize the performance of both the PU and the SUs. In the proposed mechanism, each SU submits as a bid to the PU the fraction of the power that he is willing to dedicate for relaying the PU's signal. Based on these bids, the PU computes the leasing time allocated to each SU and the duration of its direct transmission so as to optimize its performance. Furthermore, depending on the available channel state information (CSI) to the SUs, three different schemes are studied. The first scenario considers the case where only local CSI is available to the SUs, the second scenario assumes partial CSI knowledge whereas the third scenario considers that full CSI is available to each SU.

In Section 2, the system model under consideration is described whereas in Section 3, the resource allocation mechanism is analyzed both from PU’s and SUs’ point of view. Finally, Section 4 presents the numerical analysis of the proposed scheme and discussion on its performance and Section 5 concludes the paper.

II. SYSTEM MODEL

The system model under consideration is presented in Fig.1. The primary transmitter (PT) is assumed to communicate with the primary receiver (PR) in the same frequency band with N pairs of secondary transmitters (STs) and secondary receivers (SRs). In order to improve its performance, the PU decides whether and how to lease its spectral resources to the SUs.

Discrete-time, uncorrelated Nakagami-\(m\) fading channels are considered for the wireless links whereas the channels are assumed to be invariant within each timeslot but not between...
successive timeslots. The channel power gains between the primary link, the $i$th secondary link and the primary/secondary transmitter to the secondary/primary receiver are denoted as $h_p$, $h_{S_i}$, $h_{PS_i}$ and $h_{S_iP}$ correspondingly (Fig.1). Considering that the amplitude gains of the fading channels are modeled according to unit mean Nakagami distribution [6], the probability density function (pdf) of channel power gains is:

$$g(h_j) = \frac{m_j^{m_j} h_j^{m_j-1}}{\Gamma(m_j)} e^{-m_j h_j}$$

(1)

where $\Gamma(\cdot)$ is the Gamma function [7] and the parameter $m_j$ denotes the ratio of the line of sight (LoS) signal power to that of the multi-path component.

In our model, the overlay approach is considered, in the sense that the SUs are able to successfully decode the PU’s signal and to transmit it to the PR simultaneously with their own signal transmission [8], [9]. More specifically, each ST$_i$ is considered to be able to split its transmit power $P_s$ in two parts: the power’s fraction $a_i P_s$ is used for relaying the PU’s message while the remaining power’s fraction $(1 - a_i) P_s$ is used for its own transmission to the SR$_i$ ($0 \leq a_i \leq 1$).

Focusing on the distributed nature of CRs, a novel auction-based mechanism for the joint allocation of SUs’ power and the leasing time of PU’s spectrum is proposed. The overall procedure is depicted in Fig.1 for the duration of one timeslot. In particular, at the beginning of each timeslot, the SUs decide the fraction of their power that they are willing to dedicate for relaying PU’s signal and they send their power bids to the PU. Based on these bids, the PU decides whether to cooperate with the SUs, the duration of the spectrum leasing process ($t_{\text{leas}}$) as well as the time fraction which should be allocated to each SU ($t_i$). More specifically, in case that the PU wants to cooperate ($t_{\text{leas}} > 0$), each timeslot is separated in two individual phases. During the first phase, the PT transmits its data to the PR and to the STs that have better channel conditions of PT – ST$_i$ link compared to the primary link ($h_{PS_i} > h_p$). This phase is referred as the “STs-Receive Phase” and it has duration $t_{PU}$. In the second phase, that is referred as the “STs-Transmit Phase”, each ST$_i$ transmits both its own data and the PU’s data to the PR and the corresponding SR$_i$, utilizing the licensed spectrum for specific time $t_i$, based on a TDMA scheme.

III. AUCTION-BASED RESOURCE ALLOCATION MECHANISM

In this section, the proposed auction-based resource allocation mechanism is analyzed. In the first part, the auction mechanism is described analytically. Afterwards, three different scenarios for the non cooperative power control game among the SUs are presented depending on the CSI knowledge of the SUs. Finally, in the last subsection, the resource allocation problem from PU’s side is described.

A. Description of Auction Mechanism

In order to allocate the spectrum leasing time among the SUs in a distributed way, we model the resource allocation problem as an auction with specific rules. Each SU (bidder) places a bid $a_i \in [0, 1]$ equal to the transmit power’s fraction that he is willing to devote for relaying the PU’s message. Specifically, to compute their bids, the SUs participate in a non-cooperative power control game where each SU chooses its transmit power’s fraction $a_i$ so as to maximize its utility. In our approach, we study three different scenarios for the specific power control game depending on the degree of SUs’ CSI knowledge. From the other side, the PT has the role of the auctioneer deciding the time duration of the leasing process ($t_{\text{leas}}$) as well as the allocated time $t_i$ to each SU$_i$, based on the collected bids. Specifically, considering that the total duration of a timeslot is equal to $T = t_{PU} + t_{\text{leas}}$ and that $t_{\text{leas}} = \sum_{i \in N} t_i$, we assume that the PU leases its spectrum to the individual SU$_i$ for time equal to $t_i = \frac{a_i}{\sum_{i \in N} a_i} t_{\text{leas}}$. Thus, the spectrum leasing time of each SU is proportional to its contribution to the cooperative process of relaying PU’s message. For readability reasons, the allocation mechanism from the SUs’ side is presented first.

B. SUs’ Power Control Game

To study the power control problem among the SUs, we employ principles from non-cooperative game theory. Game theory is a broad field of applied mathematics which provides analytical tools to predict the outcome of interactions among subjects with conflicting objectives. Traditionally, the main areas of application of game theory have been economics. However, recently, it has also been widely used in telecommunications and particularly in the area of resource management.

In this subsection, three different scenarios for the non cooperative power control game among the SUs are described depending on the CSI that is available to them. In all the scenarios, each SU decides the fraction of the power $a_i$ that is willing to dedicate for relaying the PU’s message so as to maximize its utility function. Thus, the power control game can be defined as: $(N, a, \{U_S\}_{i=1}^N)$ where $N$ is the set of the selected SUs, $a = \{a_i\}_{i=1}^N$ is the set of allowed power strategies (different for each scenario) and $U_S$ denotes the $i$th SU’s utility function which is defined as the normalized
rate (expressed in bits/sec) that the SU \( i \) can achieve during a specific timeslot:
\[
U_{S_i}(a_i, \alpha_{-i}) = \left( t_i/T_s \right) R_{S_i} = \sum_{i \in N} a_i \frac{t_{i_{teas}}}{T_s} B \log_2 \left( 1 + \frac{(1-a_i)P_R h_{S_i}}{N_0 B} \right)
\]  

(2)

It should be noted that \( a_i \) represents the strategy of SU \( i \), whereas \( \alpha_{-i} \) denotes the vector of the strategies played by all the players except player \( i \). The parameters \( P_R \) and \( N_0 \) represent the licensed spectral bandwidth and noise power spectral density, correspondingly. Moreover, it has been considered that the SUs can subtract the PUs signal from the received signal at the SR using interference cancellation techniques [9].

Similarly to [10] and based on Theorems 1 and 2 of [11], it can be proven that the proposed games have a unique Nash equilibrium. In order to achieve the equilibrium point, each SU computes the solution of the equation \( \partial U_{S_i}/\partial a_i = 0 \) which is independent from the parameter \( t_{i_{teas}} \). Following, we describe the optimization problems that the SUs have to solve for the three considered scenarios.

1) Scenario A - Local CSI: In the specific scenario, we consider that each SU knows only the CSI of its own link (\( h_S \)) and its goal is to solve the following optimization problem:
\[
\max_{a_i} U_{S_i}(a_i, \alpha_{-i}) \\
\text{s.t. } (1-a_i)P_R h_{S_i} \leq Q
\]

(3)

which results in the strategy set \( a = \left\{ [a_i]_{i \in N} : (1 - Q/P_R h_{S_i})+ \leq a_i \leq 1 \right\} \)

2) Scenario B - Partial CSI: Each SU is considered to know the CSI of its own link and to be also aware of the interfering CSI from its transmitter to the primary receiver (\( h_{S,P} \)). In this scenario, in order to exploit this knowledge so as not to harm the communication of the primary link, the SUs optimize their utility function given specific interference power constraints. Thus, the following optimization problem has to be solved:
\[
\max_{a_i} U_{S_i}(a_i, \alpha_{-i}) \\
\text{s.t. } (1 - a_i)P_R h_{S,P} \leq Q
\]

(4)

3) Scenario C - Full CSI: This scenario assumes that the SUs are aware of the CSI in Scenario B together with the primary link’s CSI (\( h_P \)). The knowledge of both \( h_{S,P} \) and \( h_P \) allows the SUs to maximize their utility without deteriorating the signal to noise ratio (SNR) of the primary link. Hence, the optimization problem can be defined as:
\[
\max_{a_i} U_{S_i}(a_i, \alpha_{-i}) \\
\text{s.t. } (1-a_i)P_R h_{S,P} + h_P \leq Q
\]

(5)

leading to \( a = \left\{ [a_i]_{i \in N} : (1-a_i)h_{S,P} \leq a_i \leq 1 \right\} \). In this case, the SUs with \( a_i = 1 \) are considered to leave the game since their utility has a zero value.

It should be noted that the primary link’s and the interference ST-PR link’s channel gains can be obtained directly by listening to a feedback from the PU as proposed in [12]. Obviously, moving from Scenario A to Scenario C, the SUs have more complete knowledge of their environment’s CSI. However, at the same time, they are further restricted so as not to harm the communication of the PU. Therefore, each scenario has specific advantages and disadvantages for the PU and the SUs, as it is verified in the numerical analysis’ section.

C. PU’s Resource Allocation

In the previous subsection, the resource allocation scheme from the SUs’ side has been presented, describing in details how the SUs decide the optimal value of \( a_i \) to set as their bid. In this section, we focus on the resource management scheme from the PU’s side.

Specifically, it is considered that the PU computes the spectrum leasing time so as to optimize its performance in terms of throughput. The utility function of the PU is defined as its transmission rate and it can be expressed as \( U_P(t_{PU}) = R_P(t_{PU}) \) where \( R_P(t_{PU}) \) is the transmission rate of the primary link.

In order to be able to compute the total transmission rate of the primary link, we focus on the cooperative links PT-STs-PR considering that each ST plays the role of a relay (see Fig.2).

According to the system model, the achievable rates between PT – ST\(_1\) and ST\(_1\) – PR are correspondingly given by:
\[
R_{PS_i} = B \log_2 \left( 1 + \frac{h_{PS_i} P_P}{N_0 B} \right)
\]

(6)

and
\[
R_{S,P} = B \log_2 \left( 1 + \frac{a_i h_{S,P} P_s}{(1-a_i)h_{S,P} P_s + N_0 B} \right)
\]

(7)

where \( P_P \) denotes the transmit power of the PU. To simplify the proposed model, the PT-STs-PR links can be considered as a single link (PT – ST\(_{all}\) – PR) as it is depicted in Fig.2.

The achievable rate of PT – ST\(_{all}\) link is dominated by the worst channel gain \( h_{PS_i} \) and it can be given by:
\[
R_{PS_i} = \min_{i \in N} \left\{ R_{PS_i} \right\} = B \log_2 \left( 1 + \min_{i \in N} \frac{h_{PS_i} P_P}{N_0 B} \right)
\]

(8)

whereas the rate of the link between ST\(_{all}\) – PR can be expressed as the weighted sum of the ST\(_{all}\) – PR links’ rates:
\[
R_{S_{all} P} = \frac{t_1}{t_{teas}} R_{S_1 P} + \cdots + \frac{t_N}{t_{teas}} R_{S_N P} = \sum_{i \in N} \frac{t_i}{t_{teas}} R_{S_i P}
\]

(9)

It should be noted here that since only the SUs with better channel conditions of the PT-ST link compared to the primary

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Fig. 2. Simplification of the proposed system model
link have been selected \((h_{PS_i} > h_P)\), it holds that \(R_{PS_{\text{sat}}} \geq R_P\) where \(R_P = B \log_2 \left(1 + h_P P / (N_0 B)\right)\) denotes the rate of the direct link PT-PR.

Hence, the initial system model can be simplified to the half-duplex relay channel model depicted in Fig.2. Given that the “STs-Receive Phase” and the “STs-Transmit Phase” have not the same duration; the total achievable rate is given by [13]:

\[
R(t_{PU}) = \frac{1}{T} \min \left\{ t_{PU} R_{PS_{\text{sat}}} t_{PU} R_P + (T_s - t_{PU}) R_{PS_{\text{sat}}} P \right\} = \frac{1}{T} \min \left\{ t_{PU} R_{PS_{\text{sat}}} t_{PU} R_P + (T_s - t_{PU}) \sum_{i \in N} \frac{t_i}{t_i + s} R_{SP} \right\}.
\]

Thus, the PU’s resource allocation problem boils down to the following optimization problem (P1):

\[
\begin{aligned}
\max_{t_{PU}} & \min \{ f_1(t_{PU}), f_2(t_{PU}) \} \\
\text{s.t.} & \ 0 \leq t_{PU} \leq T_s
\end{aligned}
\]

**Theorem:** The optimal solution \(t_{PU}^*\) of (P1) is given by the following expression:

\[
t_{PU}^* = \begin{cases} 
T_s & \text{if } R_P > R_{PS_{\text{sat}}} \\
\frac{R_{PS_{\text{sat}}} P T_s}{R_{PS_{\text{sat}}} P t_{PU} + R_{PS_{\text{sat}}} P} & \text{if } R_P \leq R_{PS_{\text{sat}}} 
\end{cases}
\]

**Proof:** To find the optimal solution of (P1), we define the functions: \(f_1(t_{PU}) = t_{PU} R_{PS_{\text{sat}}} \) and \(f_2(t_{PU}) = t_{PU} R_P + (T_s - t_{PU}) R_{PS_{\text{sat}}} P\). Hence, the optimization problem can be defined as:

\[
\max_{t_{PU}} \left\{ \min \left\{ f_1(t_{PU}), f_2(t_{PU}) \right\} \right\} \\
\text{s.t. } 0 \leq t_{PU} \leq T_s
\]

As it can be seen, \(f_1, f_2\) are linear functions of \(t_{PU}\). Furthermore, \(f_1\) constitutes a strictly increasing function of \(t_{PU}\) whereas \(f_2\) can be either a strictly increasing function of \(t_{PU}\) (in case \(R_P > R_{PS_{\text{sat}}} P\)) or a decreasing function of \(t_{PU}\) (in case \(R_P \leq R_{PS_{\text{sat}}} P\)). In the first case, the optimal \(t_{PU}^*\) is equal to \(T_s\) which means that, in this case, the direct communication is more beneficial for the PU. Otherwise, the optimal value of \(t_{PU}\) can be found from the intersection of \(f_1\) and \(f_2\). Given that \(R_{PS_{\text{sat}}} \geq R_P\), the following conditions hold:

\[
\begin{aligned}
f_1(0) &= 0 < T_s R_{PS_{\text{sat}}} = f_2(0) \quad \text{(14)} \\
f_1(T_s) &= T_s R_{PS_{\text{sat}}} \geq T_s R_P = f_2(T_s) \quad \text{(15)}
\end{aligned}
\]

Thus, there is exactly one optimal \(t_{PU}^*\) such that \(f_1(t_{PU}^*) = f_2(t_{PU}^*)\) that is given by \(t_{PU}^* = \frac{R_{PS_{\text{sat}}} P T_s}{R_{PS_{\text{sat}}} P t_{PU} + R_{PS_{\text{sat}}} P}\). Combining both cases, Theorem 1 is proven.

**IV. Numerical Analysis**

In this section, the performance of the proposed resource allocation mechanism is investigated through the numerical analysis. For the simulation scenario in Matlab, we have considered a CR network with one PU and 5 SUs’ pairs. Typical values such as \(T_s = 10^{-3}\)s, \(B = 10^6\)Hz and \(N_0 B = 1\) have been considered. Moreover, without loss of generality, we have assumed that the Nakagami parameters of all the links have the same value \(m_s = m_{SP} = m_{PS} = m_P = 1\) (Rayleigh fading) whereas the transmit powers of both the PU and the SUs are considered equal to \(P_S = P_P = 1W\). Finally, the interference constraint for the Scenario B is considered equal to \(Q = 0dB\) unless it is otherwise stated.

In Fig.3, the main characteristics of the proposed auction (bids/allocations/utility) at a specific timeslot are presented, for all the scenarios. The x-axis represents the five SUs sorted with ascending values of channel gain \(h_{SP}\). The first subfigure depicts the bids of each SU representing the transmit power’s fraction that each SU chooses to dedicate for relaying PU’s data. As it can be seen, for the Scenario A, where the SUs are aware only of their own CSI, the power bids are almost equal for all the SUs. In case of the Scenario B, we can observe that the imposed interference constraint has led the SUs with better channel conditions of the ST-PR link, to devote the greatest part of their transmission power for cooperation in order not to interfere to the primary link. On the other hand, in Scenario C, the SUs with the worst values of channel gain \(h_{SP}\) are those who place higher bids so as to maintain the primary link’s communication quality.

The second subfigure represents the normalized allocated time allocated to each SU \((t_i/T_s)\). As it is expected the distribution of the allocated time among the SUs depends on the power bids of each SU and the total leasing time of each scenario. Finally, the last subfigure depicts the utility of the SUs. As it can be observed, Scenario A leads to the highest values of the instantaneous utility for all the SUs whereas Scenario B results in the worst values of SU’s utility. This remark can be explained considering that in Scenario A, the SUs may not be aware of the CSI but on the same time they do not have any constraints allowing them to employ higher power levels for their own transmission and to increase their utility. On the contrary, the other two scenarios are stricter for the SUs as they are forced to use higher power levels for cooperation in order not to harm the communication of the primary link.

Fig.4 and Table I show the average performance of the proposed mechanism during a frame \(T_f = 300 \cdot T_s\) and con-
considering \( Q = -5 \text{dB} \). The parameter defined as “PU’s Gain” represents the ratio of the PU’s utility using the proposed mechanism to its utility employing direct communication. Furthermore, the “cooperation index” is defined as the percentage of the time that the PU chooses to cooperate with the SUs. The first basic remark, observing the gain of PU in Table I, is that the proposed resource allocation mechanism leads to better performance compared to the direct PU’s communication for all the scenarios. Moreover, from Fig. 4, it can be observed that Scenario A may lead to higher instantaneous total utility but there is higher probability that the PU chooses not to cooperate leading to zero utility for the SUs. This remark can be verified from the low value of the cooperation index in Table I. On the other hand, Scenario B results in lower values of total utility but higher probability of cooperation leading in higher gain for the PU. Finally, Scenario C has the best performance in terms of SUs’ utility as it is also proven from the high values of cooperation index and the total SUs’ utility in Table I. Thus, it can be seen that the additional knowledge of CSI is more advantageous in terms of average performance, either for the PU or the SUs.

### V. Conclusion

This paper presents a novel resource allocation mechanism for overlay CR networks. Focusing on the distributed nature of CR, an auction-based mechanism is proposed for the joint allocation of SUs’ power and the leasing time of PU’s spectral resources. The PU has the role of the auctioneer allocating the leasing time to the SUs so as to optimize its performance. On the other hand, the bidders of the auction are represented by the SUs whereas the bid of each SU denotes the optimal power level that he is willing to dedicate for relaying the PU’s message. In order to present a complete analysis, three different scenarios are presented depending on the available CSI to the SUs. The numerical results show the efficiency of the proposed mechanism highlighting the advantages and disadvantages of each scenario. In the future, we plan to study the leasing problem considering an energy-efficient mechanism and also to compare the proposed mechanism with other schemes in the literature, so as to depict the benefits and weaknesses of each method. In the future, the leasing problem considering an energy-efficient mechanism will be studied and a comparison of the proposed mechanism with other schemes in the literature will be performed.

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