High resolution image reconstruction in ultrasound computer tomography using deconvolution

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ABSTRACT

Ultrasound computer tomography is an imaging method capable of producing volume images with high spatial resolution. The imaged object is enclosed by a cylindrical array of transducers. While one transducer emits a spherical wavefront (pulse), all other transducers are recording the radiofrequency (RF) a–scans simultaneously. Then another transducer acts as the emitter and so on.

In this paper we describe the image reconstruction method and an enhanced algorithm for the a–scan preprocessing. The image reconstruction is based on a ”full aperture sum–and–delay” algorithm evaluating the reflected and scattered signals in the a–scans. The a–scans are modelled as the tissue response of the imaged object convoluted with the shape of the ultrasound pulse, which is determined by the transfer function of the transducers and the excitation. Spiking deconvolution and blind deconvolution with different parameters are used to build inverse filters of the ultrasound pulse. Applying the inverse filters to the a–scans results in sharper signals which are used for image reconstruction. Smallest scatterers of 0.1 mm size corresponding to one fifth of the used ultrasound wavelength are visible in the reconstructed images. Compared to conventional b–scans the resulting images show an approximately tenfold better resolution.

Keywords: ultrasound, tomographic reconstruction, ultrasound computer tomography, deconvolution, signal processing, RF data, breast imaging

1. INTRODUCTION

Ultrasound imaging of the female breast is a widespread accepted method in breast cancer diagnosis.\textsuperscript{1–3} It is a low–cost imaging method applicable even for young women without tissue damaging radiation. Disadvantages of nowadays systems are the operator dependent image quality, which is poor compared to X–ray imaging. Ultrasound computer tomography (USCT) as an enhanced imaging methodology to produce high–quality and reproducible ultrasound images has been discussed for the last three decades.\textsuperscript{4–10} A few academic experimental set–ups have been built.\textsuperscript{4, 10–14} The high demands to the transducers technology to produce thousands of transducer systems with identical characteristics in combination with the high demands on the data processing hardware with huge data rates (> 100 GBytes/s) and the high demands on the data processing algorithms have prevented the clinical use until today.

In our institute we are developing a three–dimensional ultrasound computer tomography system (USCT)\textsuperscript{15, 16} including the necessary transducer technology, the analog and digital hardware for ultra–high data rates, the signal processing and image reconstruction algorithms and software.

In this paper we present a method for signal processing based on deconvolution resulting in the reconstruction of significantly enhanced images. In section 2 briefly the USCT architecture, the data recording and the basic signal processing and image reconstruction algorithms are described. Spiking deconvolution and blind deconvolution for signal enhancement are well known from the literature. The fundamentals and their adaption to USCT are presented in section 3. In section 4 we show and compare deconvolution results of USCT signals, which are discussed in section 5.

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2. ULTRASOUND COMPUTER TOMOGRAPHY

2.1. Architecture and data acquisition

In conventional ultrasound imaging a linear transducer array is operated manually by the medical doctor and only the reflections are recorded. In ultrasound computer tomography (USCT) the object, e.g. the female breast, is placed in a tank filled with water as coupling medium. The tank walls are covered with transducers in a fixed geometry, building a cylindrical array surrounding the imaged object. One transducer emits a short ultrasound pulse which is scattered by the structures inside the object. Ideally every transducer has a spherical emission characteristic. All other transducers measure the transmitted, reflected and scattered signals (a–scans) in parallel. The radiofrequency (RF) a–scans are amplified, digitized and stored. Then a different transducer will emit an ultrasound pulse while all others receive the signals and so on. In figure 1 the principle of the USCT architecture is shown.

![Figure 1. Architecture of the USCT system built in our institute, shown in 2D. A ring (cylinder) of ultrasound transducers encloses the object (left). One transducer emits a short ultrasound pulse, all other transducers receive simultaneously. The a–scan at the right side shows both the directly transmitted and the scattered signals.](image)

2.2. Signal processing

The used transducers have a resonance frequency of 3 MHz and a bandwidth of approximately 50 %. The signal processing of a recorded a–scan \( x(t, \vec{e}, \vec{r}) \) consists of the following steps:

- Analog low–pass filtering with a limiting frequency at 5 MHz,
- Digitization: sampling frequency 10 MHz, quantization depth 12 bit,
- Calculation of the envelope function based on the analytical continuation

\[
E(t, \vec{e}, \vec{r}) = |x(t, \vec{e}, \vec{r}) - iH(x(t, \vec{e}, \vec{r}))|.
\]

\( x(t, \vec{e}, \vec{r}) \) is the a–scan measured by a voltage equivalent to the sound pressure dependent on the time \( t \), the position \( \vec{e} \) of the emitting transducer and the position \( \vec{r} \) of the receiving transducer. \( H(x(t, \vec{e}, \vec{r})) \) denotes the Hilbert transform of the a–scan. In figure 2 a typical a–scan and its envelope function is shown.

2.3. Image reconstruction

For simplification we assume constant sound speed \( c \) in water and the object. Furthermore, no corrections to the angle–dependent sensitivity of the transducers are applied. The reconstruction itself is based on a full aperture sum–and–delay algorithm.\(^5,15\) To determine the reflective amplitude \( R(\vec{x}) \) of a position \( \vec{x} \) the envelope functions of all available combinations of emitting positions \( \vec{e} \) and receiving positions \( \vec{r} \) are accumulated:

\[
R(\vec{x}) = \sum_{\vec{e}, \vec{r}} E\left(\frac{|\vec{e} - \vec{x}| + |\vec{x} - \vec{r}|}{c}, \vec{e}, \vec{r}\right)
\]  \( (1) \)
Figure 2. A–scan and its envelope function. The thin line shows the a–scan and the thick line the corresponding envelope function. The first peak is caused by the signal travelling directly from the emitting to the receiving transducer. The following peaks are reflected and scattered by the object inside the tank.

The quality of the reconstructed images depends highly on the time resolution of the reflected and scattered signals. The resolution of the system is dominated by the length of the measured ultrasound pulse $w(t)$ after an impulse excitation and the resulting length of the envelope function used for image reconstruction. The ultrasound pulse used in figure 2 has a length of at least three wavelengths corresponding to a spatial length of 1.5 mm assuming a sound velocity in water of 1500 m/s and an emitter frequency of 3 MHz. The influence of the pulse length on the image quality is demonstrated in figure 3. Small nylon threads are imaged as blurred points with a diameter of approximately 1.5 mm. To enhance the the quality of the reconstructed images the pulse

Figure 3. Phantom and reconstructed images. Left: Rough plan of the phantom. The smallest structures consist of nylon threads with a diameter of 0.1 mm and a spacing of 0.5 mm shown in the region enlargement below. The threads can be regarded as point sources. Right: Reconstruction using a full aperture sum–and–delay algorithm using the envelope function. The nylon threads are visible, but appear blurred and are not separable.
length cannot be reduced directly, since the transfer function of the ultrasound transducers and the attached electronics is band limited. But the information about the position and amplitude of the reflectors and scatterers within the tissue is contained in the a–scans. For that reason we replace the calculation of the envelope function by an inverse filter (deconvolution) to eliminate the influence of the pulse length and to sharpen the a–scans.

3. SIGNAL DECONVOLUTION

Sharpening signals by inverse and deconvolution filters has a long history in seismic data analysis in geophysical exploration,\textsuperscript{17,18} channel equalization in digital communications\textsuperscript{19} and image restoration\textsuperscript{20} in astrophysics.\textsuperscript{21} In medical imaging several researchers applied the techniques described below to ultrasound a–scans and b–scan images.\textsuperscript{22–27}

3.1. The model

The generation of a–scans is modelled as a linear system (figure 4). The recording is triggered by an external Dirac pulse $\delta(t)$. Omitting the noise the recorded a–scan $x(t)$ may be described by

$$x(t) = w(t) * s(t),$$

where * represents the convolution operator. The shape of the emitted ultrasound pulse $w(t)$ is determined by the ultrasound excitation generated by the electronics and the transfer function of the transducers. $s(t)$ is the tissue response, also referred as spatial reflectivity function, source function, medium response or spatial reflectance distribution.

In the discrete formulation the signals $w(t)$ and $s(t)$ have been digitized with a uniform sample rate to $w(t) = [w(0), w(1), \cdots w(A-1)]^T$ and $s(t) = [s(0), s(1), \cdots s(B-1)]^T$. The discrete convolution is based on the assumption that the signals are periodic with periods $A$ and $B$ respectively. To avoid an overlap of the periods by discrete convolution the individual periods are elongated to $M \geq A + B - 1$ by zero padding.\textsuperscript{20}

The tissue response $s(t)$ can be considered as a series of pulses with different time–shifts and amplitudes corresponding to the formation of reflective boundaries and scatterers within the tissue. The ultrasound pulse $w(t)$ can be regarded as a causal and band–limited function and the resulting a–scan $x(t)$ as a ”blurred” version of the tissue response. The general idea in deconvolution is to construct an inverse filter $w^{-1}(t)$ that compensates the influence of the ultrasound pulse to reconstruct the unknown tissue response $s(t) = x(t) * w^{-1}(t)$ and to sharpen the a–scan.

We apply two different methods to determine an inverse filter. If the shape of the ultrasound pulse is known we apply the algorithms for ”spiking deconvolution”, if not we estimate the statistical properties of $w(t)$ and construct an inverse filter by ”blind deconvolution”.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Linear model of the generation of an a–scan. Triggered by an external Dirac pulse an ultrasound pulse is generated. Its shape depends on an arbitrary excitation function formed by the electronics convoluted with the transfer function of the transducers. The tissue response convoluted with the ultrasound pulse and added noise build the recorded a–scan.}
\end{figure}
3.2. Spiking deconvolution

In general the problem can be regarded as the determination of a digital filter \( f(t) \) which solves the following equation:

\[
d(t) = w(t) * f(t),
\]

where \( d(t) \) is a desired vector of length \( N \leq M \). In spiking deconvolution \( d(t) \) is defined as a spiking function with an arbitrary phase shift \( k \)

\[
d(t) = \delta(t - k) = \begin{cases} 
1 & : t = k \\
0 & : \text{else}
\end{cases}
\]

\( f(t) \) can be regarded as an inverse filter of \( w(t) \) with an intern shift \( k \). In particular, if the phase shift \( k = 0 \), \( d(t) = \delta(t) \) is the Dirac pulse and the digital filter becomes \( f(t) = w^{-1}(t) \), as defined in the last section. Equation 3 can be written as

\[
\delta(t - k) = \sum_{i=0}^{N-1} w(t - i)f(i),
\]

respectively in matrix notation:

\[
\delta(t - k) = Wf(t),
\]

with the circular matrix:

\[
W = \begin{bmatrix}
w(0) & w(-1) & w(-2) & \cdots & w(-N + 1) \\
w(1) & w(0) & w(-1) & \cdots & w(-N + 2) \\
w(2) & w(1) & w(0) & \cdots & w(-N + 3) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
w(N - 1) & w(N - 2) & w(N - 3) & \cdots & w(0)
\end{bmatrix}
\]

All values of \( w(t \geq A) = 0 \) (see last section) and all values \( w(t < 0) \) vanish, since \( w(t) \) is a causal function. With that \( W \) changes into a banded matrix with

\[
W = \begin{bmatrix}
w(0) & 0 & 0 & \cdots & 0 \\
w(1) & w(0) & 0 & \cdots & 0 \\
w(2) & w(1) & w(0) & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
w(N - 1) & w(N - 2) & w(N - 3) & \cdots & w(0)
\end{bmatrix}
\]

The digital filter \( f(t) \) should minimize the square error cost function

\[
\varepsilon^2 = (Wf(t) - \delta(t - k))^2 + \alpha f(t)^2,
\]

where the square of a vector is defined as \( f(t)^2 = f(t)^Tf(t) \). The constraint \( \alpha \) denotes a small positive constant that penalizes values of \( f(t) \) with large magnitudes. The solution is given by the least square Wiener filter with

\[
f(t) = (W^TW + \alpha I)^{-1}W^T\delta(t - k).
\]

\( W^TW = [w(-t) * w(t)] \) denotes the auto–correlation matrix of \( w(t) \), \( W^T\delta(t - k) \) the cross–correlation between \( w(t) \) and \( \delta(t - k) \), and \( I \) is the identity matrix. The tissue response can now be estimated as

\[
s(t) = f(t) * x(t)
\]

\[
= \left( (W^TW + \alpha I)^{-1}W^T\delta(t - k) \right) * x(t)
\]
To build an inverse filter the ultrasound pulse is required. In our ultrasound computer tomography system we use a calibration measurement with no object inside the tank. The directly transmitted signal from an emitting transducer to a recording one can be regarded as a convolution of the ultrasound pulse with a shifted Dirac pulse. The ultrasound pulse is segmented manually and assigned to $w(t)$. To evaluate equation 7 the parameters have to be assigned: The length $N$ of the inverse filter, the phase shift $k$ of the spiking function and the damping factor $\alpha$ of the penalty term. To optimize $N$ and $k$ we predefine the constraint $\alpha$ to a fixed value, e.g. $\alpha = 0$, and we alter $N$ within a given range $N = [1, 2, \ldots, N_{\text{max}}]$ and for each $N$ we alter $k$ within the range $k = [1, \ldots, N]$. We choose the filter with the parameter combination minimizing the square error $\varepsilon^2 = (WF(t) - \delta(t - k))^2$. After the parameters $N$ and $k$ are set we alter $\alpha$ manually within the range $0.00 - 0.02$ to reduce the noise in the deconvoluted signals significantly without eliminating the desired signals.

It has to be pointed out that in spiking deconvolution an optimum inverse filter for the predefined ultrasound pulse is defined. If the pulse is modified marginally by angle–dependent frequency shifts of the transducers, variations in the characteristics of different transducers or by dispersion within the scanned object, the inverse filter may produce disturbed and noisy results. Taking these constraints into account it is difficult to determine an ideal ultrasound pulse $w(t)$ and the corresponding ideal inverse filter for the whole system.

### 3.3. Blind deconvolution

In blind deconvolution, both the ultrasound pulse and the tissue response are regarded as unknown. Only the $a$–scans $x(t)$ are available. To remodel the Wiener filter (equation 7), which depends on $w(t)$, in such a way that it depends only on $x(t)$, an additional assumption is made. If the tissue response $s(t)$ is considered as an arbitrary series of pulses its statistical properties can be regarded similar to those of white noise. All frequencies in the power spectrum can occur with the same probability.

An important characteristic of white noise $n(t)$ is its auto–correlation function $n(-t) * n(t) = \delta(t)$. Thus the auto–correlation of the tissue response resembles the auto–correlation function of white noise as a scaled Dirac pulse $s(-t) * s(t) = 1/r_0\delta(t)$, where $1/r_0$ denotes the scaling factor. The auto–correlation $x(-t) * x(t)$ of an $a$–scan becomes:

\[
x(-t) * x(t) = (w(-t) * s(-t)) * (w(t) * s(t)) = (w(-t) * w(t)) * (s(-t) * s(t)) = \frac{1}{r_0^2}(w(-t) * w(t)),
\]

which is a scaled version of the auto–correlation $w(t) * w(-t)$ of the ultrasound pulse. The Wiener filter (equation 7) can now be described as

\[
f(t) = (X^T X + \alpha I)^{-1} r_0 \delta(t - k).
\]

$X^T X = [x(-t) * x(t)]$ denotes the auto–correlation matrix of $x(t)$. The factor $r_0$ is composed of the scaling factor $1/r_0^2$ and an additional factor $w_k$ left from the cross–correlation $w(-t) * \delta(t - k) = w_k \delta(t - k)$. Equation 10 determines an inverse filter only based on the knowledge of the auto–correlation function of the $a$–scans, the parameters $N$, $k$ and $\alpha$. The parameter $r_0$ influences only the scale of the deconvoluted tissue response and can be neglected.

The auto–correlation function of the $a$–scans can be easily determined using an auto–correlation window of a fixed length on the base of all recorded $a$–scans of an USCT measurement. It has the advantage, that the influence of all statistical variations of the the ultrasound pulse caused by angle–dependent frequency shifts of the transducers, different transducers and dispersion are incorporated.

Unfortunately the optimization of the parameters $N$, $k$ and $\alpha$ by minimizing the mean square error is not longer possible, since the ultrasound pulse $w(t)$ is not explicitly known. At the moment we choose the parameters more or less arbitrarily based on the knowledge gained by spiking deconvolution. In future we will explore an optimization criterion based on the statistical properties of the estimated tissue response functions in relation to white noise.
4. RESULTS

We tested both methods, spiking deconvolution and blind deconvolution, to a–scans recorded by our experimental set–up of ultrasound computer tomography.\textsuperscript{15}

![Figure 5. The effects of differently shaped ultrasound pulses to spiking and blind deconvolution. a) Ultrasound pulse \( w(t) \) recorded at the opposite transducer position \( \gamma = 0^\circ \), which was used to build a spiking deconvolution filter. b) Different ultrasound pulse \( \tilde{w}(t) \) recorded at position \( \gamma = 45^\circ \). c) Deconvoluted \( w(t) \) using the spiking deconvolution filter. d) Deconvoluted \( \tilde{w}(t) \) using the spiking deconvolution filter optimized for \( w(t) \) resulting in a distorted signal with side–peaks. e) Deconvoluted \( w(t) \) using the blind deconvolution filter. f) Deconvoluted \( \tilde{w}(t) \) using the blind deconvolution filter. In blind deconvolution the quality of both deconvoluted signals is similar.]

For spiking deconvolution the ultrasound pulse \( w(t) \) had to be measured. We chose two directly facing transducers placed on opposite positions (angle \( \gamma = 0^\circ \)) in the USCT tank from the calibration measurement. An example of a manually segmented ultrasound pulse \( w(t) \) is shown in figure 5 a), starting at approximately 79.5 µs. According to equation 7 an inverse filter was determined with the length \( N = 5.4 \mu s \) and phase shift \( k = 2.2 \mu s \) by minimizing the square error. To reduce the high–frequency noise amplification of the inverse

\[
\text{Amplitude [V]} \quad \text{Time [\mu s]}
\]
filter a damping factor $\alpha = 0.01$ was introduced. In figure 5 c), the application of the inverse filter to the used ultrasound pulse $w(t)$ results in a sharp peak with minor distortions and almost no lingering. In figure 5 d), the inverse filter was applied to an ultrasound pulse $\tilde{w}(t)$ (figure 5 b)) recorded from a different receiver position at angle $\gamma = 45^\circ$. Since the characteristics of the transducers are angle–dependent, $\tilde{w}(t)$ diverges significantly from $w(t)$ resulting in significant distortions with relatively high side–peaks in the deconvoluted signal.

![Figure 6. A–scan and its estimated tissue response by blind deconvolution. The thin line shows the a–scan $x(t)$ and the thick line the corresponding tissue response $\hat{s}(t)$.](image)

For blind deconvolution the auto–correlation function $x(-t) \ast x(t)$ was estimated from 100 randomly chosen a–scans using an auto–correlation window of 12.8 $\mu$s length, corresponding to approximately 5 times of the pulse length $w(t)$. The parameters $N = 3.2 \mu$s, no phase shift $k = 0 \mu$s and damping factor $\alpha = 0.01$ were chosen arbitrarily. The blind deconvolution filter was applied to the ultrasound pulses $w(t)$ at angle $\gamma = 0^\circ$ and $\tilde{w}(t)$ at angle $\gamma = 45^\circ$ in figure 5 c) and f). Both deconvolutions show a sharp peak at the beginning of the ultrasound pulse with minor side peaks. Because of the smaller length of the inverse filter the lingering effect is more pronounced. The quality of both blind deconvoluted signals c) and f) is similar as expected in the last section. The shown examples in figure 5 represent the typical effects caused by spiking and blind deconvolution. For that reason we applied the blind deconvolution filter in the following signal processing and image reconstruction steps.

For image reconstruction the calculation of the envelope function is replaced by deconvolution. According to the assumption that the tissue response is regarded as a series of pulses with different time–shifts and amplitudes, only the non–negative values are used:

$$\hat{s}(t + k, \vec{e}, \vec{r}) = \begin{cases} f(t) \ast x(t, \vec{e}, \vec{r}) & : f(t) \ast x(t, \vec{e}, \vec{r}) > 0 \\ 0 & : \text{else} \end{cases}$$  \hfill (11)

$\hat{s}(t + k, \vec{e}, \vec{r})$ denotes the deconvoluted tissue response of the emitting transducer at position $\vec{e}$ and the receiving transducer at position $\vec{r}$. The phase shift $k$ of the inverse filter is compensated by a time shift $k$ in the tissue response to align the spiking response with the beginning of the ultrasound pulse. In figure 6 an a–scan $x(t)$ and its corresponding tissue response $\hat{s}(t)$ using the blind deconvolution filter is shown. Compared to the envelope function represented in figure 2 the tissue response shows significantly sharper peaks and by that a higher temporal resolution.

The image reconstruction algorithm according to equation 1 becomes now:

$$R(\vec{x}) = \sum_{\vec{e}, \vec{r}} \hat{s}\left(\frac{|\vec{x} - \vec{e}| + |\vec{x} - \vec{r}|}{c}, \vec{e}, \vec{r}\right)$$  \hfill (12)
Using equations 11 and 12 the phantom sketched in figure 3 was reconstructed. The results are shown in figure 7 (right). The reconstructed images appear sharper containing more detailed information. The nylon threads with a size of approximately one fifth of the ultrasound wavelength are clearly visible and separable. The higher temporal resolution of the deconvoluted tissue response ensures the enhanced spatial resolution (< 0.25 mm) of the reconstructed images.

![Figure 7. Reconstructed images. Left: Using the envelope function. Right: After blind signal deconvolution. The use of the estimated tissue responses results in significantly sharper images. The nylon threads with a diameter of 0.1 mm and a spacing of 0.5 mm are clearly visible and separable.](image)

**5. DISCUSSION**

We have shown that deconvolution of the a–scans can be used to enhance the quality of the reconstructed images. Spiking and blind deconvolution have been implemented in the USCT image reconstruction software written in Matlab. Small scatterers of 0.1 mm size corresponding to one fifth of the used ultrasound wavelength are visible in the reconstructed images. Compared to conventional b–scan ultrasound the resulting images show an approximately tenfold better resolution.

Generally the discrete deconvolution of band–limited ultrasound pulses requires deconvolution filters of infinite length. Since the described methods use truncated filters of finite length, an error remains. We estimate the ultrasound pulse \( w(t) \) in spiking deconvolution and the auto–correlation function \( x(-t) \ast x(t) \) in blind deconvolution on the basis of measured a–scans, which are disturbed with noise, introducing an additional error. Furthermore we assume in our reconstruction algorithm ideal ultrasound transducers without angle dependent frequency shifts, similar ultrasound characteristics for different transducers and no dispersion within the tissue. These errors lead to distorted tissue responses in spiking deconvolution, if the optimized deconvolution filter is applied to differently shaped ultrasound pulses. In our experiments blind deconvolution produced better inverse filters, since it uses the auto–correlation function estimated on the basis of many a–scans: The ultrasound pulse is not directly defined and additive noise is reduced by averaging.

The optimization of the parameters for blind deconvolution inverse filters is currently done manually. In future we plan to explore a least square error optimization criterion based on the statistical properties of the estimated tissue responses to determine the parameters filter length \( N \), phase shift \( k \) and damping factor \( \alpha \). Since we assume the tissue response consisting of pulses with different time–shifts and amplitudes, its statistical properties should resemble those of white noise.
Furthermore we plan to implement different deconvolution filters in the FPGAs of the USCT data acquisition hardware\textsuperscript{16,29} to enhance the signals. Since the deconvoluted tissue response consists of a series of pulses, it provides a natural basis for ultrasound a–scan compression. We will explore different lossless and lossy compression schemes and their effect to the quality of the reconstructed images. If the data rate of 20 MBytes/s per a–scan can be reduced by one or two orders of magnitude without loosing relevant image information ultrasound computer tomography will be less expensive and it will be easier to establish this new imaging method.

REFERENCES


