VI. CONCLUSION

Multiparametric quadratic and linear programming theory has been applied with success for implementing deterministic MPC controllers. In this note, we have proposed to apply the approximate multiparametric convex programming solver of [18] to the robust MPC control scheme proposed in [7]. An explicit description of the control law is obtained for ease of implementation of robust MPC. The control law assures robust constraint handling and robust convergence to a given bounded set. Also, an alternative approach has been given in order to assure convergence to the origin.

REFERENCES


Comments on “On the Global Stability of Delayed Neural Networks”
Chuandong Li and Xiaofeng Liao

I. INTRODUCTION

Consider the delayed neural network given in [1]

\[
\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(u_j(t)) + \sum_{j=1}^{n} b_{ij} \psi_j(x_j(t - \tau_j)), \quad i = 1, 2, \ldots, n. \tag{1}
\]

Assume that \( u^* = (u^*_1, \ldots, u^*_n)^T \) is an equilibrium point of system (1), the transformation \( x(t) = u(t) - u^* \) puts (1) into the following form:

\[
\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} \phi_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} \psi_j(x_j(t - \tau_j)), \quad i = 1, 2, \ldots, n. \tag{2}
\]

The main theorem presented in [1] is restated as follows.

**Theorem 1:** The equilibrium \( u^* \) of (1) is globally asymptotically stable if there exist constants \( p_k > 0 (k = 1, 2, \ldots, L) \), \( q_k > 0 (k = 1, 2, \ldots, L) \), \( \gamma_j > 0, \alpha_{ij}, \beta_{ij}, \gamma_{ij}, \xi_{ij}, \eta_{ij} \) \( \in \mathbb{R}, i, j = 1, 2, \ldots, n \) such that

\[
\sum_{k=1}^{L} \left( \sum_{k=1}^{L} p_k |a_{ij}| r_{ij}^{\alpha_{ij}} + \frac{\gamma_{ij}}{\alpha_{ij}} r_{ij}^{\frac{\gamma_{ij}}{\alpha_{ij}}} |b_{ij}| r_{ij}^{\beta_{ij}} \right) + \sum_{k=1}^{L} q_k |b_{ij}| r_{ij}^{\eta_{ij}} + \frac{\gamma_{ij}}{\alpha_{ij}} r_{ij}^{\frac{\gamma_{ij}}{\alpha_{ij}}} |b_{ij}| r_{ij}^{\beta_{ij}} < r c_i \tag{3}
\]

holds for each \( i = 1, 2, \ldots, n \), in which \( L_1 \alpha_{ij} + \alpha_{ij} = 1, L_1 \xi_{ij} + \xi_{ij} = 1, L_2 \beta_{ij} + \beta_{ij} = 1, L_1 q_{ij} + \eta_{ij} = 1 \) for all \( i, j = 1, 2, \ldots, n \) and

\[
r = 1 = \sum_{k=1}^{L_1} p_k = \sum_{k=1}^{L_2} q_k.
\]

Two errors appear in the proof of Theorem 1 when the authors attempt to obtain (7) by using [1, eq. (5)]. Concisely, the following

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incorrect inequalities were obtained by using (5): For all \(i, j = 1, 2, \ldots, n\)

\[
\begin{align*}
&\sum_{k=1}^{l-1} p_k |a_{ij}| r_{k,i}^{\xi_{ij}} m_{k,j}^{\epsilon_{ij}} |x_i(t)|^r + |a_{ij}| r_{k,i}^{\xi_{ij}} m_{k,j}^{\epsilon_{ij}} |x_j(t)|^r \\
&\sum_{k=1}^{l-2} q_k |b_{ij}| r_{k,i}^{\gamma_{ij}} n_{k,j}^{\eta_{ij}} |x_i(t)|^r + |b_{ij}| r_{k,i}^{\gamma_{ij}} n_{k,j}^{\eta_{ij}} |x_j(t) - \tau_j|^{r}. \\
&\sum_{k=1}^{l-2} q_k |b_{ij}| r_{k,i}^{\gamma_{ij}} n_{k,j}^{\eta_{ij}} |x_i(t)|^r + |b_{ij}| r_{k,i}^{\gamma_{ij}} n_{k,j}^{\eta_{ij}} |x_j(t) - \tau_j|^{r}.
\end{align*}
\]

(4)

A simple example can show that (4) does not hold in general. Let \(r = 3, \alpha_1 = 4, \xi = 1/6, \epsilon = 1/3, |a_{ij}| = 1, m_{ij} = 4, |x_i(t)| = 1, p_k = (1/2)(k = 1, 2, \ldots, L_1).\) Then, calculating the inequality (4) yields a contradiction, i.e., \(\leq 8.\)

A correct version for (4) is presented as follows:

\[
\begin{align*}
&\sum_{k=1}^{l-1} p_k |a_{ij}| r_{k,i}^{\xi_{ij}} m_{k,j}^{\epsilon_{ij}} |x_i(t)|^r + |a_{ij}| r_{k,i}^{\xi_{ij}} m_{k,j}^{\epsilon_{ij}} |x_j(t)|^r \\
&\sum_{k=1}^{l-2} q_k |b_{ij}| r_{k,i}^{\gamma_{ij}} n_{k,j}^{\eta_{ij}} |x_i(t)|^r + |b_{ij}| r_{k,i}^{\gamma_{ij}} n_{k,j}^{\eta_{ij}} |x_j(t) - \tau_j|^{r}. \\
&\sum_{k=1}^{l-2} q_k |b_{ij}| r_{k,i}^{\gamma_{ij}} n_{k,j}^{\eta_{ij}} |x_i(t)|^r + |b_{ij}| r_{k,i}^{\gamma_{ij}} n_{k,j}^{\eta_{ij}} |x_j(t) - \tau_j|^{r}.
\end{align*}
\]

Using the inequality (5) to the context of [1], we can obtain the condition of Theorem 1 in which the fractions \(p_k\) and \(q_k\) are added in comparison with (3)

\[
\sum_{i=1}^{n} \sum_{k=1}^{l-1} p_k |a_{ij}| r_{k,i}^{\xi_{ij}} m_{k,j}^{\epsilon_{ij}} + \gamma_{ij} m_{k,j}^{\epsilon_{ij}} |a_{ij}|^{r} \delta_i \\
\sum_{k=1}^{l-2} q_k |b_{ij}| r_{k,i}^{\gamma_{ij}} n_{k,j}^{\eta_{ij}} + \gamma_{ij} n_{k,j}^{\eta_{ij}} |b_{ij}|^{r} \delta_i < c_i.
\]

(6)

The corresponding revision should be made for [1, Cor. 1].

REFERENCES


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**Correction to “Quadratic Stability and Stabilization of Dynamic Interval Systems”**

Wei-Jie Mao and Jian Chu

Abstract—It is pointed out that the counterexample in a previous comment is not a counterexample. A new counterexample is presented to show that the necessity part of Theorem 1 of the above paper is not valid in some cases. Furthermore, based on the \(\mathcal{M}\)-matrix theory, a new quadratic stability condition for dynamic interval systems is proposed to correct the corresponding result in the above paper.

**Index Terms**—Interval systems, linear matrix inequality (LMI), \(\mathcal{M}\)-matrix, quadratic stability.

I. COUNTEREXAMPLE

We agree with the authors of [2] that the necessary condition of [1, Th. 1] does not hold in some cases. The reason for this mistake is that Lemma 3 was misused in the proof of Theorem 1. For example, if \(T_{11} < 0\) and \(|x^T T_1 x| > 4 \Delta x^T t_1 P e_1 e_1^T P x x^T e_1^T e_1^T x\) for all nonzero \(x \in R^n\), there does not always exist a real scalar \(\lambda_1 > 0\) such that \(\lambda_1^2 \Delta x^T t_1 P e_1 e_1^T P + \lambda_1 T_{11} + e_1 e_1^T < 0\) because \(T_{11}\) depends on some uncertain parameters. Lemma 3 is only applicable to the case of \(T_{11}\) to be a constant matrix.

However, we do not agree with the counterexample given in [2] for the following reason. Consider an interval matrix

\[
A = \begin{bmatrix}
-3 & 2 + \delta_1 & 0 \\
-10 & -2 & -1 \\
0.5 & 1 + \delta_2 & -3
\end{bmatrix}
\]

(1)

which is regarded as a counterexample in [2]. For this interval matrix, we can compute the upper bound of quadratic stability in two ways.

**Method 1:** Use the LMI condition for vertex matrices, which gives the necessary and sufficient condition. In detail, an interval matrix \(A\) of \(n \times n\) dimension is quadratically stable if and only if there exists a symmetric positive-definite matrix \(X\) satisfying \(X A^T + A X < 0, \quad i = 1, 2, \ldots, n\), where \(A, x \in R^n\), for all vertex matrices of \(A\).

**Method 2:** Use the LMI condition by [1, Th. 1], which is a sufficient condition in general.

For (1), we can numerically verify that both methods give the same upper bound of quadratic stability \(\delta = 2.05\). This means that the sufficient condition of [1, Th. 1] is exact in this example. This also shows that the conclusion in [2] “the condition in [1] is essentially conservative when the number of the uncertain entries is greater than 1” is not correct, since the number of the uncertain entries is 2 in (1).

Nevertheless, [1, Th. 1] is shown to be incorrect by the following counterexample. Consider an interval matrix similar to (2) described by

\[
A = \begin{bmatrix}
-3 & 2 + \delta_1 & 0 \\
-10 + \delta_1 & -2 & -1 + \delta_4 \\
0.5 + \delta_2 & -3
\end{bmatrix}
\]

(2)

Subject to \(|\delta_1| \leq 0.5\beta, |\delta_2| \leq \beta, |\delta_3| \leq 0.5 \beta\) and \(|\delta_4| \leq 0.5 \beta\).

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