A note on Doppler processing of coherent radar backscatter from the water surface: With application to ocean surface wave measurements

Paul A. Hwang, Mark A. Sletten, and Jakov V. Toporkov

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1. Introduction

The close correlation of ocean surface waves and radar backscatter has been recognized since the early days of radar development [e.g., Crombie, 1955]. The main scattering mechanisms of radar sea returns were clarified by a series of theoretical and experimental research [e.g., Wright, 1966, 1968]. By the mid-1980s, extraction of wave spectral information reaches essentially a mature stage as reflected in the sophisticated 3-D spectral analysis of Young et al. [1985]. The results show convincingly that aside a scaling factor of the modulation transfer function (MTF), the wave-number-frequency spectrum derived from the spatiotemporal images of radar backscattering intensity is essentially the ocean surface wave spectrum. Excellent agreement in wave period and wave propagation direction is illustrated. The 3-D spectral processing technique continues to generate exciting results of spatial and temporal evolution of ocean surface waves [e.g., Dankert and Rosenthal, 2004]. The extensive information contained in the 3-D spectrum can even be used to derive the current velocity vector and bathymetry through the dispersion relation [Young et al., 1985; Trizna, 2001]. The issue of MTF relating the radar backscattering intensity to surface wave height is a much more difficult problem because the magnitude and phase of MTF are influenced by many factors, including the radar wavelength, look angle, look direction, slope of surface roughness spectrum, long wave period and direction, surface currents, wind speed, and wind direction [e.g., Alpers and Hasselmann, 1978; Plant et al., 1978, 1983, 1987; Plant, 1986; Thompson and Gasparovic, 1986; Hwang and Shemdin, 1990], although some simplified parameterizations have been suggested for operational application [e.g., Dankert and Rosenthal, 2004; Nieto Borge et al., 2004].

One way to circumvent the thorny issues of the MTF is to use coherent radar that preserves the phase information of the return signal. The phase information is related to the Doppler frequency shift that can be processed to yield the line-of-sight (radial) velocity of the scattering elements [e.g., Plant et al., 1987; Plant and Keller, 1983; Plant, 1997; Johnson et al., 2009]. Although the derived radial velocity has many contributing factors, including the phase velocity of Bragg scattering waves and currents of all sources, the oscillatory portion of the radial velocity is generated primarily by ocean surface waves. Spectral analysis of the radial velocity measured by radar thus yields the wave-induced velocity spectrum, the conversion of which to surface wave elevation spectrum is much more straightforward. Plant [1997] presents a detailed investigation of microwave Doppler velocity of Ku band sea return covering incidence angles between 50 and 80 degrees. A bound wave hypothesis is used to explain the observed large value of the Doppler velocity, especially in the horizontal-transmit-horizontal-receive (HH) polarization. Of special interest to this study is the observation of the prominent low-frequency component of the Doppler spectrum as incidence angle increases [Plant, 1997, Figure 5]. Particularly, at 50° incidence angle the measured Doppler velocity spectrum resembles a typical wind sea spectrum and the...
spectral peak frequency is in very good agreement with in situ measurement by a nearby buoy. As incidence angle increases, a low-frequency spectral component of the Doppler signal becomes more prominent although the spectral component corresponding to the wind sea remains identifiable. The source of the low-frequency component in the Doppler velocity spectrum is not known.

[4] A recent numerical study by Johnson et al. [2009] describes in detail the retrieval of sea surface height profile from simulated low grazing angle coherent and incoherent radar data. In addition to the phase signal of Doppler velocity, they also discuss the phase signal from vertical interferometry [Eshbaugh and Frasier, 2002]. The study confirms that the retrieval of surface height using phase information (either from vertical interferometry or Doppler frequency shift) does not require empirical parameter as in the case of height retrieval using normalized radar cross section. Their numerical studies demand significant computational power; Johnson et al. [2009] report that a complete case simulation of two incidence polarizations requires approximately 18 h of parallel processing using a supercomputer with 64 processors.

[5] In this paper, we present an analysis of the wave properties derived from the Doppler velocity data measured by an X band coherent in receive radar from a tower about 60 km offshore in the Atlantic Ocean. The experimental conditions and instrumentation have been described by Hwang et al. [2008a, 2008b], a brief summary is given in section 2. A discussion of the Doppler processing using a pulse pair approach and a comparison with more conventional Fourier spectral analysis is presented in section 3. The main purpose of the paper is to illustrate the recovery of wave height information using the Doppler signal from the coherent radar return. As discussed in the first paragraph, wave height retrieval is the weakest aspect of surface wave measurement using noncoherent radars. The method of surface wave analysis is described in section 4. The result of application to field measurements is presented in section 5. For experimental setup with nonrotating radar antennas (upwind pointing) used in the present study, reliable information of dominant wave period and significant wave height can be derived from radar data as short as a few seconds long with range coverage on the order of 10 wavelengths. Section 6 is a summary.

2. A Brief Description of the Field Experiment

[6] Dual-polarization X band coherent radar measurements were collected from a tower about 60 km off the Georgia coast (Station SPAG1 at 31.38°N, 80.57°W, local water depth 25 m; see http://www.ndbc.noaa.gov/station_page.php?station=SPAG1). The primary purpose of the radar experiment was to explore the relationship between sea spikes and breaking waves [Hwang et al., 2008a, 2008b]. Details of the radar system and experimental conditions have been described in the papers cited above so only a brief outline is given here. The radar frequency is 9.3 GHz. The transmitted pulse waveform from the magnetron is recorded to remove the random phase difference from pulse to pulse during post processing using the cross correlation method. The magnetron pulse width is approximately 50 ns. The return signal is down converted to an intermediate frequency of 20 MHz and digitally sampled at 100 MHz, resulting in a range sampling spacing of about 1.5 m. Pulse-to-pulse switching between two antennas is used to collect HH and vertical-transmit-vertical-receive (VV) backscatter on alternate pulses at a per polarization pulse repetition frequency (PRF) of 1200 Hz. The radar antennas are mounted on a railing 12 m above the mean water level and point to north. The range of grazing angles, θg, in the data set presented here is from 1° to 6.3°.

[7] Data available on the SPAG1 web site are hourly wind direction, wind speed, wind gust, significant wave height, and air and water temperatures. Additional information of spectral peak wave period and surface wave spectrum is available from a nearby National Data Buoy Center (NDBC) buoy 41008 (at 31.40°N, 80.87°W, 18 m depth, http://www.ndbc.noaa.gov/station_page.php?station=41008), about 28.5 km to the west of SPAG1. Radar measurements were collected on 9, 10, and 11 April 2006. For each day, the radar data acquisition was manually started at approximately hourly intervals. The data length of each collection episode is about 2 min. Figure 1 shows the relevant wind and wave measurements from the two stations for the experimental period. The wind direction remains relatively steady from NNE the whole time. With the radar antennas pointing toward north, the condition of the collected data set is mainly upwind looking. The wind speed (7–15 m/s) fluctuates about diurnally, but the variation of wave height (1.43–2.75 m) and wave period (4.6–7.4 s) is much more complicated [Hwang et al., 2008a, 2008b].

3. Doppler Velocity Analysis

[8] Two approaches can be used to derive the Doppler velocity from the coherent radar return. The first is by computing the time derivative of the phase of return signal. The phase of the radar signal can be obtained through Hilbert transformation of the radar return and the Doppler velocity from the coherent radar return. The first is by measuring the Doppler velocity spectrum is available from a nearby National Data Buoy Center (NDBC) buoy 41008 (at 31.40°N, 80.87°W, 18 m depth, http://www.ndbc.noaa.gov/station_page.php?station=41008), about 28.5 km to the west of SPAG1. Radar measurements were collected on 9, 10, and 11 April 2006. For each day, the radar data acquisition was manually started at approximately hourly intervals. The data length of each collection episode is about 2 min. Figure 1 shows the relevant wind and wave measurements from the two stations for the experimental period. The wind direction remains relatively steady from NNE the whole time. With the radar antennas pointing toward north, the condition of the collected data set is mainly upwind looking. The wind speed (7–15 m/s) fluctuates about diurnally, but the variation of wave height (1.43–2.75 m) and wave period (4.6–7.4 s) is much more complicated [Hwang et al., 2008a, 2008b].

[9] With the pulse pair approach, the complex expression of the received radar signal, V(t, r), is constructed by applying the Hilbert transform to each pulse of radar echo,

\[ V(t, r) = a(t, r) e^{i\phi(t, r)}, \]

where \( t \) is time, \( r \) ground range, \( a \) amplitude and \( \phi \) phase. The normalized radar cross section (NRCS) is derived from the square of the amplitude taking into account the cubic range falloff,

\[ \sigma_0(t, r) = C(r) \frac{d^2(t, r)}{r^4}, \]

where the factor \( C \) includes the antenna pattern and relevant calibration reference. The instantaneous Doppler frequency
of surface scattering element is calculated by the temporal derivative of the unwrapped phase of the complex signal,

\[
\omega_D(t, r) = \frac{\partial \phi(t, r)}{\partial t}.
\]  

For the data presented here, the differential time is 1/1200 s. To reduce data noise, running average with a window of 200 temporal pixels is performed, yielding an equivalent integration time of 1/6 s.

[11] The Doppler frequency is caused by the motion of the scattering element. The horizontal surface velocity projected in the radar look direction can be calculated from the

**Figure 1.** Wind and wave conditions relevant to the experiment: (a) wind speed, (b) wind direction, (c) significant wave height, and (d) peak wave period. Measurements from both stations SPAG1 and 41008 are shown.

**Figure 2.** An example of the spatial evolution of Doppler spectrum along the ground range: (a) VV and (b) HH.
measured radial component of the advection velocity (the Doppler velocity) of the scattering element by

$$u_D(t_r) = \frac{\omega_D(t_r)}{2k_r \cos \theta_g}$$

where $k_r$ is the radar wave number and $\theta_g$ the grazing angle. For the radar frequency used in this experiment, $k_r = 197$ rad/m, and the denominator on the right hand side of (4) is 394 rad/m at $\theta_g = 1^\circ$ and 392 rad/m at $\theta_g = 6^\circ$.

The Doppler frequency of radar backscatter is more conventionally investigated by its spectrum computed from short segments of the complex radar return signal [e.g., Plant et al., 1987; Plant and Keller, 1983; Plant, 1997]. Figure 2 shows the sequences of VV and HH Doppler spectra calculated with nonoverlapping segments of 200 data points (1/6 s time series) using the FFT approach. For plotting purpose, the spectra are normalized to have its peak equal to one and with an offset increases sequentially with ground range, which is noted on each curve in Figure 2a. The quasi-sinusoidal oscillation along range reflects the modulation by the long-scale surface gravity waves.

The Doppler frequency computed by (3) is the equivalent of the mean Doppler frequency defined by the first moment of the Doppler frequency spectrum [Thompson and Jensen, 1993]. Figures 3a and 3c represent an example of the spatiotemporal images of the VV and HH Doppler velocity fields of the scattering roughness elements modulated by large-scale ocean surface waves processed by the pulse pair procedure. The corresponding images computed from the first moment of the Doppler spectra using the FFT procedure are shown in Figures 3b and 3d. The spatial coverage of the data presented is about 625 m (ground range from 96.8 to 721.4 m) and temporal coverage 5 s. The event near range 140–160 m displays the typical properties of a sea spike with sharply increased mean Doppler frequency, broadened spectral bandwidth (see Figure 2), and a duration as long as a few seconds. More quantitative comparisons between the pulse pair and FFT approaches are illustrated with temporal cuts through the images at four different ranges in Figure 4 and spatial (range) cuts at two different times in Figure 5. From these comparisons, it is confirmed that the Doppler velocity information useful for the retrieval of surface wave information (mainly the oscillatory fluctuation) can be derived from either the pulse pair or FFT procedure. From data processing point of consideration, the former is much simpler to implement.

4. Ocean Surface Wave Analysis

Applying spectral analysis to the spatial series (along range) of the Doppler velocity record such as the one shown in Figure 3 or Figure 5, a wave number spectrum of wave velocity, $S_u(k)$, is obtained. Because the record length of each spatial series is relatively short (on the order of 10 dominant wavelengths), detrending of signal prior to Fourier transformation is important. Low-pass procedure is
adopted here for signal detrending. The results presented below uses a second-order Butterworth digital filter with a low-pass wavelength of 120 m (the dominant wavelengths in the data set are between 30 and 90 m). The passband can be adjusted for different sea states. The low-pass trend is then subtracted to yield the high-pass Doppler velocity data.

Test runs using high-pass wavelengths from 90 to 210 m show considerable differences so some judgment of setting the high-pass wavelength is crucial. We adopt the practice of estimating the dominant length scale from the space-time images of the Doppler returns such as those shown in Figure 3. The sensitivity of setting the high-pass wavelength can also be inferred from the analysis of, e.g., Plant [1997, Figure 5].

From linear wave theory, the surface displacement spectrum, $S_h$, is related to the radar measured velocity spectrum, $S_u$, by

$$S_u(k) = D(k)S_h(k),$$

where $D$ is directional distribution, $\theta$ and $\theta_r$ are the wave propagation and radar look directions with respect to the mean wind direction (assumed to be 0), and $\omega$ the angular frequency, which is related to the wave number $k$ by the linear dispersion relation,

$$\omega = (gk \tanh kh)^{1/2},$$

and $h$ the local water depth (25 m). For an ideal unidirectional wave train, the relationship between the directionally averaged spectra of surface elevation and wave-induced velocity would be simple, but directional distribution of the wavefield in nature introduces a directional factor, $X_D$, and

$$S_p(k) = X DS_h(k)/\omega^2.$$

Figure 6 is an example of the wave number spectra processed with VV and HH Doppler velocity. For comparison, the equivalent spectrum converted from the frequency spectrum measured by the NDBC buoy at about the same time is also shown. $X_D = 1$ is applied in the conversion of frequency to wave number spectrum using (7) since there are several more complicated factors (to be further discussed in the last paragraph of this section) that would modify the ideal conversion equation (5).

For ocean surface wave research involving momentum and energy exchanges across the air-sea interface, the key parameters of the surface wavefield are the significant wave height and spectral peak frequency. Together with the reference wind speed (typically the neutral wind speed at 10 m elevation is employed), the three environmental variables form the dimensionless parameters quantifying ocean surface processes such as the growth of wind-generated waves and air-sea momentum and energy exchanges [Hwang, 2009]. The spectral peak wave component can be obtained using the method suggested by Young [1999, p. 239],

$$\omega_p = \frac{\int_0^\infty \omega S_p^4(\omega) d\omega}{\int_0^\infty S_p^4(\omega) d\omega}.$$
Figure 7. Comparison of (a) peak wave period and (b) significant wave height calculated with VV radar Doppler velocity and buoy measurements. Results using 1 and 5 s of radar data are shown. Line segments of 1:1, 1.2:1, and 1:1.2 slopes are superimposed for reference. (c and d) Same as Figures 7a and 7b but for HH Doppler velocity.

The result is in excellent agreement with the peak wave period reported on the Web site, with a proportionality coefficient of 0.998, correlation coefficient 0.937, normalized bias 1.6%, and normalized root-mean-square (RMS) difference 4.1%. Young’s procedure is borrowed to process the wave number spectrum of surface elevation derived from radar Doppler velocity, that is,

\[
k_{p} = \frac{\int_{0}^{\infty} kS^{2}(k)dk}{\int_{0}^{\infty} S^{2}(k)dk},
\]

and linear dispersion relation (6) is used to convert the spectral peak wave number to spectral peak wave frequency for comparison with NDBC report. It is recognized that the two spectral peak quantities are not identical [Plant, 2009] and further discussion is presented in section 5.

As have been discussed in many earlier publications [e.g., Pidgeon, 1968; Lee et al., 1995, 1996; Hwang et al., 2008a, 2008b], the sea return of horizontal polarization contains considerable breaking wave contribution. The resulting Doppler velocity is generally larger than the counterpart of vertical polarization. The spectral peak of HH Doppler velocity is apparently at a lower wave number than that of the VV Doppler velocity (Figure 6a). These spectra compare well with in situ measurement from a nearby NDBC buoy especially in the energetic spectral peak region. Interestingly, the spectrum of the Doppler velocity computed with 1 s of data, while spikier, is very similar to that computed with 5 s data (Figure 6b).

[19] For low grazing angle configuration, shadowing effect is obviously serious. It causes the trough region and lower waves behind a large wave crest to be underrepresented in the backscatter measurement. While this may pose only minor problem on the resolution of the spectral peak wavelength, we do expect some impact on the spectral density at the spectral peak and more distortion away from the peak region. Here we seek an empirical relation between the spectrally integrated surface wave height and particle velocity computed from the Doppler signal, that is, to obtain significant wave height, \( H_s \), or RMS wave displacement, \( \eta_{RMS} \), from radar Doppler velocity measurement,

\[
\eta_{RMS} = \frac{H_s}{4} = X \frac{u_{DRMS}}{\omega_p},
\]

where \( \omega_p \) is introduced for dimensional consistency, \( u_{DRMS} \) the RMS value of the high-passed Doppler velocity, and \( X \) an empirical coefficient accounting for various uncertainties including directional distribution, shadowing effect, radar look direction with respect to wave propagation, swell modification, and difference between spatial and temporal measurements.

5. Application and Result

[20] All together, 32 cases are available for analysis (see Figure 1). The results are presented in Figure 7, showing the comparison of peak wave period and significant wave height calculated from radar Doppler velocity and in situ wave sensors on the tower and the buoy. The peak wave period derived from the present procedure is about 1 to 1.5 s larger than in situ measurements. Accounting for this bias, most wave period and wave height data points are confined within an envelope bounded by ±20% from perfect agreement; for reference, line segments of 1:1, 1.2:1 and 1:1.2 slopes are superimposed in Figure 7. Table 1 lists the statistical parameters, including normalized and unnormalized
bias, \( B_a \) and \( B \), proportionality coefficient (slope) from orthogonal fitting, \( s \), normalized and unnormalized RMS difference, \( D_n \) and \( D \), and correlation coefficient, \( R \), computed from linear regression (for a discussion of orthogonal fitting [see Hwang et al., 1998, Appendix]).

[21] As mentioned in the last paragraph, the wave period derived from radar with 1 s or 5 s data length biases high by about 1–1.5 s or 20–27%. Several reasons can contribute to this bias.

[22] 1. The inherent difference of the spectral peak components in frequency and wave number spectra due to the additional \( k^{-0.5} \) weighting in the wave number spectrum [Plant, 2009], i.e.,

\[
S(k) = S(\omega) \frac{d\omega}{dk} \approx \frac{1}{2} S(\omega)g^{0.5}k^{-0.5}
\]  

(11)

[23] As expected from the functional dependence shown in (11), the spectral peak wave number is at a lower wave number than that computed from the spectral peak frequency using the linear dispersion relation.

[24] 2. Shadowing effect due to low grazing angle application causing the underrepresentation of lower, and presumably shorter, waves behind higher wave crests [e.g., Nieto Borge et al., 2004; Johnson et al., 2009].

[25] 3. The primary reason of the overestimation of wave period is considered to be the nature of strong low-frequency component in the radar Doppler velocity signal, especially near low grazing angles [see, e.g., Plant, 1997, Figure 5]. The exact source of the low-frequency behavior of the radar Doppler velocity signal is not determined at this stage but it may be related to sea spikes that produce significant increment in the magnitude of both VV and HH Doppler velocity [Hwang et al., 2008b]. The Doppler velocity of breaking scatterers is considerably larger than that of the nonbreaking ones for both polarizations. The difference is about 0.7 to 0.8 m/s at 5.5° grazing angle and about 0.4 to 0.5 m/s at 1.4°. These differences are a substantial fraction of the mean Doppler velocities of the nonbreaking subpopulations, between 1.2 to 1.8 m/s for VV and 1.6 to 2.2 m/s for HH. Because sea spikes occur at a much lower frequency than the dominant waves, the spectral signature of Doppler return thus has a strong low-frequency component.

[26] The slope of linear regression is very close to unity (Table 1). The relative RMS difference is on the order of 20% using 5 s data and up to 56% using 1 s data with only minor differences between VV and HH polarizations. As a result of large relative RMS difference, the correlation coefficient is somewhat poor, 0.06 to 0.36 using 1 and 5 s radar data. This is partially caused by the difficulty in determining the spectral peak component in spiky spectra, especially when the spectral degree of freedom is low. The problem is alleviated somewhat with the integral definition of spectral peak [Young, 1999] adopted here (9).

[27] The regression parameters for significant wave height are somewhat better than those of the peak wave period. The bias is between 0 and 5%, good linear proportionality, 19 to 37% RMS difference and correlation coefficient of 0.36 and 0.46 using 1 and 5 s radar data. The empirical parameter, \( X \), in (10) is found to be about 1.3 for VV and 1.0 for HH from the present data set, reflecting some mutual cancellation effects among the various factors causing distortion of the measured Doppler velocity from the true orbital velocity of the wavefield. The exact quantification of the distortion factors, however, requires substantial theoretical analyses that are beyond the scope of this paper.

[28] Slightly improved performance can be achieved by fine-tuning the low-pass wavelength for data detrending during the spectral analysis procedure. Figure 8 illustrates the comparison of wave spectra obtained with 120 m and 165 m low-pass detrending, and Table 1 lists the statistics of

![Figure 8](image)

Figure 8. Same as Figure 6 but the detrending low-pass wavelength is 165 m. (a) \( S_u(k) \) and (b) \( S_u(k)/k \), which is approximately \( S_h(k) \).
using 90 m low-pass detrending for comparison with the result of 120 m detrending described in this section.

6. Summary

In this paper, we investigate the issue of retrieving surface wave height using coherent radar output. It is well accepted that the periodic feature of the radar backscattering cross section is the result of surface wave modulation and the technique for deriving wave period and wave direction from radar backscattering intensity measurement is well developed [e.g., Young et al., 1985]. The determination of spectral density or wave height, however, is a much more difficult issue because the complex MTF varies with many factors. In contrast to the backscattering intensity, Doppler frequency output from a coherent radar is caused by the radial velocity of the scattering roughness, which is short waves on the ocean surface. These short waves propagate with their own phase speeds and in the mean time been advected by mean and oscillatory currents of all sources. The analysis presented here confirms that surface waves are the primary contributor of the oscillatory component of the Doppler velocity. From analysis of field data, we found that with radar range coverage on the order of 10 dominant wavelengths, a reasonably good assessment of peak wave period and significant wave height can be obtained with radar data as short as 1 s. With a scanning coherent radar system, the wave direction can also be determined with short data records, on the order of 1 min [Trizna, 2009].

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References


