

Localizing global hedgehogs on the brane

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Abstract

We investigate the localization of 4D topological global defects on the brane embedded in 5D. The defects are induced by 5D scalar fields with a symmetry-breaking potential. Taking an ansatz which separates the scalar field into the 4D and the extra-D part, we find that the static-hedgehog configuration is accomplished and the defects are formed only in the AdS_4/AdS_5 background. In the extra dimension, the localization amplitude for the 4D defects is high where the warp factor is high.

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I. INTRODUCTION

Since the large extra dimension scenario was introduced [1,2], a lot of attention has been paid to localizing matter fields on the brane worlds (see, for example, Refs. [3–5]). The matter fields one can consider in the field theory are the scalar field, gauge field, and spinor field. For two-brane models, the localization (normalization) of such fields is always possible by an appropriate compactification of the extra dimensions bounded by the branes. For one-brane models, however, the success of localization depends on the background bulk geometry and the field configuration in the bulk.

As an example in 5D, for free-boson fields of spin zero and one, the action is

$$S = \int d^4x dy \sqrt{-g} \left(-\frac{1}{2} \partial_A \Phi \partial^A \Phi - \frac{1}{4} F_{AB} F^{AB} \right), \quad (1)$$

where the indices run over the whole five dimensions, and the field strength of the spin one field is determined by the gauge field, $F_{AB} = \partial_A W_B - \partial_B W_A$. In order to consider localization, one usually applies a “separation ansatz” on the fields,

$$\Phi(x^\mu, y) = \phi(x^\mu) p(y), \quad (2)$$

$$W_\alpha(x^\mu, y) = \omega_\alpha(x^\mu) q(y), \quad W_y = 0. \quad (3)$$

With a metric, for example,

$$ds^2 = B(y) \left(\bar{g}_{\mu\nu} dx^\mu dx^\nu + dy^2 \right), \quad (4)$$

one can saturate the 4D part in the action as

$$\begin{aligned} S = & \int dy B^{3/2}(y) p^2(y) \int d^4x \sqrt{-\bar{g}} \left(-\frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) + \dots \\ & + \int dy B^{1/2}(y) q^2(y) \int d^4x \sqrt{-\bar{g}} \left(-\frac{1}{4} \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta} f_{\mu\nu} f_{\alpha\beta} \right) + \dots, \end{aligned} \quad (5)$$

where $f_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ is the usual 4D field strength. The normalization of these 4D fields is thus determined by the warp geometry $B(y)$ and the transverse profile of the fields, $p(y)$ and $q(y)$. The field is normalizable if the y -integration is finite.

In this paper, we are interested in the localization of 4D topological global defects which are induced by 5D scalar-field multiplets Φ^a , with a symmetry-breaking potential $V(\Phi^a)$. Being different from the “usual” scalar field, a special ansatz for the scalar field, so called, the “hedgehog configuration” is imposed in order to realize topological defects; the scalar field asymptotically approaches the vacuum-expectation value (the symmetry-breaking scale) at large distances from the center while it remains in the unbroken-symmetry state at the center [6]. It is interesting to see if such extended bodies can be induced in 4D by 5D scalar fields, and to see how they are localized in the extra dimension. We shall impose this ansatz along the 4D part of the 5D scalar field, and investigate the localization behavior by the transverse part of the scalar field.

As usual in the investigation of the localization of the other matter fields, we do not consider self-gravity of the scalar field. The background geometry is fixed.

We require the field to be “separable” into the 4D part and the transverse part. Then as we observed above, the localization of 4D defects will be described by the transverse part of Φ^a and the background geometry.

II. FORMATION OF 4D HEDGEHOGS INDUCED BY 5D SCALAR FIELDS

We extend the usual 4D action for the scalar field with a symmetry-breaking potential, which gives rise to topological defects. The 5D action then reads

$$S = \int d^4x dy \sqrt{-g} \left[-\frac{1}{2} \partial_A \Phi^a \partial^A \Phi^a - \frac{\lambda}{4} (\Phi^a \Phi^a - \eta^2)^2 \right] - \int_i d^4x \sqrt{-h} \Sigma_i(\Phi^a), \quad (6)$$

where the spacetime index A runs over 5D, a represents the field-space index, and η is the symmetry-breaking scale. The boundary term appears when the model involves nonzero-tension branes located at $y = y_i$.

We use a conformal nonfactorizable metric ansatz,

$$ds^2 = B(y) (\bar{g}_{\mu\nu} dx^\mu dx^\nu + dy^2). \quad (7)$$

We do not consider self-gravity of the scalar field, so the background geometry [$B(y)$ and $\bar{g}_{\mu\nu}$] is given by introducing other matter fields such as a bulk cosmological constant, or the brane tension.

For the scalar field, we apply a strictly “separable” ansatz as usual,

$$\Phi^a(x^\mu, y) = \phi^a(x^\mu)p(y), \quad (8)$$

and apply a “static-hedgehog” ansatz on the 4D part in order to realize topological defects in 4D,

$$\phi^a(x^\mu) = f(r)\hat{\Omega}^a. \quad (9)$$

Here, $\hat{\Omega}^a$ is the unit vector surfacing on a sphere S^{a-1} , and $a = 1, 2, 3$ for domain walls, cosmic strings, and monopoles, respectively. The static radial configuration $f(r)$ asymptotically approaches a constant, while it takes $f(0) = 0$. This is a nontrivial configuration which gives rise to the hedgehog configuration.

With the above ansätze for the metric and the scalar field, the 4D part of the action saturates as we had seen in Introduction,

$$S = \int dy \mathcal{A}(y) \int d^4x \sqrt{-g} \left(-\frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^a \right) + \dots, \quad (10)$$

where $\mathcal{A}(y) = B^{3/2}(y)p^2(y)$ is the normalization factor, and is interpreted as the amplitude of finding the 4D configuration at the given location in the transverse direction. This quantity $\mathcal{A}(y)$ determines the localization after the y -profile $p(y)$ is obtained.

The field equation for Φ^a is given by

$$\nabla^A \partial_A \Phi^a = \frac{\partial V(\Phi^a)}{\partial \Phi^a} + \frac{\sqrt{-h}}{\sqrt{-g}} \frac{\partial \Sigma_i(\Phi^a)}{\partial \Phi^a} \delta(y - y_i), \quad (11)$$

where h is the 4D metric density including $B(y)$. This equation reduces to

$$\frac{p''}{p} + \frac{3}{2} \frac{B'}{B} \frac{p'}{p} + \frac{\square_{(4)} \phi^a(x^\mu)}{\phi^a(x^\mu)} = B \lambda (p^2 f^2 - \eta^2) + B \frac{\sqrt{-h}}{\sqrt{-g}} \left(\frac{1}{\Phi^a} \frac{\partial \Sigma_i}{\partial \Phi^a} \right) \delta(y - y_i), \quad (12)$$

where the prime denotes the derivative with respect to y .

Concerning the separation of variables, each term in the above equation is a function of only x^μ , or y except the one $\lambda B(y)p^2(y)f^2(r)$. The way to complete the separation is to make this term depend on only one variable. First, consider $f(r) = \text{constant}$, which makes the term only y -dependent. However, $f(r) = \text{constant}$ is a trivial solution which does not produce the hedgehog configuration in 4D. Therefore, we require a condition for the separation,

$$B(y)p^2(y) = \text{constant} \equiv c_1. \quad (13)$$

Then the separation becomes complete, and Eq. (12) separates into two parts,

$$\frac{p''}{p} + \frac{3}{2} \frac{B'}{B} \frac{p'}{p} + \lambda \eta^2 B - \sqrt{B} \left(\frac{1}{\Phi^a} \frac{\partial \Sigma_i}{\partial \Phi^a} \right) \delta(y - y_i) \equiv m^2, \quad (14)$$

$$\frac{\square_{(4)} \phi^a(x^\mu)}{\phi^a(x^\mu)} - \lambda c_1 f^2(r) = -m^2, \quad (15)$$

where m^2 is a constant.

First, note that the 4D equation (15) is nothing but the usual 4D scalar-field equation with a 4D effective symmetry-breaking potential,

$$\overline{V}(\phi^a) = \frac{\overline{\lambda}}{4} (\phi^a \phi^a - \overline{\eta}^2)^2. \quad (16)$$

We can identify

$$\overline{\lambda} = \lambda c_1, \quad (17)$$

$$m^2 = \overline{\lambda} \overline{\eta}^2. \quad (18)$$

The latter relation (18) states that m defined in Eqs. (14) and (15) behaves like the mass of the 4D perturbed-scalar field where the symmetry is broken. We will see later that the sign of m^2 plays an important role in defect formation.

Second, we can cast the extra-D equation (14) in a differential equation only for B by using the separation condition $Bp^2 = c_1$ [Eq. (13)]. Then the equation leads to

$$B'' + 2m^2 B - 2\lambda \eta^2 B^2 + 2B^{3/2} \left(\frac{1}{\Phi^a} \frac{\partial \Sigma_i}{\partial \Phi^a} \right) \delta(y - y_i) = 0. \quad (19)$$

As a result, the separation condition converted the scalar-field equation to an Einstein equation.

From the beginning we have ignored self-gravity of the scalar field, and have assumed that the background geometry is given and fixed by some other matter fields. Such a given background, however, does not necessarily satisfy the above Einstein equation (19). Therefore, what is remaining is to investigate which backgrounds have Eq. (19) be consistent with their Einstein equations.

Let us analyze Eq. (19). The last term is the boundary term which arises when there exist nonzero-tension branes. The brane tension σ_i is related as

$$\kappa^2 \sigma_i = 3 \left[\frac{1}{\Phi^a} \frac{\partial \Sigma_i(\Phi^a)}{\partial \Phi^a} \right]_{y=y_i}, \quad (20)$$

where $\kappa^2 = 8\pi G_5$.

The quadratic term in B arises when there exists a bulk cosmological constant Λ_5 which is to be related to the parameters from Eq. (19),

$$\Lambda_5 = -2\lambda\eta^2. \quad (21)$$

The linear term corresponds to the 4D curvature term, and the parameter is related as

$$\bar{R}_{(4)} = -6m^2. \quad (22)$$

If the background-matter fields are provided in such a way to satisfy the above relations, Eq. (19) is completely satisfied.

From $\Lambda_5 = -2\lambda\eta^2$, Λ_5 can be either positive, or negative depending on the sign of λ . If we keep $V(\Phi^a)$ in the conventional shape of the symmetry-breaking potential, i.e., $\lambda > 0$ and $\eta^2 > 0$, Λ_5 is negative. Therefore, the bulk geometry is AdS_5 . Λ_5 is a free parameter, and its value is not restricted.

From $\bar{R}_{(4)} = -6m^2$, the 4D world-volume has a constant curvature. There are several types of manifold of which curvature is constant. First, we can consider the 4D Minkowski manifold of which curvature vanishes, $\bar{R}_{(4)} = -6m^2 = 0$. Second, we can consider a manifold

with a 4D cosmological constant, and the curvature is $\bar{R}_{(4)} = -6m^2 = 4\Lambda_4$. Therefore, the possible 4D manifold is one of M_4 , dS_4 , and AdS_4 .

Let us examine these three 4D manifolds. The 4D curvature is related with the mass parameter, $m^2 = \bar{\lambda}\bar{\eta}^2$ from Eq. (18).

(i) M_4 : For the Minkowski manifold, $m^2 = 0$. In the effective 4D potential $\bar{V}(\phi^a)$ in Eq. (16), the mass term (quadratic term) is missing, and there exist only the constant term and the quartic term of ϕ^a . Therefore, the nonzero vacuum-expectation value (VEV) of ϕ^a does not develop, and there is no symmetry breaking along the world-volume direction. Defects are not formed.

(ii) dS_4 : For the de Sitter manifold, $m^2 = \bar{\lambda}\bar{\eta}^2 = -\frac{2}{3}\Lambda_4 < 0$. First, if $\bar{\lambda} < 0$ while $\bar{\eta}^2 > 0$, the effective 4D potential \bar{V} is an upside-down form of the usual symmetry-breaking potential. It is hard to expect that a stable hedgehog configuration is achieved with this type of potential. Second, if $\bar{\eta}^2 < 0$ while $\bar{\lambda} > 0$, the potential has the absolute minimum at $|\phi^a| = 0$ with a nonzero-vacuum energy.¹ There does not develop a nonzero VEV, and the situation is similar to M_4 . No defects are formed.

(iii) AdS_4 : For the Anti-de Sitter manifold, now the potential \bar{V} has the shape of the usual symmetry-breaking potential. The nonzero VEV, $|\phi^a| = \bar{\eta}$, develops and the defects are formed.

As a result, in generating the hedgehog configuration along the 4D world-volume with the 5D scalar field and the symmetry-breaking potential, the only possible manifold is AdS_4/AdS_5 .

We have assumed that self-gravity of the scalar field is negligible, and the background

¹If we apply a perturbation on the scalar field about the vacuum, $|\phi^a| = 0$, in this case, the perturbed-scalar field has $(\text{mass})^2 = -\bar{\lambda}\bar{\eta}^2/2 > 0$. Therefore, m identified as in Eq. (18) does not exactly represent the mass of the perturbed-scalar field in all cases. We had better interpret Eq. (18) as just a relation between the parameters.

geometry is given by other matter fields. In this case, the background is AdS_4/AdS_5 and it is generated by Λ_5 and Λ_4 .² Meanwhile, as we observed above, these quantities are related with the parameters of the scalar field by $\Lambda_5 = -2\lambda\eta^2$ and $\Lambda_4 = -\frac{3}{2}\bar{\lambda}\bar{\eta}^2$. Then one may wonder how consistently we can keep the parameters Λ_5 and Λ_4 as the source of curvature, while we are ignoring the corresponding scalar-field parameters. The clue is that the scalar-field contribution to the curvature (to the energy-momentum tensor, in other words) is suppressed by the Planck-mass scale, so the condition, $\eta \ll \text{Planck-scale}$, is sufficient to ignore its effect, and the same reasoning is true for Λ_4 and the 4D parameters.

III. REVIEW OF AdS_4/AdS_5

In the previous section, we observed that the 4D hedgehog configuration is realized in the AdS_4/AdS_5 background. In this section, we review the localization of gravity in this background, which was investigated by Karch and Randall [7],³ and Kogan, *et. al.* [8]. We shall focus on the two-brane model later on.

The Einstein-Hilbert action for gravity is

$$S_g = \int d^4x dy \sqrt{-g} \left[\frac{R - 2\Lambda_5}{2\kappa^2} - \frac{\sqrt{-h}}{\sqrt{-g}} \sigma_i \delta(y - y_i) \right]. \quad (23)$$

With the metric ansatz,

$$ds^2 = B(y)(\bar{g}_{\mu\nu}^{AdS} dx^\mu dx^\nu + dy^2), \quad (24)$$

the solution to Einstein equations is given by

²In a general sense, Λ_4 is not regarded as a source of the curvature. Instead, the brane tension is a source, and Λ_4 is an induced quantity by this tension and Λ_5 by matching boundary conditions. Here, however, we are not going to be bothered by such a discretion.

³Localization of the ordinary-matter fields in this set-up (one-brane model) was investigated in Ref. [9].

$$B(y) = 2 \frac{\Lambda_4}{\Lambda_5} \sec^2 \left[\sqrt{-\frac{\Lambda_4}{3}} (|y| - y_0) \right], \quad (25)$$

where y_0 is an integration constant which can be fixed if we apply a normalization condition on B [e.g., $B(0) = 1$]. However, we will not do this, but leave y_0 free.

The 4D cosmological constant Λ_4 is determined by Λ_5 and the brane tension,

$$\sigma_I = \frac{\sqrt{-6\Lambda_5}}{\kappa^2} \sin \left(\sqrt{-\frac{\Lambda_4}{3}} y_0 \right), \quad (26)$$

$$\sigma_{II} = \frac{\sqrt{-6\Lambda_5}}{\kappa^2} \sin \left[\sqrt{-\frac{\Lambda_4}{3}} (y_* - y_0) \right], \quad (27)$$

where brane I (II) is located at $y = 0$ ($y = y_*$).

The warp factor $B(y)$ is a periodic function which diverges at

$$\sqrt{-\frac{\Lambda_4}{3}} (|y| - y_0) = \left(n + \frac{1}{2} \right) \pi. \quad (28)$$

In order to avoid the difficulties coming from the divergence, we compactify the bulk to contain only the regular region bounded by the branes. Without loss of generality, we require $y_0 > 0$. The warp factor $B(y)$ falls as y increases from brane I at $y = 0$, and reaches the minimum at $y = y_0$, and then increases. The tension of brane I is positive ($\sigma_I > 0$), and the tension of brane II changes its sign depending on the the brane location, $\sigma_{II} >=< 0$ for $y_* >=< y_0$. As was indicated in the previous section, these tensions are related with the boundary terms in the scalar-field action (6) by the relation (20), for consistency.

To investigate the localization of gravitons, introduce a perturbation to the background metric,

$$ds^2 = g_{AB}^{(bg)} dx^A dx^B + h_{\mu\nu} dx^\mu dx^\nu, \quad (29)$$

where $g_{AB}^{(bg)}$ represents the background metric (24), and

$$h_{\mu\nu}(x^\mu, y) = h(y) \hat{e}_{\mu\nu}(x^\mu). \quad (30)$$

From the field equations,

$$\square_{(bg)} h_{MN} + 2R_{MANB}^{(bg)} h^{AB} = 0, \quad (31)$$

$$\square_{(4)} \hat{e}_{\mu\nu}(x^\mu) = \left(m_g^2 + \frac{2}{3} \Lambda_4 \right) \hat{e}_{\mu\nu}(x^\mu), \quad (32)$$

and by rescaling $\hat{h}(y) = B^{-1/4} h(y)$, we obtain the nonrelativistic Schrödinger-type equation for the graviton,

$$\left[-\frac{1}{2} \frac{d^2}{dy^2} + U(y) \right] \hat{h}(y) = \frac{m_g^2}{2} \hat{h}(y), \quad (33)$$

where

$$U(y) = -\frac{\Lambda_4}{3} \left\{ -\frac{9}{8} + \frac{15}{8} \sec^2 \left[\sqrt{-\frac{\Lambda_4}{3}} (|y| - y_0) \right] \right\} + \tilde{\Sigma}_I \delta(y) + \tilde{\Sigma}_{II} \delta(y - y_*), \quad (34)$$

$$\tilde{\Sigma}_I = \frac{3}{2} \sqrt{-\frac{\Lambda_4}{3}} \tan \left(\sqrt{-\frac{\Lambda_4}{3}} y_0 \right), \quad (35)$$

$$\tilde{\Sigma}_{II} = \frac{3}{2} \sqrt{-\frac{\Lambda_4}{3}} \tan \left[\sqrt{-\frac{\Lambda_4}{3}} (y_* - y_0) \right]. \quad (36)$$

The zero-mode solution ($m_g^2 = 0$) to this equation is given by

$$\hat{h}_0(y) = a_1 \sec^{3/2} \left[\sqrt{-\frac{\Lambda_4}{3}} (|y| - y_0) \right], \quad (37)$$

where a_1 is the normalization constant, and the massive-mode solutions are described the hypergeometric function.

The zero-mode (37) is again a *sec*-function which diverges where the warp factor B diverges. Therefore, in the one-brane model which bounds to the singular points, the zero-mode is not normalizable. This zero-mode does not recover 4D gravity [7]. However, in the two-brane model, it is manifestly normalizable by placing the second brane at an appropriate position [8].

In addition to the zero-mode, the graviton has an almost massless mode which is the first-excited massive mode, \hat{h}_1 . In the one brane model of Ref. [7], this ultra-light mode was responsible for 4D gravity while the zero mode is absent (because it is not normalizable). In the two brane model of Ref. [8], this mode contributes to 4D gravity almost equally to the normalizable zero-mode. In the latter, authors placed the second brane at the symmetric

point about the minimum of B , i.e., at $y = y_* = 2y_0$. And, they also discovered that 4D gravity remains in the same picture even when the second brane is displaced very far from the symmetric point, $y_* \gg 2y_0$.

In the next section, we shall discuss the hierarchies in the two-brane set-up. We shall place the second brane very far from the minimum of B , i.e., $y_* \gg y_0$ in order to achieve proper hierarchies of physical quantities between the two branes. With this brane separation, we shall discuss the localization of the 4D defects by looking at the amplitude $\mathcal{A}(y)$. Meanwhile, gravity will be localized in a similar manner to the symmetric case $y_* = 2y_0$ as was stated above, so we will not be concerned any further.

IV. HIERARCHIES AND DEFECT LOCALIZATION

In this section, we discuss the hierarchies of various physical quantities in the literature, and the localization amplitude of defects. We assume a two-brane model where the second brane is displaced far from the symmetric point. For the symmetric case ($y_* = 2y_0$) in Ref. [8], there is no hierarchy between the two branes since the warp factor B (which controls the hierarchy) is the same at the two branes.

A. Particle mass

The hierarchy is achieved as the warp factor flows along the transverse direction. The mass hierarchy of Higgs-type particles is explicitly described as (for derivation, readers see Ref. [2])

$$v_{eff} \propto \sqrt{B(y = y_i)}, \quad (38)$$

where v_{eff} is the mass scale and y_i is the location of the brane. Therefore, the particle-mass hierarchy between the two branes is given by

$$\frac{v_{eff}(y = 0)}{v_{eff}(y = y_*)} = \left[\frac{B(0)}{B(y_*)} \right]^{1/2} \equiv \mu. \quad (39)$$

For example, to achieve the TeV/Planck-scale hierarchy, $\mu \approx 10^{-15}$. In this work, however, we will not restrict the absolute scale of hierarchies between the branes. Instead, we shall discuss qualitatively how physical properties flow along the extra dimension, and compare their relative picture between the branes.

B. G_4

The 4D gravitational constant at $y = y_i$ is given usually by the zero-mode graviton,

$$G_4(y = y_i) = G_5 |\hat{h}_0(y = y_i)|^2 / e_l \propto B^{3/2}(y_i), \quad (40)$$

where the length unit e_l was included to match the dimension. In AdS_4/AdS_5 , the ultra-light massive mode \hat{h}_1 will contribute to G_4 almost equally, but the above proportionality will not be altered much. Therefore,

$$\frac{G_4(0)}{G_4(y_*)} = \left[\frac{B(0)}{B(y_*)} \right]^{3/2} = \mu^3. \quad (41)$$

The flowing picture is similar to that of the particle mass with only a different power.

C. Effective 4D cosmological constant

Now let us discuss the effective 4D cosmological constant on the brane. For $(A)dS_4$ branes, the 4D metric at $y = y_i$ is given by

$$ds_4^2(y = y_i) = B(y) \left[\bar{g}_{\mu\nu}^{(A)dS} dx^\mu dx^\nu \right] \quad (42)$$

$$= B(y_i) \left(1 + \frac{\Lambda_4}{12} l^2 \right)^{-2} (-dt^2 + \gamma_{ij} dx_i dx_j). \quad (43)$$

where l^2 is the Lorentzian-length element,⁴ and $\gamma_{ij}dx^i dx^j$ is the 3D flat metric. Absorb $B(y_i)$ by rescaling the coordinates,⁵ then the metric leads to

$$ds_4^2(y = y_i) = \left[1 + \frac{\Lambda_4}{12B(y_i)}L^2 \right]^{-2} (-dT^2 + \Gamma_{ij}dX^i dX^j), \quad (44)$$

where $\Gamma_{ij}dX^i dX^j$ is again the 3D flat metric in the rescaled coordinates. In this new coordinates, the effective 4D cosmological constant is given by

$$\Lambda_4^{eff}(y = y_i) = \frac{\Lambda_4}{B(y_i)}. \quad (45)$$

This Λ_4^{eff} is normalized if we normalize $B(y)$. For example, if we normalize as $B(0) = 1$, then $\Lambda_4^{eff}(0) = \Lambda_4$. However, since we are interested only in the relative scales between the two branes, we will not do such a normalization.

The resulting effective vacuum-energy density can be obtained by

$$\rho_{\Lambda_4}^{eff}(y = y_i) = \frac{\Lambda_4^{eff}(y_i)}{8\pi G_4(y_i)} \propto \frac{\Lambda_4}{B^{5/2}(y_i)}, \quad (46)$$

and we get the ratio,

$$\frac{\rho_{\Lambda_4}^{eff}(0)}{\rho_{\Lambda_4}^{eff}(y_*)} = \left[\frac{B(0)}{B(y_*)} \right]^{-5/2} = \mu^{-5}. \quad (47)$$

The effective vacuum-energy density as well as the effective 4D cosmological constant flows along the the extra dimension in the opposite way to the other physical quantities discussed before. Their scale is small where the warp factor is large.

Wherever our universe is located in the extra dimension, the effective 4D cosmological constant should be bounded by some limit. The observational data show that the recent

⁴For cartesian coordinates (x_1, x_2, x_3) , $l^2 = -t^2 + x_1^2 + x_2^2 + x_3^2$, for cylindrical coordinates (r, θ, z) , $l^2 = -t^2 + r^2 + z^2$, and for spherical coordinates (r, θ, φ) , $l^2 = -t^2 + r^2$.

⁵The coordinates are rescaled by $X^\mu = \sqrt{B(y_i)}x^\mu$ (the angular coordinates are not rescaled, but remain unchanged), thereby $L = l\sqrt{B(y_i)}$.

accelerating expansion of our universe prefers a positive cosmological constant. However, milder restriction on the cosmological constant is given by the Weinberg window [10],

$$-10^{-120} M_{Pl}^4 < \rho_{\Lambda_4}^{eff} < 10^{-118} M_{Pl}^4, \quad (48)$$

which admits negative values, but very small.

D. $\mathcal{A}(y)$

Finally let us discuss the localization amplitude $\mathcal{A}(y)$ of the 4D defects. \mathcal{A} was defined from the normalization (10) of the scalar field along the extra dimension. Imposing the separation condition (13), it becomes

$$\mathcal{A}(y) = B^{3/2}(y)p^2(y) = c_1 \sqrt{B(y)}. \quad (49)$$

The ratio between the two branes becomes

$$\frac{\mathcal{A}(0)}{\mathcal{A}(y_*)} = \left[\frac{B(0)}{B(y_*)} \right]^{1/2} = \mu. \quad (50)$$

The flow-pattern of the amplitude is the same with that of the particle mass v_{eff} .

We may interpret the amplitude $\mathcal{A}(y)$ as the probability density to find 4D defects at a given location y . Then, from the above result the defects can be more likely localized where the warp factor is large.

Suppose that our universe is located on brane II where the warp factor is large. Compared with brane I, the effective 4D cosmological constant can be relatively very small, while the localization amplitude of defects is very high. However, in this case, gravity becomes strong compared with that on brane I.

In this work, we have not been very interested in the absolute values of the physical quantities (G_4 , Λ_4^{eff} , etc.), since we have been concerned mainly on the qualitative picture of the hierarchy and the defect localization. Therefore, we do not need to evaluate the values of the constants involved with the model such as y_0 , y_* , and a_1 , but let us complete this

section with the derivation of c_1 , the constant appearing in the separation condition. It is evaluated from the normalization,

$$N = \int_{-y_*}^{y_*} \mathcal{A}(y) dy = c_1 \int_{-y_*}^{y_*} \sqrt{B(y)} dy = 1, \quad (51)$$

where we used Eq. (49). After integration, we get

$$c_1^{-1} = \sqrt{-\frac{6}{\Lambda_5}} \ln \left[\frac{(1 + \sin Y_*) \cos Y_0}{(1 - \sin Y_0) \cos Y_*} \right], \quad (52)$$

where we defined, $Y \equiv y\sqrt{-\Lambda_4/3}$.

V. CONCLUSIONS

In this paper, we have investigated the localization of 4D topological global defects induced by 5D scalar fields. We adopted an ansatz for the scalar field Φ^a , which strictly “separates” Φ^a into the 4D part $\phi^a(x^\mu)$ and the extra-D part $p(y)$. The action involved the usual kinetic and symmetry-breaking potential terms, which are covariantly extended to higher dimensions. We also assumed a static ansatz for the fields.

We ignored self-gravity of the scalar field, and assumed that the background geometry is given by some other matter fields such as Λ_5 . This is guaranteed if the scale of scalar-field parameters is small compared with the corresponding Planck scale (for example, $\eta \ll 5\text{D Planck scale}$, etc.).

Starting with the separation ansatz for the scalar field, we found that the scalar-field equation requires a specific condition to make the equation separable; the warp factor $B(y)$ and the y -profile of the scalar field, should satisfy $B(y)p^2(y) = \text{constant}$. The resulting two equations from the separation are the 4D equation and the extra-D equation.

The separation condition can convert the extra-D equation into an Einstein equation. To be consistent with the resulting Einstein equation, the background requires a bulk cosmological constant and the constant 4D curvature. The bulk cosmological constant is taken to be negative in order to keep the symmetry-breaking potential in a conventional shape.

The 4D equation is nothing but the usual scalar-field equation for defects. The solutions to this equation are the usual-defect solutions in the literature. The mass parameter m^2 of the effective potential is controlled by the 4D curvature $\bar{R}_{(4)}$. Depending on this curvature, the realization of the hedgehog configuration is determined. For the zero-curvature Minkowski and the positive-curvature de Sitter 4D manifolds, the potential does not allow a nonzero vacuum-expectation value of the 4D scalar field. The hedgehog configuration is not achieved, and the defects are not formed in the 4D world-volume. For the negative-curvature anti-de Sitter 4D manifold, however, the defects are possibly formed. Therefore, the possible geometry to accomplish 4D topological defects in the 4D world is AdS_4/AdS_5 .

In a warped geometry, physical scales flow along the extra dimension. We found that the localization amplitude of the defects is high where the warp factor is high. There, the magnitude of the effective 4D cosmological constant is relatively very small, but gravity is strongly coupled.

All the results in this work rely crucially on the “separation condition”. If we lift the separation ansatz, we may be able to get the defect formation consistently even in the Minkowski and the de Sitter manifold. However, the techniques involved in getting the solutions must be very tough.

Including self-gravity of the scalar field seems also to make the problem complicated. The separation ansatz in the self-gravitating system does not look very feasible.

Any type of global defects share the results of this work. The localization picture is the same for domain walls, cosmic strings, and monopoles. However, it is not very plausible to assume that the domain walls are static [11].

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