Design of state estimator for neural networks of neutral-type

Ju H. Park a,*, O.M. Kwon b

a Robust Control and Nonlinear Dynamics Laboratory, Department of Electrical Engineering, Yeungnam University, 214-1 Dae-Dong, Kyongsan 712-749, Republic of Korea

b School of Electrical and Computer Engineering, 12 Gaeshin-Dong, Heungduk-Gu, Chungbuk National University, Cheonjyu, Republic of Korea

Abstract

In this paper, the design problem of state estimator for a class of neural networks of neutral-type is studied. A delay-dependent linear matrix inequality (LMI) criterion for existence of the estimator is proposed by using the Lyapunov method. The criterion can be easily solved by various convex optimization algorithms. A numerical example with simulation results is given to show the effectiveness of proposed method.

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1. Introduction

Since cellular neural networks (CNNs) have been introduced by Chua and Yang [1], various aspects of different neural networks such as Hopfield-type neural networks, cellular neural networks, Lotka–Volterra neural networks, Cohen–Grossberg neural networks, and bidirectional associative memory neural networks have received a great deal of interest due to their extensive applications in the fields of signal processing, pattern recognition, fixed-point computation, combinatorial optimization and associative memories [1–5]. As is well known, time delay may occur in the process of information storage and transmission in neural networks. This leads to the model of delayed cellular neural networks (DCNNs). Thus the stability analysis of DCNNs has become an important topic of theoretical studies in neural networks during the last decade [6–21]. It is well known that the existence of time delays is often a source of oscillation and instability. Due to the complicated dynamic properties of the neural cells in the real world, the existing neural network models in many cases cannot characterize the properties of a neural reaction process precisely. It is natural and important that systems will contain some information about the derivative of the past state to further describe and model the dynamics for such complex neural reactions [22–25].

* Corresponding author.
E-mail address: jessie@ynu.ac.kr (J.H. Park).

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On the other hand, since the neuron states are not often fully available in the network outputs in some applications, the state estimation problem of neural networks becomes an important topic for real applications recently [26,27]. Especially, Wang et al. [26] studied the problem of the state estimator for DCNNs and derived some delay-independent sufficient conditions for the existence of the estimator. Also, some delay-dependent conditions for the problem are presented by Huang et al. [27]. However, to the best of authors’ knowledge, the problem on design of state estimator for neural networks of neutral-type has not been investigated.

In this paper, we consider a general class of neural networks with time delays described by a nonlinear delay differential equation of neutral-type. Then, the main objective of this work is to estimate the neuron states of the network through available output measurements such that the dynamics of the closed-loop error system is globally stable. By constructing a suitable Lyapunov functional and utilizing LMI framework, a novel delay-dependent criterion for the existence of proposed state estimator of the network is given in terms of LMI. The advantage of the proposed approach is that resulting stability criterion can be used efficiently via existing numerical convex optimization algorithms such as the interior-point algorithms for solving LMIs [28].

Notation: $\mathbb{R}^n$ denotes the $n$ dimensional Euclidean space, and $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices. $I$ denotes the identity matrix with appropriate dimensions. The superscript “T” represents the transpose of given vector or matrix. $\| \cdot \|$ denotes the Euclidean norm of given vector. * denotes the elements below the main diagonal of a symmetric block matrix. For symmetric matrices $X$ and $Y$, the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite, (respectively, nonnegative). diag{⋯} denotes the block diagonal matrix.

2. Problem statement

In this paper, the model of neural networks of neutral-type is described by the following state equation:

$$
\begin{align*}
\dot{x}(t) &= -Ax(t) + W_1 f(x(t)) + W_2 g(x(t-h)) + V \dot{x}(t-h) + J, \\
y(t) &= Cx(t) + z(t,x(t)),
\end{align*}
$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector associated with $n$ neurons, $f(x(t)) = [f_1(x_1(t)), \ldots, f_n(x_n(t))]^T \in \mathbb{R}^n$ and $g(x(t-h)) = [g_1(x_1(t-h)), \ldots, g_n(x_n(t-h))]^T \in \mathbb{R}^n$ is the neuron activation functions, $J = [J_1, J_2, \ldots, J_n]^T$ is the external input vector at time $t$, $y(t) \in \mathbb{R}^m$ is the measurement output, $h > 0$ corresponds to finite speed of axonal signal transmission delay, $A = \text{diag}(a_i)$ is a positive diagonal matrix, $C \in \mathbb{R}^{m \times n}$ is a known constant matrix, $W_1 = (w_{ij}^1)_{n \times n}$, $W_2 = (w_{ij}^2)_{n \times n}$ and $V = (v_{ij})_{n \times n}$ are the interconnection matrices representing the weight coefficients of the neurons, and $z : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^m$ is the neuron-dependent nonlinear disturbances on the network outputs.

Throughout the paper, it is assumed that the functions $f, g, z$ satisfy the following conditions:

(A) The neurons activation functions $f$, and $g$ and the disturbance function $z$ are Lipschiz continuous:

$$
|f(x_1) - f(x_2)| \leq |F(x_1 - x_2)|, \quad |g(x_1) - g(x_2)| \leq |G(x_1 - x_2)|, \quad |z(x_1) - z(x_2)| \leq |Z(x_1 - x_2)|,
$$

where $F \in \mathbb{R}^{n \times n}$, $G \in \mathbb{R}^{n \times n}$ and $Z \in \mathbb{R}^{n \times n}$ is the known constant matrices.

The purpose of this paper is to present an efficient estimation algorithm to observe the neuron states from the available network output. For this end, the following full-order observer is proposed:

$$
\dot{\hat{x}}(t) = -A\hat{x}(t) + W_1 f(\hat{x}(t)) + W_2 g(\hat{x}(t-h)) + V \dot{\hat{x}}(t-h) + J + K[y(t) - Cx(t) - z(t, \hat{x}(t))],
$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimation of the neuron state, and $K \in \mathbb{R}^{n \times m}$ is the gain matrix of the estimator to be designed later.

Define the error state to be

$$
e(t) = x(t) - \hat{x}(t)
$$

and

$$
\phi(t) = f(x(t)) - f(\hat{x}(t)), \quad \varphi(t) = g(x(t)) - g(\hat{x}(t)), \quad \psi(t) = z(t, x(t)) - z(t, \hat{x}(t)).
$$
Then, the error dynamical system is expressed by
\[ \dot{e}(t) = -(A + KC)e(t) + W_2\phi(t) + W_2\varphi(t-h) + V\dot{e}(t-h) - K\psi(t). \] (6)

The following facts and lemma will be used for deriving main result.

**Fact 1** *(Schur complement).* Given constant symmetric matrices \( \Sigma_1, \Sigma_2, \Sigma_3 \) where \( \Sigma_1 = \Sigma_1^T \) and \( 0 < \Sigma_2 = \Sigma_2^T \), then \( \Sigma_1 + \Sigma_3\Sigma_2^{-1}\Sigma_3 < 0 \) if and only if
\[
\begin{bmatrix}
\Sigma_1 & \Sigma_3^T \\
\Sigma_3 & -\Sigma_2
\end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix}
-\Sigma_2 & \Sigma_3 \\
\Sigma_3^T & \Sigma_1
\end{bmatrix} < 0.
\]

**Lemma 1** *[25].* For a positive matrix \( U > 0 \), any matrices \( H \), and scalar \( h \geq 0 \), the following inequality holds:
\[
-\int_{t-h}^{t} \dot{e}^T(s)U\dot{e}(s) ds \leq \zeta^T(t) M \zeta(t) + h\zeta^T(t)M^T U^{-1} M \zeta(t),
\]
where
\[
M = \begin{bmatrix} 0 & 0 & H & 0 & 0 & 0 & 0 \end{bmatrix},
\]
\[
\zeta(t) = \begin{bmatrix} e^T(t) & e^T(t-h) & (\int_{t-h}^{t} \dot{e}(s) ds)^T & \dot{e}^T(t-h) & \phi(t) & \varphi^T(t-h) & \psi(t) \end{bmatrix}. \]

3. Main result

In this section, we derive a new delay-dependent criterion for asymptotic stability of the system (6) using the Lyapunov method combining with linear matrix inequality framework.

Then we have the following theorem.

**Theorem 1.** For given a positive scalar \( h \) and matrices \( F, G, Z \), the error system (6) is globally asymptotically stable if there exist positive definite matrices \( P, Q_i \) \( (i = 0, 1, 2) \), positive diagonal matrix \( Q_3 \), and any matrices \( M_i \) \( (i = 1, 2, \ldots, 7) \), positive scalars \( \alpha_i \) \( (i = 1, 2, 3) \) and any matrix \( Y \) satisfying the following LMI:

\[
\begin{bmatrix}
Q_1 & -M_1 + M_2^T & -M_1 + M_3^T & PV + M_4^T & PW_1 + M_5^T \\
* & Q_2 & -M_2 - M_3^T & -M_4^T & -M_5^T \\
* & * & Q_3 & -M_4^T & -M_5^T \\
* & * & * & -Q_2 & 0 \\
* & * & * & * & -\alpha_1 I \\
* & * & * & * & * \\
* & * & * & * & * \\
PW_2 + M_6^T & -Y + M_7^T & 0 & -h(A^TP + C^TY) & -(A^TP + C^TY)^T \\
-M_6^T & -M_7^T & 0 & 0 & 0 \\
-M_6^T & -M_7^T & hH & 0 & 0 \\
0 & 0 & 0 & hV^TP & V^TP \\
0 & 0 & 0 & hW_1^TP & W_1^TP \\
-Q_3 - \alpha_2 I & 0 & 0 & hW_2^TP & W_2^TP \\
* & -\alpha_3 I & 0 & -hY^T & -Y^T \\
* & * & -hQ_1 & 0 & 0 \\
* & * & * & -h(2P - Q_1) & 0 \\
* & * & * & * & -2P + Q_2
\end{bmatrix} < 0,
\]

(7)
where
\[ \Omega_1 = -PA - A^T P - YC - C^T Y T + Q_0 + G^T Q_3 G + z_1 F^T F + z_3 Z^T Z + M_1 + M_1^T, \]
\[ \Omega_2 = -Q_0 + z_2 G^T G - M_2 - M_2^T, \]
\[ \Omega_3 = H + H^T - M_3 - M_3^T. \]

Then, the gain matrix \( K \) of the state estimator (3) is given by
\[ K = P^{-1} Y. \]

**Proof.** Let us consider the Lyapunov functional candidate:
\[ V = V_1 + V_2 + V_3 + V_4 + V_5, \]
where
\[ V_1 = e^T(t) Pe(t), \]
\[ V_2 = \int_{t-h}^t e^T(s) Q_0 e(s) ds, \]
\[ V_3 = \int_{t-h}^t \int_s^t \dot{e}^T(u) Q_1 \dot{e}(u) du ds, \]
\[ V_4 = \int_{t-h}^t \dot{e}^T(s) Q_2 \dot{e}(s) ds, \]
\[ V_5 = \int_{t-h}^t \phi^T(s) Q_3 \phi(s) ds. \]

First, differentiating \( V_1 \) leads to
\[ \dot{V}_1 = 2e^T(t) P (-(A + KC) e(t) + W_1 \phi(t) + W_2 \varphi(t-h) + V \dot{e}(t-h) - K \dot{\psi}(t)). \]

Next, by differentiating \( V_i \) \( (i = 2, 3, 4, 5) \), we have
\[ \dot{V}_2 = e^T(t) Q_0 e(t) - e^T(t-h) Q_0 e(t-h), \]
\[ \dot{V}_3 = h \dot{e}^T(t) Q_1 \dot{e}(t) - \int_{t-h}^t \dot{e}^T(s) Q_1 \dot{e}(s) ds, \]
\[ \leq h \dot{e}^T(t) Q_1 \dot{e}(t) + \dot{\zeta}^T(t) \mathcal{H} \dot{\zeta}(t) + h \dot{\zeta}^T(t) \Pi^T \Pi_1^{-1} \Pi \dot{\zeta}(t), \]
\[ \dot{V}_4 = e^T(t) Q_2 \dot{e}(t) - e^T(t-h) Q_2 \dot{e}(t-h), \]
\[ \dot{V}_5 = \phi^T(t) Q_3 \phi(t) - \phi^T(t-h) Q_3 \phi(t-h), \]
where in deriving Eq. (12), the following inequality is used by Lemma 1:
\[ - \int_{t-h}^t \dot{e}^T(s) Q_1 \dot{e}(s) \leq \dot{\zeta}^T(t) \mathcal{H} \dot{\zeta}(t) + h \dot{\zeta}^T(t) \Pi^T \Pi_1^{-1} \Pi \dot{\zeta}(t). \]

From (2) and (5), it is clear that
\[ \phi^T(t) \phi(t) = |f(x(t) - f(\tilde{x}(t))| \leq |F e(t)|^2 = e^T(t) F^T F e(t), \]
\[ \phi^T(t) \varphi(t) = |g(x(t) - g(\tilde{x}(t))| \leq |G e(t)|^2 = e^T(t) G^T G e(t), \]
\[ \psi^T(t) \psi(t) = |z(x(t) - z(\tilde{x}(t))| \leq |Z e(t)|^2 = e^T(t) Z^T Z e(t). \]

Then, for positive scalars \( z_i \) \( (i = 1, 2, 3) \), we have
\[ z_1 [e(t)^T F^T F e(t) - \phi^T(t) \phi(t)] \geq 0, \]
\[ z_2 [e^T(t-h) G^T G e(t-h) - \phi^T(t-h) \phi(t-h)] \geq 0, \]
\[ z_3 [e^T(t) Z^T Z e(t) - \psi^T(t) \psi(t)] \geq 0. \]
As a tool of deriving a less conservative stability criterion, we add the following zero equation with any matrices $M_i$ ($i = 1, 2, \ldots, 7$) to be chosen as

$$2\left[ e^T(t)M_1 + e^T(t - h)M_2 + \left( \int_{t-h}^t \dot{e}(s)ds \right)^T M_3 + e^T(t - h)M_4 + \phi^T(t)M_5 + \phi^T(t - h)M_6 + \psi^T(t)M_7 \right]$$

$$\times \left[ e(t) - e(t - h) - \int_{t-h}^t \dot{e}(s)ds \right] = 0.$$  

(18)

Here note that

$$\phi^T(t)Q_3\phi(t) \leq e^T(t)G^TQ_3Ge(t).$$  

(19)

Utilizing the relationship (10)-(19), thus we have the following a new bound for $\dot{V}$:

$$\dot{V} \leq \zeta^T(t) \left( \Sigma + h\bar{H}^TQ_1^{-1}\bar{H} + \Gamma^T(hQ_1 + Q_2)\Gamma \right) \zeta(t),$$  

(20)

where

$$\Sigma = \begin{bmatrix} (1, 1) & -M_1 + M_2^T & -M_1 + M_3^T & PV + M_4^T & PW_1 + M_5^T & PW_2 + M_6^T & -PK + M_7^T \\ * & \Omega_2 & \Omega_3 & -M_4^T & -M_5^T & -M_6^T & -M_7^T \\ * & * & \Omega_3 & -M_4^T & -M_5^T & -M_6^T & -M_7^T \\ * & * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & * & -\alpha_1I & 0 & 0 \\ * & * & * & * & * & -Q_3 - \alpha_2I & 0 \\ * & * & * & * & * & * & -\alpha_3I \end{bmatrix},$$  

(21)

with

$$(1, 1) = -P(A + KC) - (A + KC)^T P + Q_0 + G^TQ_3G + \alpha_1F^T F + \alpha_3Z^T Z + M_1 + M_1^T,$$

$$\Gamma = [- (A + KC) \ 0 \ 0 \ V \ W_1 \ W_2 \ -K].$$  

(22)

If the matrix inequality $\Sigma + h\bar{H}^TQ_1^{-1}\bar{H} + \Gamma^T(hQ_1 + Q_2)\Gamma$ is a negative definite matrix, then $\dot{V} < 0$.

By Fact 1, the inequality $\Sigma + h\bar{H}^TQ_1^{-1}\bar{H} + \Gamma^T(hQ_1 + Q_2)\Gamma < 0$ is equivalent to the following inequality:

$$\begin{bmatrix} (1, 1) & -M_1 + M_2^T & -M_1 + M_3^T & PV + M_4^T & PW_1 + M_5^T & PW_2 + M_6^T & -PK + M_7^T \\ * & \Sigma_2 & \Sigma_3 & -M_4^T & -M_5^T & -M_6^T & -M_7^T \\ * & * & \Sigma_3 & -M_4^T & -M_5^T & -M_6^T & -M_7^T \\ * & * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & * & -\alpha_1I & 0 & 0 \\ * & * & * & * & * & -Q_3 - \alpha_2I & 0 \\ * & * & * & * & * & * & -\alpha_3I \\ PW_2 + M_6^T & -PK + M_7^T & 0 & -h(A + KC)^T Q_1 & -(A + KC)^T Q_2 \\ -M_6^T & -M_7^T & 0 & 0 & 0 & 0 & 0 \\ -M_6^T & -M_7^T & hH & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & hV^T Q_1 & V^T Q_2 & 0 & 0 \\ 0 & 0 & 0 & hW_1^T Q_1 & W_1^T Q_2 & 0 & 0 \\ -Q_3 - \alpha_2I & 0 & 0 & hW_2^T Q_1 & W_2^T Q_2 & 0 & 0 \\ * & -\alpha_3I & 0 & -hK^T Q_1 & -K^T Q_2 & 0 & 0 \\ * & * & * & -hQ_1 & 0 & 0 & 0 \\ * & * & * & * & -Q_2 & 0 \end{bmatrix} < 0.$$  

(23)
Pre- and post-multiplying $\text{diag}\{I, I, I, I, I, I, I, P Q_1^{-1}, P Q_2^{-1}\}$ and $\text{diag}\{I, I, I, I, I, I, I, Q_1^{-1}, P, Q_2^{-1} P\}$, respectively, we have

\[
\begin{pmatrix}
(1, 1) & -M_1 + M_2^T & -M_1 + M_3^T & PV + M_4^T & PW_1 + M_5^T \\
* & \Sigma_2 & -M_2 - M_3^T & -M_4^T & -M_5^T \\
* & * & \Sigma_3 & -M_4^T & -M_5^T \\
* & * & * & -Q_2 & 0 \\
* & * & * & * & -\alpha_1 I \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
PW_2 + M_6^T & -PK + M_7^T & 0 & -h(A + KC)^T P & -(A + KC)^T P \\
-M_6^T & -M_7^T & 0 & 0 & 0 \\
-M_6^T & -M_7^T & 0 & hH & 0 \\
0 & 0 & 0 & hV^T P & V^T P \\
0 & 0 & 0 & hW_1^T P & W_1^T P \\
-\alpha_2 I & 0 & 0 & -hK^T P & -K^T P \\
* & -\alpha_3 I & 0 & -hQ_1 & 0 \\
* & * & -hQ_1 & 0 & 0 \\
* & * & * & -hP Q_1^{-1} P & 0 \\
* & * & * & * & -P Q_2^{-1} P
\end{pmatrix} < 0. \tag{24}
\]

It is noted that (24) is not an LMI because of two nonlinear terms $-hP Q_1^{-1} P$ and $-P Q_2^{-1} P$. In view of the inequality [27]

\[ P Q_1^{-1} P \geq 2P - Q_1, \quad P Q_2^{-1} P \geq 2P - Q_2, \]

and change of variable as $Y = PK$, the LMI condition (7) can guarantee that inequality (24) holds. This implies that the error dynamics (6) is asymptotically stable by the Lyapunov theory. This completes our proof. \hfill \square

**Remark 1.** The criterion given in Theorem 1 is delay-dependent. It is well known that the delay-dependent criteria are less conservative than delay-independent criteria when the delay is small. The solutions of Theorem 1 can be obtained by solving the eigenvalue problem with respect to solution variables, $P, Y, Q_i$ ($i = 0, 1, 2, 3), \alpha_i$ ($i = 1, 2, 3)$, and $M_i$ ($i = 1, 2, \ldots, 7)$, which is a convex optimization problem [28]. In this paper, we utilize Matlab’s LMI Control Toolbox [29] which implements interior-point algorithm. This algorithm is significantly faster than classical convex optimization algorithms [28].

**Remark 2.** The parameter $K$ of the estimator (3) can be determined by the relationship $K = P^{-1} Y$ after obtaining the LMI solutions, $P$ and $Y$.

**Remark 3.** By iteratively solving the LMI given in Theorem 1 with respect to $h$, one can find the maximum upper bound of time delay $h$ for guaranteeing asymptotic stability of error system (6).

**Remark 4.** In this paper, the parametric uncertainties in system matrices, $A, W_1, W_2, C, V$, do not considered for simplicity. When it comes to consider the uncertainty like $\Delta W = D\Delta(t)\tilde{E}$ with $\|\Delta(t)\| \leq 1$ and known constant matrices $D$ and $E$, it can easily extend the approach used for obtaining main result without loss of generality.
Example 1. A numerical example with simulation results is provided to demonstrate the effectiveness of the proposed method.

Consider the delayed neural network with the following parameters:

\[
A = \text{diag}\{3, 3, 4\}, \quad W_1 = \begin{bmatrix}
0.2 & -0.1 & 0 \\
0.1 & 0.3 & -0.2 \\
-0.2 & 0.1 & 0.2
\end{bmatrix},
\]

\[
W_2 = \begin{bmatrix}
0.1 & 1 & 0.2 \\
-0.1 & 0.2 & 0 \\
-0.2 & 0.1 & 0.2
\end{bmatrix}, \quad V = \begin{bmatrix}
0.1 & 0.05 & 0.05 \\
0.05 & 0.1 & 0.05 \\
0.05 & 0.1 & 0.1
\end{bmatrix},
\]

\[
C = I, \quad f(x) = 0.5 \sin(x(t)), \quad g(x(t - h)) = 0.5 \sin(x(t - h)), \quad h = 0.5,
\]

\[
J = \begin{bmatrix}
\sin(4t) + 0.005t^2 \\
-\sin(4t) - 0.004t^2 \\
1.2 \sin(4t) + 0.01t^2
\end{bmatrix}, \quad z(x(t)) = 0.2 \sin(4x(t)), \quad x(0) = [1 \; -1 \; 0.5]^T.
\]

From the functions \(f, g, z\), it is easy to see that \(F = 0.5I, \; G = 0.5I, \; Z = 0.2I\).

Now, by applying Theorem 1 to above system, one can see that the LMI given in Theorem 1 is feasible and the following solutions are obtained:

\[
P = \begin{bmatrix}
0.7058 & -0.0612 & -0.0152 \\
-0.0612 & 0.9844 & -0.0012 \\
-0.0152 & -0.0012 & 0.5498
\end{bmatrix}, \quad Y = \begin{bmatrix}
-1.3131 & 0.1202 & 0.0408 \\
0.1186 & -1.8189 & -0.0028 \\
0.0253 & -0.0032 & -1.5392
\end{bmatrix},
\]

Fig. 1. The true state \(x_1(t)\) and its estimate \(\hat{x}_1(t)\).
$M_1 = \begin{bmatrix} -0.2088 & 182.5932 & 130.9147 \\ 182.6004 & -0.2151 & -27.6990 \\ -130.9110 & 27.7016 & -0.1914 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0.1453 & 182.6066 & -130.9094 \\ -182.5868 & 0.1347 & 27.7018 \\ 130.9163 & -27.6987 & 0.1658 \end{bmatrix},$

$M_3 = \begin{bmatrix} 0.0434 & 182.6097 & -130.9077 \\ -182.5836 & 0.0181 & 27.7032 \\ 130.9180 & -27.6976 & 0.0862 \end{bmatrix}, \quad M_4 = \begin{bmatrix} 0.0014 & -0.0016 & 0.0036 \\ 0.0084 & 0.0008 & -0.0015 \\ 0.0025 & -0.0001 & 0.0078 \end{bmatrix},$

$M_5 = \begin{bmatrix} 0.0150 & 0.0034 & -0.0009 \\ -0.0041 & 0.0226 & -0.0011 \\ 0.0006 & -0.0029 & 0.0031 \end{bmatrix}, \quad M_6 = \begin{bmatrix} 0.0171 & 0.0079 & 0.1986 \\ -0.0171 & 0.0079 & 0.1986 \\ 0.3098 & -0.0489 & -0.0171 \end{bmatrix}$

$M_7 = \begin{bmatrix} -0.0020 & 0.0004 & 0.0177 \\ 0.0150 & -0.0034 & -0.0009 \\ 0.0006 & 0.0000 & -0.0002 \end{bmatrix}$

$H = \begin{bmatrix} -0.3538 & 0.0266 & 0.0109 \\ 0.0271 & -0.4038 & 0.0053 \\ 0.0110 & 0.0055 & -0.2556 \end{bmatrix}$

$Q_0 = \begin{bmatrix} 0.1986 & -0.0226 & 0.0048 \\ -0.0226 & 0.2772 & 0.0184 \\ 0.048 & 0.0184 & 0.1070 \end{bmatrix}$

$Q_1 = \begin{bmatrix} 0.3132 & -0.0582 & -0.0164 \\ -0.0582 & 0.4143 & -0.0090 \\ -0.0164 & -0.0090 & 0.1794 \end{bmatrix}$

$Q_3 = \text{diag}\{0.4286, 1.3952, 0.3234\}, \quad x_1 = 0.6036, \quad x_2 = 0.4649, \quad x_3 = 5.1286.$

Fig. 2. The true state $x_2(t)$ and its estimate $\hat{x}_2(t)$. 
From the solutions $P$ and $Y$, then the gain matrix, $K$, of state estimation is

$$K = \begin{bmatrix}
-1.8601 & 0.0099 & -0.0030 \\
0.0048 & -1.8470 & -0.0064 \\
-0.0054 & -0.0095 & -2.7997
\end{bmatrix}.$$ 

By applying the state estimator with $K$ obtained above, the simulation results are shown in Figs. 1–3. From the figures, one can see that the responses of the state estimators track to true states quickly.

### 4. Conclusion

In this paper, the state estimation problem has been studied for neural networks of neutral-type. A novel delay-dependent criterion for the existence of state estimator has been presented for the network by using the Lyapunov method. The criterion is expressed in terms of LMI. The LMI can be easily solved via Matlab’s LMI Toolbox in the paper. The numerical simulation results has been shown that the estimator designed is well track the true states.

### References


