Safe, Stable and Intuitive Control for Physical Human-Robot Interaction

Vincent Duchaine and Clément Gosselin

Abstract—For physical human-robot interaction, safety and dependability are of utmost importance due to the potential risk a relatively powerful robot poses for human beings. From the control standpoint, it is possible to increase this level of safety by guaranteeing that the robot will never exhibit any unstable behaviour. However, stability is not the only concern in the design of a controller for such a robot. During human-robot interaction, the resulting cooperative motion should be truly intuitive and should not restrict in any way the human performance. For this purpose, we have designed a new variable admittance control law that guarantees the stability of the robot during constrained motion and also provides a very intuitive human interaction. The first characteristic is provided by the design of a stability observer while the other is based on a variable admittance control scheme that uses the force derivative as a way to predict human intention. The stability observer is based on a previous stability investigation of cooperative motion which implies the knowledge of the interaction stiffness. A method to accurately estimate this stiffness online using the data coming from the encoder and from a multi-axis force sensor at the end effector is also provided. The stability and intuitivity of the control law were verified in a user study during a cooperative drawing task with a 3 degree-of-freedom (dof) parallel robot.

I. INTRODUCTION

Halfway between current industrial robots and future fully autonomous humanoids one can expect in a close future, the emergence of a new generation of robots with the capability to assist directly the human in some more or less sophisticated everyday tasks. In the last few years, a significant amount of research was done in this research trend under what is now usually referred to as physical human-robot interaction (pHRI). Although this category of research covers many different topics, it is mainly focused on safety for the obvious reason that, in this concept, humans share their workspace with robots, which can be powerful relative to a person.

Under this safety consideration it is easy to understand why a control algorithm that leads to a stable and therefore safe physical interaction is recognized as being one of the main research challenges[1]. However, almost all schemes for control of constrained motion are known to potentially exhibit unstable behaviour when facing rigid environments [2]. One of the most commonly used control schemes in pHRI that does not make exception to this rule is probably the so-called impedance/admittance control.

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Using the idea that instability is linked to the stiffness of the interaction interface, which is a human in this case, Tsumugiwa et al. have proposed in [3] a new variable impedance control law based on an estimation of the human stiffness. In this work the authors have used the human stiffness to proportionally adjust the virtual damping coefficient. The experimental validation they conducted has demonstrated a completely stable robot behaviour with also an improved cooperative performance comparatively to conventional impedance control. In [4], the authors circumvent the potential instability of admittance control by instead, using a procedure of human intent estimation and use this identified motion to move in cooperation with the human operator.

One other solution for stability of controllers for constrained motion is the so-called concept of passivity. In this approach, instead of using an explicit model of the process, the idea is to look at the power flow. This concept has been mainly used in haptics [5], but also more recently in the context of pHRI [6]. This idea is identified in [1] as being a promising feature for stable and therefore safe pHRI. However, although this approach guarantees the complete stability of a robot controller with a very good robustness, this criteria is somehow over-conservative with potential negative repercussion on the transparency of the controller.

This last point is also of a certain importance. Indeed, the stability of the control schemes is not the only requirement in the design of good control algorithms for pHRI. The capability of following smoothly and naturally the human...
during the cooperative task is also of a major importance. In other words, the robot must not become a burden for the human and the cooperation should be very intuitive for the person involved. This concept is often referred to as transparency. In [7] the authors specifically address this problem in the context of human-robot cooperative motion.

Regarding this challenge, one popular idea has been to use a new adaptive impedance/admittance control law specially designed for human-robot cooperative motion. Even in its original form proposed by Hogan two decades and a half ago [8], impedance control has been proved to share some similarities with how a human being interacts with its own environment, including other people [9]. However, humans are known to adapt their own impedance while performing some given tasks. According to this fact and in order to provide a better human-robot interaction, it was initially proposed in [10] to use a discrete variable impedance damping coefficient based on a threshold on the velocity of the robot. Later, in [11] it was proposed to continuously adapt the virtual damping coefficient as function of the human intention, inferred from the time derivative of the force at the interaction point.

The approach presented in this paper results in a combination of a variable admittance control law based on human intention, that we presented in [11], and of a stability observer. The latter is designed with the aim that the control law would no more be a tradeoff between performance and stability and is based on the result of the investigation of the stability of the human-robot cooperative scheme using Lyapunov analysis that was presented at ICRA08 [12]. A short overview of the variable admittance control based on an estimation of the human intent is first given. Then, a procedure for the estimation of interaction stiffness using the same multi-axis force sensor than for the admittance control is presented. Using the result of this estimation, a stability observer based on a previous stability investigation and then a complete control law are provided. Finally, the stability of the control law and its capability to follow optimally human motion are validated during a cooperative drawing task between 6 human subjects and a 3-dof parallel manipulator that can be seen in figure 1.

II. VARIABLE ADMITTANCE MODEL AS FUNCTION OF HUMAN INTENTION

In a pHRI using touch as the sense of interaction, the interpretation of the human intention is typically based on the direction and the magnitude of the force measured at the operational point by the sensor. This information essentially provides knowledge about the present: the direction and the speed desired by the operator. Using the rate of change of this force could provide information about the future intentions of the human operator, similarly to the approximation of the future values of a function using a first order Taylor expansion. Since it was shown in [9] that damping parameters are the dominating impedance/admittance coefficients in the context of human-robot cooperation, the additional information provided by the time derivative of the force will be used to adjust these coefficients. The variable damping is adjusted according to the following law:

A sudden increase in the magnitude of the force \( f \) in the direction of the velocity or from zero velocity is interpreted as a human intention to accelerate. In this situation, a high damping coefficient is detrimental since it limits the acceleration and the velocity. Therefore, the damping coefficient \( c \) should be reduced. On the other hand, a sudden decrease in the magnitude of the force in the direction of the velocity is interpreted as an intention of the human operator to decelerate, stop the movement or reverse the direction. In this situation, a high damping coefficient is useful, reducing the virtual inertia of the system.

Following these ideas, a variable damping matrix can be defined as:

\[
C_{vi} = C - S
\]  

where the \( ij \) entry of matrix \( S \) is given by:

\[
s_{ij} = \alpha \delta_{ij} \text{sgn}(\dot{p}_i) \dot{f}_i
\]  

and where \( \dot{f}_i \) is the time derivative of the \( i \)th component of the force, \( \dot{p}_i \) is the \( i \)th component of the Cartesian velocity vector, \( \delta_{ij} \) is the Kronecker delta, \( \alpha \) is a weighting factor and \( \text{sgn} \) is the signum function. Including this damping matrix in the conventional admittance model leads to a variable control scheme that will be used as a basis in this work:

\[
f = M \ddot{p} + C_{vi} \dot{p}
\]

More details about this variable admittance law and how it can increase transparency during cooperation can be found in [11].

III. ONLINE ESTIMATION OF INTERACTION CARTESIAN STIFFNESS

In [12] the stability of admittance control during human-robot interaction has been investigated using Lyapunov theory. An analytic expression describing the stability frontier for this scheme has been found, from which equations for critical impedance parameters were derived. These equations are functions of some known characteristics of the system but also of the stiffness at the interaction point, which obviously cannot be directly obtained. Also this latter property is not constant over time since humans always adjust their stiffness as a function of the task to be performed. Therefore, real-time estimation of the interaction stiffness is fundamental in order to predict instability and design new adaptive feedback schemes that will guarantee stability. This section presented a method to provide this estimation mainly based on least squares methods.

A. Previous approaches

Mussa-Ivaldi et al. [13] have investigated the challenge of estimating human stiffness more than two decades ago. However, the proposed approach was only suitable for offline estimation, requiring a large amount of data and processing. Although this method is not applicable for real-time estimation of the interaction stiffness, the results of these experiments have demonstrated that the Cartesian stiffness of human arms always has the shape of an ellipsoid.
In [3], Tsumugiwa et al. proposed a way to estimate the human stiffness in real-time. In order to simplify the problem, they assumed that there is no coupling of the impedance characteristics of human arms in the 3 axes, therefore, the stiffness matrix is diagonal. This matrix was directly estimated from the data of a force sensor and of the Cartesian position (provided by the encoders via the FKP) following this equation:

\[ K = \Delta F_{est} \Delta P_{est}^{-1} \]  \hspace{1cm} (4)

with

\[ \Delta F_{est} = \text{diag} \left( f(t) - f(t-t_s) \right) \]  \hspace{1cm} (5)
\[ \Delta P_{est} = \text{diag} \left( p(t) - p(t-t_s) \right) \]  \hspace{1cm} (6)

where \( f \) is the vector of measured forces at the end effector, \( p \) is the vector of positions and \( t_s \) is the time step.

Although this method can provide good estimates in some configurations, it can also lead to very poor approximations in others, with the consequence of underestimating the real stiffness. The problem is related to the assumption of a diagonal matrix. According to this assumption, the principal directions of the estimated ellipsoid-shaped stiffness matrix is always aligned with Cartesian principal directions. Figure 2 illustrates the case of a poor estimate based on a real experiment in a plane (two-dimensional workspace).

One other drawback of the approach is that the estimation is performed using a differentiation of the force and position signals, which can be relatively noisy and also lead to the problem of a zero division if the relative displacement goes below the resolution of the encoder.

**B. Proposed method**

The Cartesian behaviour of the human arm can be characterized as an impedance resulting from a combination of inertia, damping and stiffness effects. However, it was shown in [14] that the inertial effect is almost negligible. In this work, the interaction stiffness is estimated by also neglecting the human damping by directly linking the force read at the interaction point to the displacement, thereby resulting in an overestimation of the stiffness at high velocities. However, since the stiffness is known to be very low during fast movements, the stability will not be a major concern in this case and it is very unlikely that this overestimation will have a real impact. A choice was made here to deal with the overestimation of the stiffness rather than dealing with the noise added by including velocity measurements in the estimation loop. However, even if the human damping is not directly estimated, this characteristic is further taken into account in the calculation of the critical damping, as it will be shown later.

The equation linking force (\( f \)) to position (\( p \)) is a relation that can be expressed in a \( n \) dimensional space as:

\[ f = Kp + b \]  \hspace{1cm} (7)

where \( b \) is a vector of offset which is not zero if the point of zero force does not coincide with the origin and is not necessarily constant depending on the human desired motion. It is desired to estimate matrix \( K \) such that it is a solution to eq. (7). Assuming a diagonal stiffness matrix gives us a fully constrained system, which can be easily solved using only one measurement. This is very useful in the context of an online estimation. However, since it was shown that this assumption can lead to erroneous estimations, we need to use at least \( m = n \) sets of data for position and force to estimate adequately the stiffness in a \( n \) dimensional space. However, due to measurement noise or the possibility to obtain a singular system (two consecutive identical measurements), it would be preferable to base our estimation on a larger set of data, namely \( m \) data points where \( m > n \). In this case the system becomes over-constrained and in the context, an exact solution to eq. (7) would be very unlikely. Therefore, instead of attempting to solve this equation directly we will rather try to minimize the square of the errors given by the estimation of the matrix \( K \), namely:

\[ \min_k \chi_k^2 = (Kp + (B - F))^T (Kp + (B - F)) \]  \hspace{1cm} (8)

where \( K \) is the \( n \times n \) stiffness matrix, \( P \) and \( F \) are respectively the \( n \times m \) Cartesian position and force matrix and \( B \) is the \( n \times m \) matrix containing the offset of the linear expression. Force and position data are directly obtained from sensors, however the offset matrix \( B \) must be evaluated. Using linear regression theory to approximate this parameter, one has:

\[ B_{[n\times m]} = (\bar{f} - \bar{V}^{-1} \Sigma \bar{p})_{[n\times 1]} 1_{[1\times m]} \]  \hspace{1cm} (9)

where \( \bar{f} \) and \( \bar{p} \) are the vectors of the \( n \) means value for the \( m \) past samples and \( V \) and \( \Sigma \) are the \( n \times n \) diagonal matrix of the variance of the \( n \) position signals and the covariance between the latter and the \( n \) force components, namely

\[ V_{ij} = \delta_{ij} \sigma^2_{p_i} \]  \hspace{1cm} (10)
\[ \Sigma_{ij} = \delta_{ij} \text{cov}(p_i, f_j) \]  \hspace{1cm} (11)

where \( p_i \) and \( f_j \) are respectively the \( i^{th} \) or \( j^{th} \) row vector of matrices \( P \) and \( F \) and \( \delta_{ij} \) is the Kronecker delta. Differentiating eq. (8) leads to:

\[ \text{grad} \chi_k^2 = 2 (KP - F + B) P^T. \]  \hspace{1cm} (12)
Since eq. (8) is quadratic, its minimum can be found when:
\[
\frac{\partial\chi^2}{\partial K} = 0. \tag{13}
\]
Then, the value of \( K \) that minimizes the error on the sampling horizon is given by:
\[
K = (F - B)P^T (PP^T)^{-1} \tag{14}
\]
where \( PP^T \) is simply the right inverse of \( P \).

1) Singular \( P \) matrix in real implementation: It is very unlikely to have always significant measurements in all \( n \) dimensions for a robot working in the \( n \) dimensional space. In fact, if part of the prescribed task requires only translation or if movement is momentarily constrained in a plane, matrix \( P \) or \( F \) will probably be singular or close to being singular, leading to difficulties in evaluating \((PP^T)^{-1}\). Many real tasks will lead to this kind of constrained motion. Just for example, drawing in cooperation with a robot will constrain motion on a plane.

In order to avoid this problem, some tests need to be conducted on the data collected to find which sets are significant and which are not. The variance on the position data can be used as a threshold (\( v_{thr} \)) for making a decision. The following matrix is defined:
\[
H_{[n \times n]} = \text{diag} (h) \tag{15}
\]
where \( h \) is a vector whose components are given by the Heaviside function:
\[
h_i = \begin{cases} 
0 & \text{if } V_{i,i} - v_{thr} < 0 \\
1 & \text{if } V_{i,i} - v_{thr} \geq 0.
\end{cases} \tag{16}
\]
In other words, matrix \( H \) is a diagonal matrix whose \( i \)th diagonal entry is equal to zero if the variance of the data for the corresponding coordinate is smaller than a given threshold. Otherwise, the \( i \)th diagonal entry is equal to 1.

1 Matrix \( H \) provides information on which directions should be considered and can therefore be used to build a matrix \( G \) that will change the size of matrices \( P, F \) and \( B \) in order to work in a dimension in which all data is significant. The new matrix is in fact a unitary basis of the column space of matrix \( H \) or more simply in this case, an \([n \times \text{rank}(H)]\) matrix which is the contraction of matrix \( H \) where the vanishing columns have been suppressed. For example if:
\[
H = \begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 
\end{bmatrix} \tag{17}
\]
\( G \) will be:
\[
G = \begin{bmatrix} 1 & 0 \\
0 & 1 \\
0 & 0 
\end{bmatrix}. \tag{18}
\]

Using this matrix \( G \), the interaction stiffness can now be computed in \( R^\text{rank}(H) \) without running into the singularity problem. Therefore, eq. (14) becomes:
\[
K = G^T [(F - B)P^T] G \left(G^T P P^T G \right)^{-1}. \tag{19}
\]

The threshold can have distinct values for different axis, i.e. translation vs rotation.

IV. STABILITY OBSERVER

In [12] we found an equation describing the stability boundaries of the system from which equations for critical admittance parameters were obtained. By critical admittance parameters we mean the lowest virtual admittance that can be assigned to the robot without becoming unstable for a given system. Using these equations, it is possible to build a feedback that would adapt in real-time the virtual admittance of the robot in a way such that the cooperation would always be stable. Since damping is directly related to the dissipation of energy, it is more intuitive to adjust this parameter than to play with the virtual inertia.

A. Critical damping for a \( n \) dimensional system

If we generalize the expression for critical impedance damping given in [12], for a system in a \( n \) dimensional space, one has:
\[
C_c = -\frac{1}{2T} (M + TD_h) + \frac{1}{2T} \sqrt{(M + TD_h)^T (M + TD_h) + 4T^2 KM} \tag{20}
\]
where \( K \) and \( D_h \) are respectively the human \( n \times n \) stiffness and damping matrix, \( M \) is the \( n \times n \) virtual mass impedance of the robot and \( T \) is the time delay.

However, as discussed in the preceding section, it would be very difficult to estimate the critical damping in the whole \( n \) dimensional space. So using again matrix \( G \), the critical damping in \( R^\text{rank}(H) \) is then given by:
\[
C_c = -\frac{1}{2T} \left( Z - \sqrt{Z^T Z + 4T^2 KG^T MG} \right) \tag{21}
\]
where \( Z \) is the projection of \( (M + TD_h) \) into \( G \) space, namely:
\[
Z = G^T (M + TD_h) G \tag{22}
\]

According to the physics of the problem, the space orientation of the ellipsoid given by the critical damping matrix gain should be the same as that of the stiffness matrix. However, using only eq. (21) it is not so trivial to see — at least for the authors — if this orientation will be maintained. Figure 3 shows the planar shape of the critical damping.
matrix, given by the previous equation, on a real subject in two configurations. As expected, one can see that eq. (21) maintains the orientation of the stiffness ellipsoid.

B. Feedback

Assigning directly the critical damping matrix to the impedance model could guarantee the overall stability of the cooperative system. However, this would result in a non-trivial virtual robot dynamics that would be very difficult to learn for a human, with the consequence of a loss of effectiveness. It is preferable to build such feedback using a stability observer that will keep a pre-assigned virtual impedance as long as the latter does not lead to instability. This stability observer is based on the following equation:

\[
D = G^T C_{vi} G - C_e
\]  

(23)

where \(C_{vi}\) is the variable impedance damping matrix defined in eq. (1) for natural adaptation to the human intention and \(G^T C_{vi} G\) is simply its projection into the space associated with the cooperative task. Matrix \(D\) is called the global dissipation matrix. If this matrix is positive definite or positive semi-definite, the pre-assigned virtual dynamics allows the closed-loop system to remain dissipative and the robot can keep its original parameters. However, if \(D\) becomes negative definite, the impedance damping matrix needs to become directly the critical one to prevent energy build-up.

We may want to keep a safety margin in the stability estimation in order to be sure that the robot always remains safe for the human operator. In this case we can write the global dissipation matrix as:

\[
D = G^T C_{vi} G - \beta C_e
\]  

(24)

where \(\beta\) is a scalar comprised between 1 and \(\infty\) which is used as a safety factor.

Verifying the positive definiteness of matrix \(D\) using the Sylvester criterion can be time consuming since it requires computing the determinant of all the sub-matrices. This method can also lead to the problem of infinity if the result of eq. 24 is not a full rank matrix. A more efficient way to use this criterion is to compute a LU factorization of matrix \(D\) and to use the sign of the pivots to conclude on positive definiteness. Using this approach, two scalar indices are defined which, if equal, means that matrix \(D\) is positive definite or at least semi-positive. Using Einstein’s convention for summation, these indices can be written as:

\[
e = L_{ij} \delta^{ij} + U_{ij} \delta^{ij}
\]  

(25)

and

\[
e^* = \text{abs}(L_{ij}) \delta^{ij} + \text{abs}(U_{ij}) \delta^{ij}.
\]  

(26)

Applying the Kronecker delta on these indices gives us a simple and clear numerical answer on the positive definiteness of matrix \(D\). This answer takes the form:

\[
\delta_{e,e^*} = \begin{cases} 
1, & \text{if } e = e^* \\
0, & \text{if } e \neq e^* 
\end{cases}
\]  

(27)

Finally, with the use of this relation combined with the critical damping matrix, the variable admittance law given by eq. (3) becomes:

\[
f = M \ddot{p} + \left[ \delta_{e,e^*} C_{vi} + (1 - \delta_{e,e^*}) \left[ \beta G C_e G^T + H' C_{vi} \right] \right] \dot{p}
\]  

(28)

with

\[
H' = I - H
\]  

(29)

where \(I\) is the identity matrix of order \(n\).

Pre- and post-multiplying matrix \(C_e\) by \(G\) and \(G^T\) has the effect of expanding it to an \(n \times n\) matrix with zero in the rows and columns where critical damping could not have been estimated. However, applying directly this matrix as the damping matrix for the impedance law can be hazardous. Indeed, a zero damping will appear in some Cartesian directions, leading to a potentially unstable system. Therefore, using matrix \(H'\), the corresponding initial variable damping \(C_{vi}\) can be applied in these directions. In the accompanying video, a demonstration of the procedure of stiffness estimation and its integration in the proposed controller is provided.

V. EXPERIMENTAL VALIDATION

The variable admittance controller presented in this paper is designed to exhibit stable behaviour and to provide more intuitive control during cooperative motion. The following experiment aims at demonstrating these two characteristics during a real human-robot cooperative task. The study was performed with the help of 6 different subjects of age ranging from 5 to 56. This group was formed of 3 males and 3 females all right-handed. The robot used during the cooperative drawing task is the Tripter, a fully decoupled 3-dof parallel robot with a multi-axis force sensor mounted at the end effector. Fig. (1) shows the experimental testbed. The task to be performed during this experiment involved that a subject holds a pen in cooperation with a robot and follows a fixed trajectory. The subjects all received the instruction to try to complete the task as quickly as possible, while minimizing path overshoot as much as possible. Two very different virtual admittance parameters were assigned to the robot, a high admittance corresponding to a virtual mass of 0.5 kg with a damping of 10 \(Ns/m\) and a low admittance corresponding to 5 kg and 60 \(Ns/m\). Each subject needed to perform the task for the two different dynamics twice, one with conventional fixed impedance and the other with the presented approach. These four trials were performed in a random order to reduce the impact of the learning effect in the results. For the implementation of eq. (28), \(\alpha\) and \(\beta\) parameters were set respectively to 0.5 and 1.2.

A. Results

Fig. (4) shows the results of the 6 subjects for the case where the virtual mass and damping was set to 0.5 kg and 10 \(Ns/m\). In this graph, the results for conventional admittance control as well as for the proposed control law are presented. For the system used during this experiment, these parameters were known to be close to instability with
conventional fixed admittance and therefore the robot was difficult to control. The advantage of the proposed variable admittance approach in terms of stability clearly appears on this figure by looking at the number of overshoots produced by the subjects. Due to its completely stable behaviour, even for this low virtual dynamics, the number of overshoots was significantly reduced using the presented approach.

The other case where, the virtual mass was set to 5kg and the damping to 60Ns/m, was used to demonstrate the adaptability of the control law to the human intention instead of its stability characteristic. Indeed, based on our observation that was presented in [12], these parameters were known to be above the critical stability frontier, and a fully stable behaviour was expected for both types of controls. However, such high dynamics in the context of a drawing task presented the inconvenience, with conventional fixed admittance, of being a little hard to control due to the high virtual inertia and also required a significant level of force input due to the high damping. In this context, as it was expected, the 6 subjects have performed significantly better with the robot controlled with our new control law due to its adaptation during acceleration and deceleration phases. This better performance can be observed in figure 5 in terms of shorter time to complete the task as well as by the lower number of overshoots that were produced.

VI. CONCLUSION

Taking into consideration the fact that the closed-loop stability of a robot in constrained motion is closely related to the interaction stiffness, we have designed a new adaptive admittance control law using the results of a real-time estimation of the interaction stiffness. In addition to its stability feature, the control scheme presented in this paper has also been conceived in order to have the capability to constantly adapt the robot’s virtual dynamics to match more adequately human cooperative motion. Human subject evaluation during a cooperative task confirmed that the assumption behind the design of the control law was adequately made and that the resulting scheme leads to a control solution for pHRI that is no longer a tradeoff between performance and stability.

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REFERENCES


Fig. 4. Result of the experimentation for a virtual mass of 0.5kg and 10Ns/m.

Fig. 5. Result of the experimentation for a virtual mass of 5kg and 60Ns/m.

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Fig. 4. Result of the experimentation for a virtual mass of 0.5kg and 10Ns/m.

Fig. 5. Result of the experimentation for a virtual mass of 5kg and 60Ns/m.