A Generalized Algorithm for Constrained Power Control with Capability of Temporary Removal

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Abstract

The *distributed constrained power control* (DCPC) [5] is one of the most widely accepted algorithms by the academic community. It provides guidelines in designing power control algorithms for practical cellular systems and also constitutes a building block for other radio resource management algorithms. DCPC has a property that the power reaches the maximum level when a user is experiencing degradation of channel quality. Unfortunately, this high power consumption may not lead to sufficient improvement on channel quality and may even generate severe interference, hitting other users. This undesirable phenomenon happens more often when the system is congested. In this paper, we revisit and generalize DCPC in order not to necessarily use the maximum power when the channel quality is poor. We propose the concept of temporarily removing users with low channel quality. We show how the energy consumption can be reduced through our generalized algorithm. Convergence properties of the generalized algorithm are given in this paper. Also, computational experiments are provided.

Keywords: Cellular radio system, constrained power control, connection removal, energy efficiency.

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1 Introduction

Designing cellular radio systems has become problematical, as the available radio spectrum is now scarce and the use of wireless communications is growing. To provide wireless communication services with higher capacity as well as better quality necessitates powerful and robust methods for *sharing* the radio spectrum in the most efficient way. All sharing methods in practice introduce *interference* of one sort or another which is proportional to transmitter powers. The *transmitter power control* is a key technique to better balance between received signal and interference (SIR), which in turn enables more efficient resource sharing.

During recent decades, many researchers have investigated power control from different perspectives. Especially, power control in cellular radio systems has drawn much attention since Zander's work on SIR *balancing* [1], [2]. Foschini and Miljanic [3] considered a realistic model in which a positive receiver noise and a respective target SIR were taken into account. The Foschini and Miljanic's distributed algorithm was shown to converge either synchronously [3] or asynchronously [4] to a fixed point of a *feasible* system. Based on the Foschini and Miljanic algorithm, Grandhi *et al.* [5] suggested *distributed constrained power control* (DCPC), in which the upper bound on transmission power was considered. DCPC converges to a fixed point in both feasible and infeasible systems; the fixed point supports every active transmitter in the feasible case. DCPC has become one of the most widely accepted algorithms by the academic community. It provides guidelines in designing power control algorithms for practical cellular systems. DCPC is also used as a building block for *connection removal* [6], *admission control* [7], *combined power control and base station assignment* [8], [9] and radio network simulators.

With respect to *energy efficiency*, DCPC has a drawback that the power may reach the maximum level when a user is experiencing low channel quality. Unfortunately, even if the maximum power is used, this may not necessarily lead to sufficient improvement on channel quality. The impact will be high power consumption and severe interference, hitting other users. With this in mind, in this paper, we revisit and generalize DCPC in order not to necessarily use the maximum power when the channel quality is poor. Under poor conditions, the power may even be lowered to the minimum level, which we will call *temporary removal.* In that case the user stays on the same channel and transmission will be resumed if the interference situation becomes favourable. In power control, if the power consumption level of a given algorithm were relatively low, then it would be a great advantage, especially to the mobiles that could expect a prolonged operational time. In Section 3, we explain how the generalized algorithm can improve the energy efficiency. We show that, for the feasible system, the generalized algorithm converges to the fixed point that supports every active transmitter, as DCPC does. Based on the generalization, we suggest two power control algorithms and compare them with DCPC.

When the system is infeasible so that all the active transmitters cannot be supported some sort of *permanent* removal of users, e.g. handing over to another channel or dropping of users, is necessary to maximize the network capacity. For the infeasible system, we evaluate the suggested algorithms by combining them with the so called *gradual removal* [6] and compare the combined algorithms with GRR-DCPC [6].

Computational experiments on a DS-CDMA system indicate that the suggested algorithms consume less energy while supporting more transmitters than DCPC.

2 System Model

Without loss of generality, let us consider the uplink of a cellular radio system, in which q mobiles share the same channel at a given instance. As in many other papers, we focus on the so called *snapshot* situation. A snapshot means an instant of time where the system is frozen while the power control algorithm is evaluated. We let a_i denote the base station assigned to mobile i and assume that the signal of mobile i will be received correctly if the SIR at base a_i is not less than a given target value γ_i^t. However, since the ideal situation is to make connection with the minimal transmission power, we have the following SIR constraint on mobile i:

$$\frac{g_{a_i i} p_i}{\sum_{j=1, j\neq i}^{q} g_{a_i j} p_j + \nu_{a_i}} = \gamma_i^t \tag{1}$$

In the above, p_i denotes the transmission power of mobile i, $g_{a_i j}$ is the link gain from mobile j to base a_i, and ν_{a_i} is the receiver noise at base a_i. We assume that all link gain values are positive.

Let us define a $q \times q$ matrix $H = [h_{ij}]$ such that $h_{ij} = \gamma_i^t g_{a_i j}/g_{a_i i}$ for $i \neq j$ and $h_{ij} = 0$ for $i = j$. In addition, let us define a vector $\eta = [\eta_i]$ of length q, where $\eta_i = \gamma_i^t \nu_{a_i}/g_{a_i i}$. Then, converting (1) into a matrix from, we have the following *power control problem*:

$$AP = \eta, \tag{2}$$

where $A = I - H$ and $P = [p_1, p_2, \ldots, p_q]^T$ is the *power vector*. Since the transmission power of a mobile is limited, we will consider the following constraint on the power vector:

$$0 \leq P \leq \bar{P}, \tag{3}$$

where $\bar{P} = [\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_q]^T$ denotes the maximum transmission power of each mobile.

Definition 1. *If there exists a power vector P^* that solves the problem (2) within the range of (3) at a given instant, we say that the system is feasible at the instant.*

3 Generalized DCPC

Let us consider an iterative power control algorithm and denote the power vector at the iteration n by $P(n)$. Further, we define the received SIR of mobile i at iteration n as,

$$\gamma_i(n) \equiv \frac{g_{a_i i} p_i(n)}{\sum_{j=1, j\neq i}^{q} g_{a_i j} p_j(n) + \nu_{a_i}} = \frac{\gamma_i^t p_i(n)}{\sum_{j=1}^{q} h_{ij} p_j(n) + \eta_i} \tag{4}$$

DCPC suggested by Grandhi *et al.* [5] has the form:

(**DCPC**)

$$p_i(n+1) = \bar{\mathcal{T}}_i(P(n)) \equiv \min\{\mathcal{T}_i(P(n)), \bar{p}_i\}, \;\; n = 0, 1, \ldots, \tag{5}$$

where the mapping $\mathcal{T}_i(P(n)) \equiv \frac{\gamma_i^t}{\gamma_i(n)} p_i(n) = \sum_{j=1}^{q} h_{ij} p_j(n) + \eta_i$.

Now consider the following generalized algorithm for constrained power control:

(**GDCPC**)

$$p_i(n+1) = \tilde{\mathcal{T}}_i(P(n)) \equiv \begin{cases} \mathcal{T}_i(P(n)) & \text{if } \mathcal{T}_i(P(n)) \leq \bar{p}_i, \\ \tilde{p}_i(n) & \text{if } \mathcal{T}_i(P(n)) > \bar{p}_i, \end{cases} \quad n = 0, 1, \ldots, \tag{6}$$

where the power value $\tilde{p}_i(n)$ is taken within the range of (3). If we choose $\tilde{p}_i(n) = \bar{p}_i$, GDCPC is reduced to DCPC. When setting $\tilde{p}_i(n) = 0$, it can be interpreted as a temporary connection removal, allowing the removed user to stay on the channel and power up again if the interference has decreased. By setting the transmit power to zero, the user will not waste energy mitigating bad channel conditions and other users will benefit from lower interference. When $\tilde{p}_i(n) \neq \bar{p}_i$, GDCPC violates the *monotonicity* property, and thus it is not a *standard interference function* [8] that guarantees convergence. However, we can prove the convergence of GDCPC to P^* in the feasible case, which will be given later in this section. To motivate the readers, we will first describe the energy saving property of GDCPC.

3.1 Energy Saving

Let $\bar{\mathcal{T}}_i^n(P)$ and $\tilde{\mathcal{T}}_i^n(P)$ respectively denote the power level of mobile i of DCPC and GDCPC at iteration n, starting with a power vector P. Then we have the following properties on GDCPC:

Proposition 1. *$\tilde{\mathcal{T}}_i^n(P) \leq \bar{\mathcal{T}}_i^n(P)$ for all i and n.*

Proof. From the definition of GDCPC, we have $\tilde{\mathcal{T}}_i(P) \leq \bar{\mathcal{T}}_i(P)$ for any nonnegative power vector P. Also, from the definition of $\bar{\mathcal{T}}_i(P)$, we find that if $0 \leq P_1 \leq P_2$ then $\bar{\mathcal{T}}_i(P_1) \leq \bar{\mathcal{T}}_i(P_2)$. Therefore, if $0 \leq P_1 \leq P_2$, then $\tilde{\mathcal{T}}_i(P_1) \leq \bar{\mathcal{T}}_i(P_1) \leq \bar{\mathcal{T}}_i(P_2)$. Using these relations, we have:

$$\begin{aligned} \tilde{\mathcal{T}}_i(P) &\leq \bar{\mathcal{T}}_i(P) \\ \tilde{\mathcal{T}}_i(\tilde{\mathcal{T}}_i(P)) &\leq \bar{\mathcal{T}}_i(\mathcal{T}_i(P)) \\ &\vdots \\ \tilde{\mathcal{T}}_i^n(P) &\leq \bar{\mathcal{T}}_i^n(P) \end{aligned} \tag{7}$$

□

Proposition 2. *Let us assume that there exists an iteration n_0 such that $\mathcal{T}_i^{n_0}(P) > \bar{p}_i$ and that $\tilde{p}_i(n) < \bar{p}_i$ for all i and n. Then, $\sum_{i=1}^q \tilde{\mathcal{T}}_i^n(P) < \sum_{i=1}^q \bar{\mathcal{T}}_i^n(P)$ for all $n \geq n_0$.*

Proof. Since $\tilde{\mathcal{T}}_i^n(P) \leq \bar{\mathcal{T}}_i^n(P)$ from Proposition 1, it is sufficient to show that there exists at least a mobile $j \in \mathcal{J}$ for any iteration $n \geq n_0$ where $\mathcal{J} = \{j : \tilde{\mathcal{T}}_j^n(P) < \bar{\mathcal{T}}_j^n(P)\}$. At iteration n_0, mobile $i \in \mathcal{J}$

because $\tilde{\mathcal{T}}_i^{n_0}(P) = \tilde{p}_i(n_0) < \bar{p}_i = \bar{\mathcal{T}}_i^{n_0}(P)$. At iteration $n_0 + 1$, from the definition of $\mathcal{T}_i(P)$, we can find that $\tilde{\mathcal{T}}_j^{n_0+1}(P) < \bar{\mathcal{T}}_j^{n_0+1}(P)$ for any $j \neq i$. In the same manner, we can see that there is a mobile j such that $\tilde{\mathcal{T}}_j^n(P) < \bar{\mathcal{T}}_j^n(P)$ for all $n > n_0 + 1$. □

Note that Proposition 1 and 2 are general and hold for both feasible and infeasible systems. Proposition 1 says that when starting from the same initial power vector, the power value from GDCPC is not greater than that of DCPC. Further, if there is an event that the required power is greater than the maximum allowed level, then from Proposition 2, we can expect a certain amount of energy saving from GDCPC, compared with DCPC. For $\tilde{p}_i(n)$ of GDCPC, we can use any nonnegative value less than or equal to $\bar{p}_i$. However, from the proof of Proposition 2, the setting $\tilde{p}_i(n) = 0$ will lead to the most energy-saving results. For simplicity, we will denote this version of GDCPC by GDCPC(I) throughout the paper.

3.2 Convergence in Feasible Systems

As was mentioned before, GDCPC is not a standard interference function but convergence is still guaranteed for a feasible system by the following property.

Proposition 3. *Starting with any power vector within the range of (3), GDCPC converges to P^* of a feasible system.*

Proof. From the definition of GDCPC, we have $\tilde{\mathcal{T}}_i(P) \leq \mathcal{T}_i(P)$ for any nonnegative power vector P. Also, from the definition of $\mathcal{T}_i(P)$, we find that if $0 \leq P_1 \leq P_2$ then $\mathcal{T}_i(P_1) \leq \mathcal{T}_i(P_2)$. Therefore, if $0 \leq P_1 \leq P_2$, then $\tilde{\mathcal{T}}_i(P_1) \leq \mathcal{T}_i(P_1) \leq \mathcal{T}_i(P_2)$. Using these relations, we have:

$$\begin{aligned} \tilde{\mathcal{T}}_i(P) &\leq \mathcal{T}_i(P) \\ \tilde{\mathcal{T}}_i(\tilde{\mathcal{T}}_i(P)) &\leq \mathcal{T}_i(\mathcal{T}_i(P)) \\ &\vdots \\ \tilde{\mathcal{T}}_i^n(P) &\leq \mathcal{T}_i^n(P) \end{aligned} \tag{8}$$

Let us define the dominant eigenvalue of H by $\rho(H)$. Then, it is known that $0 < \rho(H) < 1$ for a feasible system (Perron-Frobenius Theorem and Theorem 3.11 in [10]). Further, let us now consider the *weighted maximum norm* of a vector x, which is defined as $||x||_\infty^w \equiv \max_i |\frac{x_i}{w_i}|$ for a positive vector w.

If we choose the eigenvector corresponding to the dominant eigenvalue of H for w, then the *convergent* mapping $\mathcal{T} = (\mathcal{T}_i)$ fulfills $\|\mathcal{T}^n(P) - P^*\|_\infty^w \leq \rho(H)^n \|P - P^*\|_\infty^w$ (Proposition 3.1 [4]). Therefore, there exits an integer n_1 such that

$$\mathcal{T}^n(P) < \bar{P} \text{ for all } n > n_1 \tag{9}$$

By denoting $\tilde{\mathcal{T}} = (\tilde{\mathcal{T}}_i)$, it follows from Equation (8) that $\tilde{\mathcal{T}}^n(P) \leq \mathcal{T}^n(P)$, thus for $n > n_1$ we can write:

$$\begin{aligned} \tilde{\mathcal{T}}^{n+1}(P) = \mathcal{T}(\tilde{\mathcal{T}}^n(P)) \quad &\leq \quad \mathcal{T}^{n+1}(P) < \bar{P} \\ &\vdots \\ \tilde{\mathcal{T}}^{n+m+1}(P) = \mathcal{T}^m(\tilde{\mathcal{T}}^n(p)) \quad &\leq \quad \mathcal{T}^{n+m+1}(P) < \bar{P} \end{aligned} \tag{10}$$

Therefore, $\lim_{m\to\infty} \tilde{\mathcal{T}}^{n+m+1}(P) = P^*$, due to the convergent mapping $\mathcal{T}$. □

Remark 1. *So far we have focused on generalizing DCPC but Propositions 1-3 still hold even if we choose other standard interference functions [8], [9] for $\mathcal{T}$ in (5).*

From Proposition 2, we may have that $\sum_{i=1}^q \tilde{\mathcal{T}}_i^n(P) < \sum_{i=1}^q \bar{\mathcal{T}}_i^n(P)$ for all $n \geq n_0$. However, Proposition 3 says that $\lim_{n\to\infty} \sum_{i=1}^q \tilde{\mathcal{T}}_i^n(P) = \lim_{n\to\infty} \sum_{i=1}^q \bar{\mathcal{T}}_i^n(P) = \sum_{j=1}^q p_i^*$, in the feasible case. Besides the energy consumption, we are very interested in how fast the power value will converge. It has been reported that DCPC converges to P^* at a geometric rate [5], [9]. So far the convergence rate of GDCPC is an open issue. However, if we choose

$$\tilde{p}_i(n) = \max\{2\bar{p}_i - \mathcal{T}_i(P(n)), 0\} \tag{11}$$

and denote this by GDCPC(II), then we have the following:

Proposition 4. GDCPC(II) *converges to P^* of a feasible system with a same geometric rate as DCPC.*

Proof. If $\mathcal{T}_i(P(n)) < \bar{p}_i$ for mobile i, then $\left|\frac{p_i(n+1)-p_i^*}{w_i}\right| = \left|\frac{\mathcal{T}_i(P(n))-p_i^*}{w_i}\right|$ for any positive w_i. However, if $\mathcal{T}_i(P(n)) > \bar{p}_i$ for mobile i, then for any positive w_i, we have

$$\left|\frac{p_i(n+1) - p_i^*}{w_i}\right| \quad = \quad \left|\frac{\max\{2\bar{p}_i - \mathcal{T}_i(P(n)), 0\} - p_i^*}{w_i}\right| \tag{12}$$

When $\bar{p}_i < \mathcal{T}_i(P(n)) < 2\bar{p}_i$, we have

$$\left|\frac{p_i(n+1) - p_i^*}{w_i}\right| = \left|\frac{2(\bar{p}_i - \mathcal{T}_i(P(n))) + \mathcal{T}_i(P(n)) - p_i^*}{w_i}\right| < \left|\frac{\mathcal{T}_i(P(n)) - p_i^*}{w_i}\right| \tag{13}$$

and if $\mathcal{T}_i(P(n)) \geq 2\bar{p}_i$, it follows that

$$\left|\frac{p_i(n+1) - p_i^*}{w_i}\right| = \left|\frac{-p_i^*}{w_i}\right| \leq \left|\frac{\mathcal{T}_i(P(n)) - p_i^*}{w_i}\right| \tag{14}$$

Therefore, we can say that, for any mobile i and any positive w_i,

$$\left|\frac{p_i(n+1) - p_i^*}{w_i}\right| \leq \left|\frac{\mathcal{T}_i(P(n)) - p_i^*}{w_i}\right| \tag{15}$$

If we introduce the consistent matrix norm of $\|H\|_\infty^w \equiv \max_i \left|\sum_{j=1}^q \frac{h_{ij} w_j}{w_i}\right|$ and choose the eigenvector of the dominant eigenvalue of H for w, then we have

$$\begin{aligned}
||P(n+1) - P^*||_\infty^w &\leq ||\mathcal{T}(P(n)) - P^*||_\infty^w \quad \text{(from (15))} \\
&= ||H(P(n) - P^*)||_\infty^w \\
&\leq ||H||_\infty^w ||P(n) - P^*||_\infty^w \\
&= \rho(H) ||P(n) - P^*||_\infty^w \quad \text{(from Proposition 3.1 [4])}
\end{aligned} \tag{16}$$

Thus, we conclude that GDCPC(II) is a *pseudo-contraction mapping* with a geometric rate $\rho(H)(< 1)$, which is same as that of DCPC [5], [9]. □

With respect to Proposition 4, we conclude that within the class of GDCPC, there exits at least one $\tilde{p}_i(n) \neq \bar{p}_i$ which gives the same convergence rate as DCPC and possibly increases the energy efficiency. In GDCPC(II), if the required power is larger than the maximum power $\bar{p}_i$, a power lower than $\bar{p}_i$ by the amount of the gap between the required power and $\bar{p}_i$ is used. If the required power is twice larger than the maximum power, the transmitter power is set to zero. Note that $\tilde{p}_i(n) < \bar{p}_i$ in GDCPC(II), and Proposition 2 is applicable to GDCPC(II).

3.3 Convergence in Infeasible Systems

Compared to DCPC, which also converges to a fixed point in the infeasible case, the convergence properties of GDCPC(I) and GDCPC(II) will differ. In Figure 1, a two-dimensional example of a noise-limited

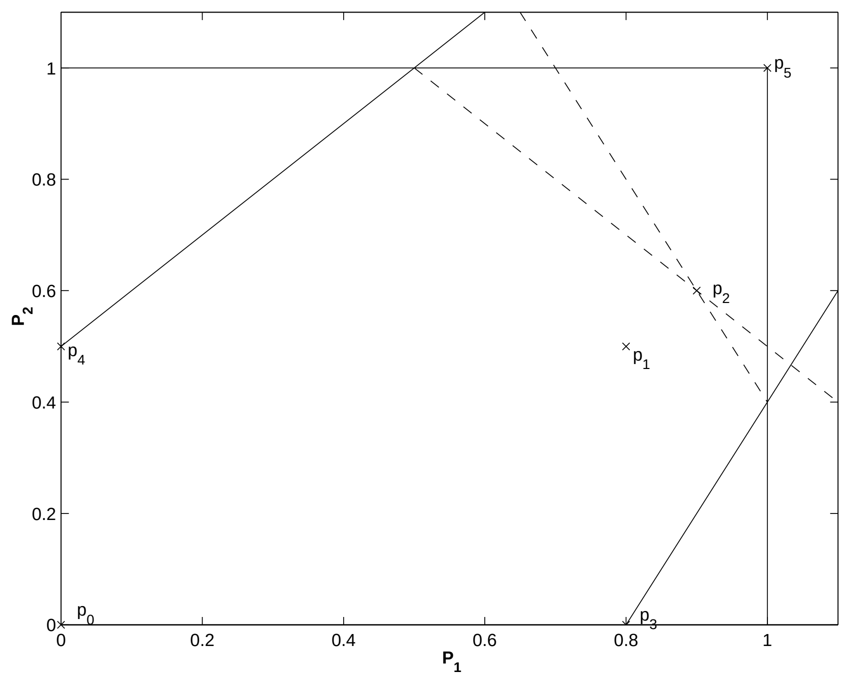

Figure 1: Fixed points of DCPC, GDCPC(I) and GDCPC(II).

infeasible system ($\rho(H) < 1$) with the maximum power of each mobile set to one, illustrates the possible fixed points. The dashed lines represent Equation (11) that could be interpreted as *virtual* targets. In this example no more than one user can be supported. Hence the optimal fixed point would be p_4 where user 2 is supported with the minimum power. The fixed point of DCPC will in this case be p_5 due to the power constraints. Clearly this point is the worst, considering no user is supported while the power usage is maximized. GDCPC(I) will, depending on the starting point, oscillate between p_0 and p_1 or converge to p_3 or p_4. In GDCPC(II), the fixed point may be the intersection between the virtual targets, p_2. However, GDCPC(II) does not always converge to a fixed point, which is exemplified in Appendix.

As illustrated in Figure 1, GDCPC(I) and GDCPC(II) may converge to a fixed point but the dynamics are more unpredictable. Due to the possibility of oscillating powers, each user may generally expect a more varying SIR and its impact on the bit error rate is not clear. Depending on power control interval, coding and interleaving strategies, the oscillation of SIR may or may not cause problems.

For an infeasible system, permanent connection removal has been utilized to increase system capacity. In GDCPC(I) and GDCPC(II), the powers may oscillate, and thus certain removal algorithms relying

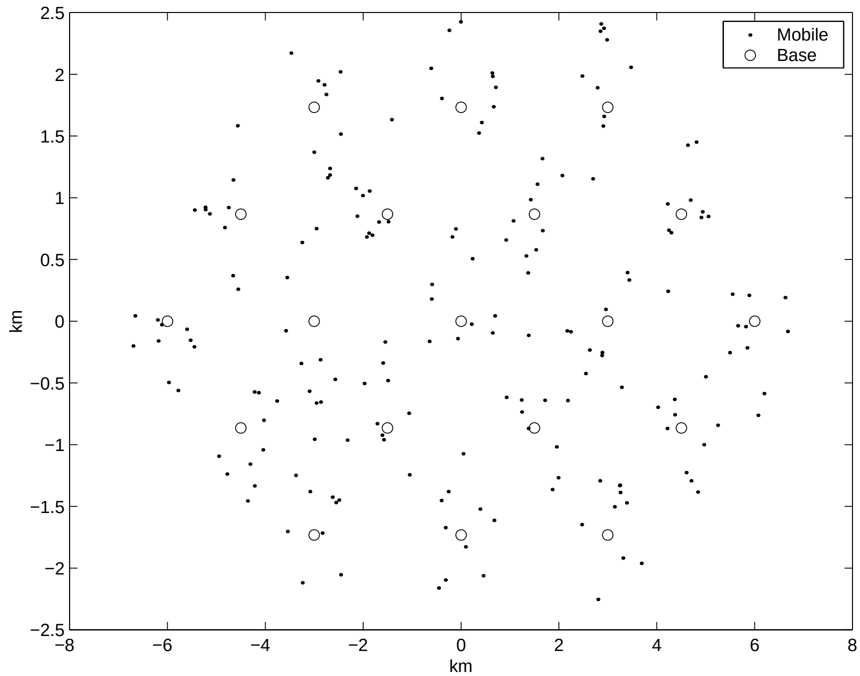

Figure 2: DS-CDMA cellular system with 19 omni-bases and 190 mobiles.

on convergence to some fixed points may not be utilized. For the purpose of permanent removal, we extend the previously suggested GRR-DCPC [6], which is an "on-the-fly" gradual removal combined with DCPC. Instead of DCPC, we combine the gradual removal with GDCPC(I) and GDCPC(II). That is, our modified gradual removal algorithm, which incorporates both temporary- and permanent removal GRR-GDCPC identifies user i as a candidate for permanent removal if $\mathcal{T}_i(P(n_0)) > \bar{p}_i$ and sets $p_i(n) = 0$, with a given probability $\delta > 0$ for all $n > n_0$. Otherwise, $p_i(n_0+1) = \tilde{\mathcal{T}}_i(P(n_0))$ and the power control proceeds with the next power iteration. In order to maximize system capacity, the removal probability δ, should be taken so that in each iteration, single removal is more probable than multiple removals. It has been shown that GRR-DCPC converges to a stationary power vector [6]. Since GRR-GDCPC uses the same decision procedure as GRR-DCPC, it is clear that GRR-GDCPC will also converge.

4 Computational Experiments

The main purpose of the experiments is to draw insight on how GDCPC(I) and GDCPC(II) perform in terms of energy saving, convergence and system capacity. To compare the performance of our proposed

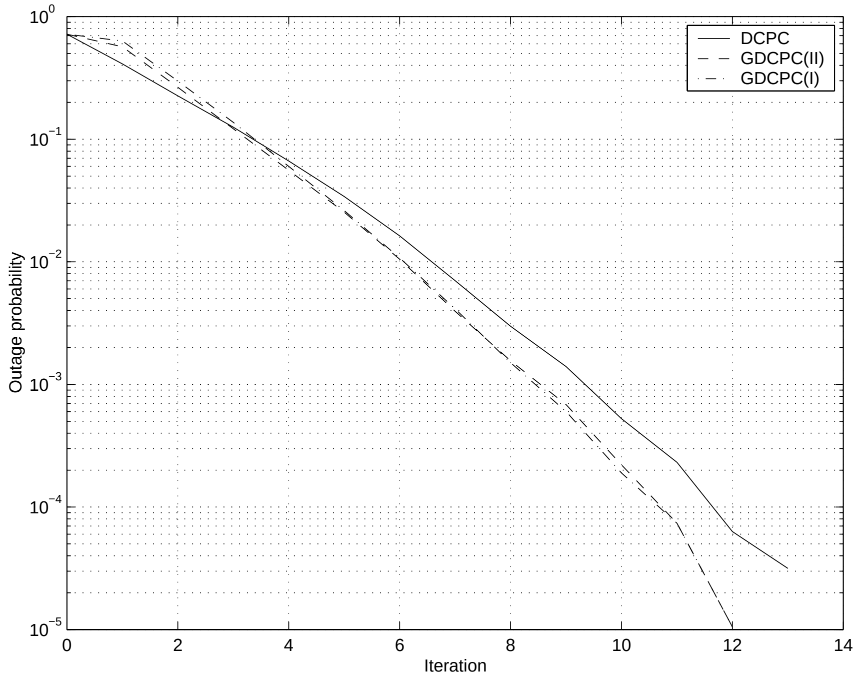

Figure 3: Outage probability in feasible systems.

algorithms, we use DCPC as a reference algorithm. A DS-CDMA system with 19 omni-bases located in the centers of 19 hexagonal cells is used as a test system (Figure 2). We consider an IS-95 example, where the *processing gain* is 21 dB. For a given instance, a total of 190 mobiles are generated, the locations of which are uniformly distributed over the 19 cells. The link gain g_{ij} is modeled as $g_{ij} = s_{ij} \cdot d_{ij}^{-4}$, where s_{ij} is the shadow fading factor and d_{ij} is the distance between base i and mobile j. The shadow fading factor is generated from a log-normal distribution with $E(s_{ij}) = 0$ dB, and $\sigma(s_{ij}) = 8$ dB. The base receiver noise is taken to be $\nu_{a_i} = 10^{-12}$ and the relative maximum mobile power is set to one. The initial power for each mobile is randomly chosen from the interval [0,1]. Each mobile is assigned to the base station that gives the lowest signal attenuation. The received E_b/I_0 from mobile i at the corresponding base is calculated by adding the processing gain to the received SIR (in dB). The target E_b/I_0 is set to 8 dB and 12 dB for each mobile when analyzing a feasible and an infeasible system, respectively.

We have considered 500 independent feasible and infeasible instances of mobile locations and shadow fadings. To check the impact on the system capacity, the *outage probability* is used as a performance measure. The outage probability at each iteration is computed over the 500 instances by counting the

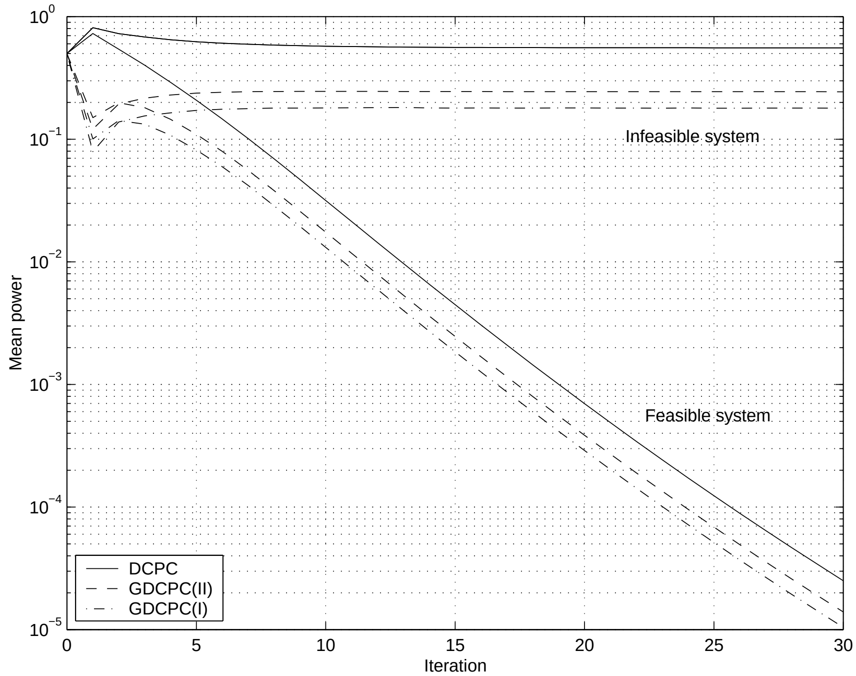

Figure 4: Mean transmission power per mobile.

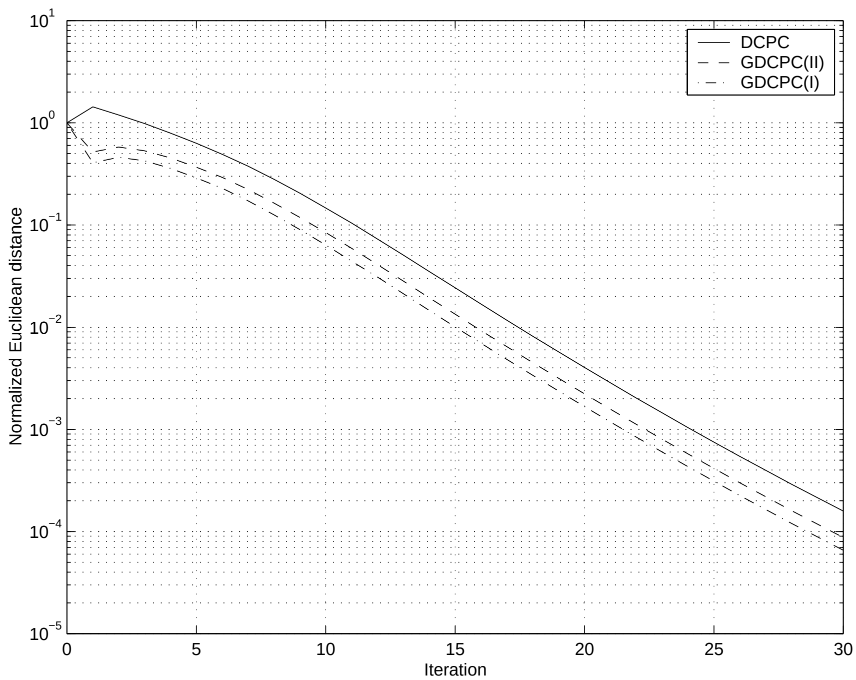

Figure 5: Convergence rate in feasible systems ($\|P(n) - P^*\|/\|P(0) - P^*\|$).

portion of the number of non-supported mobiles at the iteration. A connection is considered to be supported if E_b/I_0 is above 7.5 dB and 11.5 dB, respectively.

In Figure 3, we see that the outage probability is lower over the whole range of iterations considered (except for the initial iterations) for both GDCPC(I) and GDCPC(II), compared to DCPC. It means that GDCPC(I) and GDCPC(II) support more users in average than DCPC during the iterations. It is hard to say which of GDCPC(I) and GDCPC(II) is superior in terms of outage performance. The energy saving property is shown in Figure 4, where obviously GDCPC(I) gives the best performance in both feasible and infeasible systems. The curves of GDCPC(I) and GDCPC(II) indicate that there are many mobiles in which the required power at iteration 1 is greater than the maximum and the rest of the iterations follows Proposition 2. It can also be verified that the absolute gap among the three algorithms is decreasing with increasing iteration number for the feasible system, while it tends to be constant for the infeasible system. In Figure 5, the convergence rate, measured as $\|P(n) - P^*\|/\|P(0) - P^*\|$, where $\|\cdot\|$ denotes the Euclidean norm, empirically shows that GDCPC(I) is faster than GDCPC(II) and DCPC, both of which are proved to converge with a geometric rate. In conclusion, GDCPC(I) shows the best performance in terms of system capacity, energy saving and convergence speed, in the feasible systems.

Now let us consider the outage probability for infeasible systems. As can be seen in Figure 6, where we compare GDCPC(I), GDCPC(II) and DCPC, both GDCPC(I) and GDCPC(II) support more mobiles in average than DCPC. However, the oscillating behavior is still seen after averaging 500 snapshots in GDCPC(I) while GDCPC(II) gives smoother outage. Compared with Figure 6, the outage from DCPC and GDCPC(II) is decreasing when combined with gradual removal, while no significant difference is seen for GRR-GDCPC(I), according to Figure 7. Thus GDCPC(I) alone seems to identify a proper number of mobiles for temporary removal at an early stage. Although the outage was not improved, combining GDCPC(I) with gradual removal will guarantee convergence and therefore eliminate the oscillations in SIR. Also, note that both GRR-GDCPC(I) and GRR-GDCPC(II) are superior to GRR-DCPC that was known to be the best distributed removal algorithm [6].

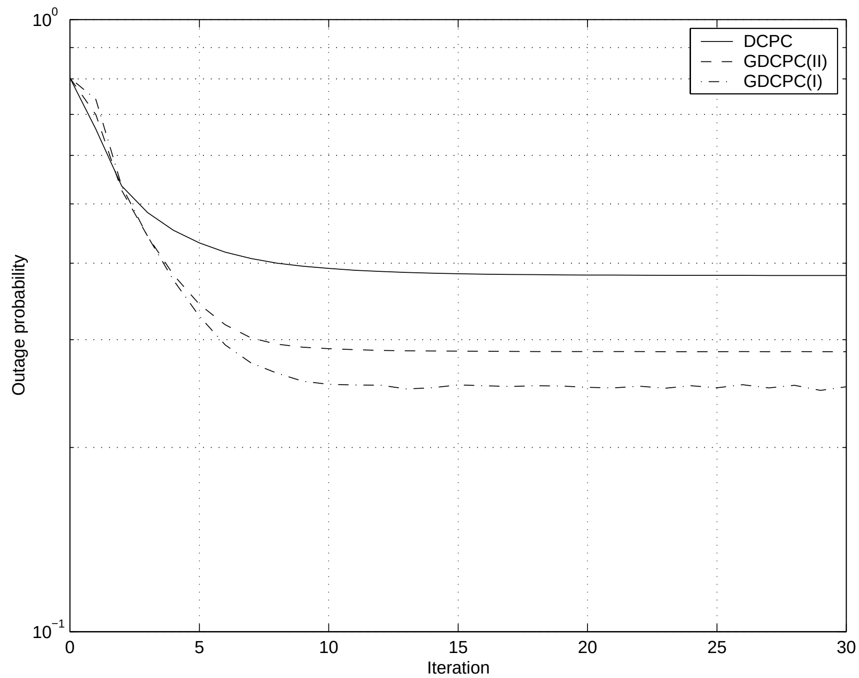

Figure 6: Outage probability for infeasible systems without permanent removal.

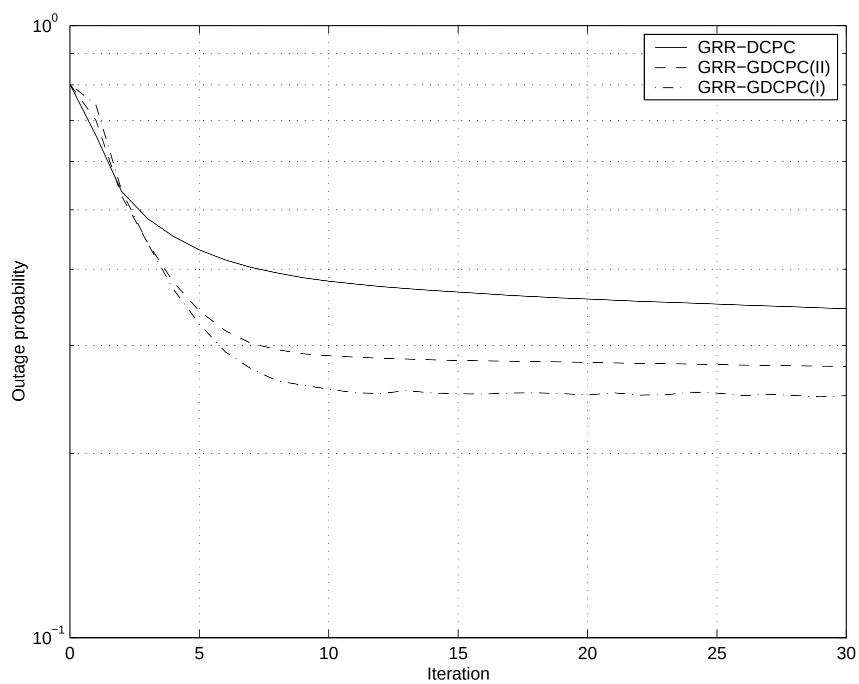

Figure 7: Outage probability for infeasible systems with permanent removal ($\delta = 0.01$).

5 Concluding Remarks

In this paper, we have proposed algorithms that are consuming less power and supporting more users than DCPC. The proposed algorithms are based on our general framework. The idea is that, when a user requires more power than the available, the power will be decreased to benefit other users under favorable situations. It was shown that our algorithms converge to the fixed point of a feasible system, supporting every active user. For an infeasible system, convergence to a fixed point was exemplified not to necessarily occur. For that case, it was seen that power oscillations may cause a rapidly varying SIR. However, this may not be a major obstacle, since the power control can be combined with a permanent removal algorithm. The difficulty with the proposed algorithms is that infeasibility may not be detected as for DCPC. This raises the question of how to combine a permanent removal algorithm with the proposed algorithms. We propose one possible approach by modifying a gradual removal algorithm that was originally designed for use with DCPC.

The practical applicability of the concept of temporary removals, which GDCPC(I) and GDCC(II) benefit from, could for example be non-real time traffic where the flexibility of handling the transmission attempts is larger. Finding necessary and sufficient conditions for convergence for infeasible systems is still an open issue. Also, there is a possibility of designing more sophisticated removal algorithms suitable for our algorithms.

Finally, we would like to mention energy management, which was emphasized in [11]. It is expected that the need for low-power design principles will increase along with the more services available. Improving energy efficiency is of interest for both the operators (downlink) as well as for the customers (uplink). Therefore such design principles include all levels of the system, e.g., network architecture, circuit design, protocols and resource management algorithms. Our work can be considered as an effort to provide an energy efficient resource management.

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Appendix

Let w be the eigenvector corresponding to $\rho(H)$. Assume that there exists a P_0 such that $\tilde{\mathcal{T}}(P_0) = P_0$ and $\mathcal{T}(P_0) > \bar{P}$. Clearly point p_2 in Figure 1 is the only such point. If we choose $P_0 + \alpha e \leq \bar{P}$ where e is the Perron-Frobenius eigenvector satisfying $He = \rho(H)e$ and $\alpha > 0$ is a constant. Then we have $\|\tilde{\mathcal{T}}(P_0 + \alpha e) - \tilde{\mathcal{T}}(P_0)\|_\infty^w = \|2\bar{P} - H(P_0 + \alpha e) - \eta - 2\bar{P} + HP_0 + \eta\|_\infty^w = \| - H\alpha e\|_\infty^w = \|\rho(H)\alpha e\|_\infty^w$. If $\rho(H) > 1$ we can write $\|\tilde{\mathcal{T}}(P_0 + \alpha e) - \tilde{\mathcal{T}}(P_0)\|_\infty^w > \|P_0 + \alpha e - P_0\|_\infty^w$, i.e., a diverging sequence. The characteristics of P_0 could be described as a *non-attractive* fixed point to which convergence is guaranteed if and only if the starting point is the point itself, P_0.

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