Abstract

We describe how techniques that were originally developed in statistical mechanics can be applied to search problems that arise commonly in artificial intelligence. This approach is useful for understanding the typical behavior of classes of problems. In particular, these techniques predict that abrupt changes in computational cost, analogous to physical phase transitions, should occur universally, as heuristic effectiveness or search space topology is varied. We also present a number of open questions raised by these studies.

Keywords: Search; Phase transitions; Constraint satisfaction

1. A statistical view of search

Search is one of the most pervasive techniques in artificial intelligence. Problem solving, diagnosis, design and planning, for example, can all be cast as a search through some space of alternatives. These search problems are often conceptually simple but computationally difficult since the solution time can grow exponentially with the size of the problem [12]. Given its central importance to the field, considerable work has been devoted to developing a variety of search methods and determining the situations for which they are best suited. In particular, heuristics [44] are often used to guide the series of choices or steps made during the search. Typically, at each decision point a heuristic evaluates a small number of potential choices, based on easily computed local properties of the search problem.
In a fundamental sense, the effectiveness of a search heuristic is determined by how the many repeated local decisions at the search steps combine to determine the global performance, e.g., the overall search cost or quality of the solution. In detail this depends on the specific problem and heuristic used. Fortunately, however, it is possible to study regularities in the typical behavior of general search methods for various classes of problems. This contrasts with the usual emphasis in computer science theory on a worst-case analysis [12]. Focusing on typical behavior is particularly appropriate for evaluating search methods in practice since one usually is not interested in how well they work for a single given problem but rather for a variety of problems likely to be encountered in the future. In such cases, there may be only limited information available about the nature of the problems to be solved so the remaining details act as unspecified degrees of freedom. It is therefore useful to treat these unspecified degrees of freedom as randomly generated by some probability distribution. In this context, one might expect that statistical techniques, which have been so successful in describing physical systems, will provide a useful framework for understanding the global behavior of these computational problems, particularly when moving beyond idiosyncratic small systems [21]. This outlook becomes even more relevant when there is uncertain information as part of the problem [45].

Statistical mechanics, based on the law of large numbers, has taught us that many universal and generic features of large systems can be quantitatively understood as approximations to the average behavior of infinite systems. Although such infinite models can be difficult to solve in detail, their overall qualitative features can be determined with a surprising degree of accuracy. Since these features are universal in character and depend only on a few general properties of the system, they can be expected to apply to a wide range of actual configurations.

The first step in the statistical approach to search is to pick a suitable ensemble of problem instances. An ensemble consists of a class of problems and an associated probability for each one to appear. Often there will be a number of plausible choices with the same basic set of parameters and the same behaviors in the limit of large problems. In such cases one is free to select that ensemble that is most easily analyzed or most readily evaluated numerically [55]. Ideally these ensembles would have the broad range of applicability as those used in statistical physics (which generally treat each microscopic state consistent with the small number of known parameters as being equally likely). Unfortunately, in spite of some work along these lines [37], the nature of realistic search problems is not yet well enough understood to make this possible. Instead, most work in this area uses simple random classes of problems such as a variety of closely related ensembles of random constraint satisfaction problems [56, Section 6.1]. Comparing the predictions of these models to real problems should help identify more realistic ensembles, in much the same way that the need to incorporate quantum mechanical effects required a revision of the ensembles used in statistical physics.

The second step of a statistical analysis is to identify suitable global properties, such as the average search cost, in terms of the model parameters. Finally, one quantitatively relates the global behaviors to the local parameters describing the model. As in the statistical physics of anything more complex than ideal gases or solids, determining
the exact form of this relation is often extremely difficult due to the many conditional probabilities involved. However, an approximate theory that assumes independent probabilities generally gives a good qualitative understanding of the global behaviors likely to be observed, and often fairly accurate quantitative values as well. These approximations give rise to so-called "mean-field" theories of physics since they correspond to assuming each component of the system interacts only with the mean behavior of the remaining components.

This technique provides a relatively simple way to identify those phenomena that are common to many statistical systems and arise mainly from the properties of large numbers rather than the details of particular systems. This is in contrast with the usual computer science theory with its focus on precise theorems. Moreover, examining the errors made by this approximation can also form the basis for more complex analyses using more detailed ensembles (indeed, the approximate theory can often be viewed as the exact behavior of some other, perhaps less realistic, ensemble of problems). Finally, one should note that the ensembles selected for empirical or theoretical work do not correspond precisely to those encountered in practice. The errors due to the choice of ensemble are likely to dominate those due to the approximations of the theory, giving another reason to focus on simple, robust phenomena rather than a complex, detailed analysis. This is a powerful advantage of the statistical mechanics approach since many behaviors are at least qualitatively similar in large size limit when detailed dependencies are ignored.

2. Phase transitions

One of the most dramatic consequences of the statistical approach to computational problems is the identification of situations where small changes in local behavior give especially large changes in global performance. A common case is exponential growth in some variable such as congestion in queues. Another is the appearance of phase transitions, analogous to those in physical systems and certain mathematical models. These abrupt transitions are the subject of this special issue.

2.1. What are phase transitions?

Matter commonly undergoes dramatic changes in its qualitative properties when certain parameters pass through particular values. A common everyday example is the melting of a solid with increasing temperature, a parameter that characterizes the average energy available to the atoms making up the solid. As temperature increases the atomic vibrations become gradually more violent, leading to the well-known phenomenon of thermal expansion. Since this increase in vibration amplitude is gradual, one would naively expect that the macroscopic properties of the substance would accordingly undergo a smooth change. While this is true over most of the temperature range, there exists a well-defined temperature for which something much more dramatic happens: a sudden change in the properties of the substance over a small temperature range, and the appearance of a qualitatively different phase, in this case a liquid. This
melting transition involves, among other things, an abrupt softening of the solid even though its average atomic energy changes only slightly. The ensuing liquid can in turn undergo another phase transition into a gas phase, where once again properties, such as the density, change discontinuously.

Other examples of phase transitions appear in magnets and superconductors when heated, the creation of self-replicating biochemicals above a certain reactive mass, and many other systems composed of a large number of components. Of particular relevance to search problems is the phase transition displayed by percolation processes. They can be easily visualized by considering a network of channels connecting sites and the water flow through these channels. An important local parameter for such a network is the average number of neighbors a site has. Given an initial concentration of fluid in a single site and a pressure gradient, a typical problem consists in determining global properties such as how many sites, on the average, get wet after a long time, or the probability that distant sites get wet. This percolation problem, which has been extensively studied in contexts such as oil extraction, the spread of epidemics, and the conductivity of electrical networks [9, 10], has a phase transition independent of the detailed geometry of the system. Specifically, for infinitely large networks, when the average number of neighbors is sufficiently small, there is unlikely to be any global flow from one side of the system to the other, i.e., the probability of finding a wet site at an arbitrarily large distance from the source is vanishingly small. This means that only a finite number of sites are connected to the source. As the number of neighbors increases however, there is a value at which global flow becomes very likely, the probability that distant sites get wet becomes nonzero, and the connected cluster becomes infinite. Here the transition is characterized by a jump in the derivative of this probability and global flow rate. At the transition the global flow is nonzero, but small. Beyond this point the global flow continues to increase. This behavior also corresponds to thresholds in the cluster size of random graphs [2]. In these cases, the transition is less dramatic than melting or boiling because the new behavior increases gradually after the transition, but nevertheless the onset is still abrupt.

These phase changes are characterized by the appearance of singularities in observables such as the density, viscosity and specific heat of matter or long range connectivity in percolation. That is, at the transition, some global behavior is not analytic, in the limit of infinitely large systems. Depending on the nature of the transition, one might also observe phenomena such as hysteresis, sluggish response to external stimuli, or the nucleation of a new phase. Studies in statistical mechanics have shown that despite the apparent diversity in the composition and underlying structure of these systems, phase transitions take place with universal quantitative characteristics, independent of the detailed nature of the interactions between individual components. This means the singular behavior of observables near the transition point is identical for many systems when appropriately scaled, defining universality classes that only depend on the range of interaction of the forces at play and the dimensionality of the problem [14, 57]. One of these common characteristics is rapidly increasing correlation lengths between parts of the system as the transition is approached, giving rise to a change from a disordered to an ordered state and particularly large variances. It is these so-called "critical phase transitions" that are most relevant to computational search.
This theoretical treatment of phase transitions relies on models that, in the limit of very many components, can be considered stochastic in nature, an approximation that works exceedingly well in characterizing their generic properties. In this so-called thermodynamic limit, extensive quantities such as the number of particles and the volume they occupy become infinitely large, while the average number per unit volume remains constant. Thus, strictly speaking, phase transitions only exist for infinitely large systems, but nevertheless give qualitative and quantitative insight to smaller cases, e.g., through the use of finite size scaling [33]. In particular, for finite systems, the transition is somewhat smoothed out.

2.2. How do phase transitions appear computationally?

Statistical mechanics has been applied in a variety of computational situations [21]. Important examples include a transition from polynomial to exponential average search cost depending on the effectiveness of heuristic pruning [27,47,51] and another from underconstrained to overconstrained problems [5]. Additional transitions have been identified in models of associative memory [30,49], automatic planning [3], optimization problems [5,31,39,58] and various cases of pattern matching and classification [15,22,36,52]. Because of the many universal features of phase transitions, their identification in a variety of computational contexts suggests there are a number of phenomenological regularities underlying these problems.

The main focus of this special issue is on transitions in constraint satisfaction problems [38] for which there has been a great deal of recent work [6,8,13,17,35,42,46,53-55]. A particularly surprising result is that hard problem instances are concentrated near the same parameter values for a wide variety of common search heuristics, on average. This location also corresponds to a transition in solubility. This behavior can be readily understood through mean-field theories [6,50,56] as due to a competition between increasing pruning of bad search paths and a decreasing number of solutions. These theories also give reasonable quantitative values for the location of transitions and can be improved by including some corrections [54]. These simple but approximate theoretical results contrast with the difficulty of deriving exact results. For example, so far the existence of the transition for the well-studied random satisfiability (SAT) problem has not been formally established, and its location can only be bounded [4,29] between clause-to-variable ratios of 3 and 4.8 for the case of 3-SAT. In addition to this transition in solubility (or more generally, from under- to overconstrained problems) there are a number of more subtle behaviors. These include transitions from polynomial to exponential average search cost, and the appearance of rare hard cases among underconstrained problems [13,25] apparently due to infrequent but extreme thrashing by many standard search techniques [1].

2.3. A simple phase transition

To help make this discussion concrete, we consider simple examples of phase transitions in search. Perhaps the simplest consists of the size of the search tree arising during a depth-first backtrack search as a function of how well the local search heuristic
is able to prune unproductive search choices \[27\]. Most of the cost in such searches is often due to attempts to recover from some early incorrect choices that preclude any possibility of a solution. In these situations, the time spent backtracking among unproductive additional choices before returning to the early choices made in the search is determined by how well the search heuristic is able to prune unproductive choices from consideration.

This problem can be formalized by supposing that the problem consists of a series of \(d\) decisions to be made and, at each decision point in the search, there are \(b\) choices to consider. This gives a search tree of depth \(d\) and branching ratio \(b\). For simplicity, we consider the case where prior search choices eliminate all possible solutions to the problem, or the problem itself has no solutions. In addition, we suppose there is some way to identify unproductive sets of choices, perhaps because they violate some constraints or through some domain knowledge incorporated into a search heuristic. A simple model is to assume that this identification eliminates each unproductive choice independently and with probability \(1 - p\). At the extremes, \(p = 0\) corresponds to perfect knowledge and \(p = 1\) to a completely ineffective heuristic. This gives an effective average branching ratio for the search of \(z = bp\). This specification of a heuristic search problem amounts to defining a simple ensemble of random problems, whose properties can be easily evaluated exactly.

Completing the analysis of this problem requires identifying some global property of interest and then relating it to the local problem parameters (\(d, b\) and \(p\) in this case). An important global property is the search cost given by the number of nodes visited during the search before concluding there is no solution. The value of the search cost will vary among the instances in the ensemble. One way to characterize the typical behavior is through the average value of the search cost. In this model, the behavior depends only on the depth of the node in the tree. Combined with the independent pruning choices in this model, we have \(\langle C_j \rangle = 1 + z \langle C_{j+1} \rangle\), where \(\langle C_j \rangle\) is the expected cost of a node at depth \(j\). The expected cost of the entire search, starting from the root, is then

\[
\langle C \rangle = \sum_{k=0}^{d} z^k = \frac{1 - z^{d+1}}{1 - z}.
\]  

This simple analytic function relates the local heuristic effectiveness (through the value of \(z\)) to the global cost, on average. However, in the limit of infinitely large problems (i.e., \(d \to \infty\)) we find very different asymptotic behaviors:

\[
\langle C \rangle \sim \begin{cases} 
\frac{1}{1 - z}, & z < 1, \\
d, & z = 1, \\
\frac{z^{d+1}}{z - 1}, & z > 1,
\end{cases}
\]  

indicating an abrupt change in character at the transition point \(z = 1\).

Because the individual search instances have different costs, another important aspect of this analysis is to determine how representative the average value is. This can be
Fig. 1. Relative deviation $r$ in the search cost as a function of the effective pruning $z$ for $b = 3$. The curves show the behavior for problems of size 20 (gray), 50 (dashed) and 100 (solid), and are indistinguishable except near the transition. Note the increasingly large fluctuations near the transition point at $z = 1$.

done by comparing the variance in costs to the average. By considering the expected value of the square of the search cost we obtain

$$\text{var}(C) = \frac{z(1 - p)}{(1 - z)^3} \left(1 - (2d + 1)z^d(1 - z) - z^{2d+1}\right).$$  \hspace{1cm} (3)

Asymptotically, the relative variance is

$$\frac{\text{var}(C)}{\langle C \rangle^2} \sim (1 - p) \begin{cases} \frac{z}{1 - z}, & z < 1, \\ \frac{1}{2d}, & z = 1, \\ \frac{1}{z - 1}, & z > 1. \end{cases}$$ \hspace{1cm} (4)

The relative deviation $r \equiv \sqrt{\text{var}(C)/\langle C \rangle}$ is just the square root of this quantity. This exhibits a peak at the transition point, as illustrated in Fig. 1. Large fluctuations at the transition point are a typical characteristic of phase transitions. As a consequence, attempts to evaluate behaviors near transitions with simulation experiments often require many more samples to accurately sample the typical range of behaviors than for parameter values far from transition points. This figure also illustrates how the singular nature of the transition in the limit of infinitely large problems arises out of the relatively smooth behavior for small sizes.

In terms of constraint satisfaction problems, $z < 1$ corresponds to very highly overconstrained problems so that incorrect choices are pruned almost immediately. This simple
example of a phase transition in a search problem can be extended to more complex situations where the number of solutions varies or the heuristic pruning effectiveness varies with depth [55]. As long as the various search choices are assumed to occur independently, exact results are relatively straightforward to obtain.

2.4. Phase transitions in constraint satisfaction problems

While the above example provides a simple description of how phase transitions can arise in search problems, more realistic models are usually not exactly solvable and one must resort either to simulations or approximate theories. To gain a more intuitive insight into the nature of the transition for constraint satisfaction problems, we examine the search trees for some small cases. Much of the recent work in this area has examined the behavior of the search as a function of the number or tightness of the constraints. Qualitatively, as more constraints are added to a problem, a series of incorrect variable assignments is more likely to violate one of the constraints allowing for earlier pruning. This tends to make the problems easier and can lead to a transition similar to that described above for pruning. However, additional constraints also reduce the number of solutions hence making it more difficult to solve the problem. Thus as constraints are added, these two factors compete against each other. In fact, the loss of solutions is the dominant effect for weakly constrained problems, leading to an increase in typical search cost. This is simply because each new constraint is more likely to eliminate states with many assigned variables (especially solutions, where all variables are assigned) than it is to eliminate states with only a few assignments. For highly constrained problems, on the other hand, the additional pruning becomes dominant leading to a decrease in search cost. This is because the highly constrained problems either have no more solutions to remove, or the remaining solutions are required to exist by the problem specification. In either case, the additional constraints do not remove any more solutions and so the increased pruning is not offset by a further reduction in the number of solutions.

This is easily illustrated with a small graph coloring example. In this problem one is given a graph and required to select, from among a specified set of colors, a color for each node so that nodes linked by an edge have different colors. In this context, each edge in the graph provides a constraint on the allowed colorings. To see the effect of increasing the number of constraints, consider a series of connected graphs constructed from a random tree [43] with 10 nodes (and 9 edges). Additional edges were added randomly to give a series of related graphs. Each graph was then searched using a simple, nonheuristic backtrack search in which nodes were colored in numerical order.

The resulting search costs, in Fig. 2, show that the sparse and dense graphs are relatively easy to search, while intermediate ones give rise to harder searches. This behavior can be readily understood from the changes in the search trees as edges are added, as shown for a few cases in Fig. 3. With few edges, there are a large number of solutions and one of them is quickly found, without any need for backtracking. As more edges are added, solutions are rapidly eliminated, while partial states with fewer assignments are removed more slowly. This results in a search tree in which many states at intermediate levels do not lead to solutions, increasing the need for backtracking. At some point, the last solutions are eliminated, giving a large increase in the search cost.
Fig. 2. Search cost \( c \) to find the first solution, if any, or determine there are none for 3-coloring a randomly generated set of connected graphs with 10 nodes, versus the number of edges \( e \) in the graph. The search used nonheuristic backtrack. The graphs range from a random tree, with 9 edges, to a complete graph (each node linked to every other node) with 45 edges. A solution exists for those cases with 19 or fewer edges. Thus the peak in search cost occurs at the point where the solutions just disappear.

Fig. 3. Search trees for coloring some of the graphs. Starting from the upper left and continuing clockwise these correspond to the graphs with 35, 20, 19 and 12 edges respectively. Black lines show the states searched to find the first solution or determine there are none (which is the corresponding search cost shown in Fig. 2). Gray lines show additional consistent partial states that would need to be searched to find all solutions. The direction of the branches indicates the color choice: left, center and right corresponding to colors 1, 2 and 3, respectively. In the cases with at least 20 edges, there are no solutions and the search must examine all consistent assignments with this particular choice of variable ordering before concluding the problem is not soluble.
as the backtracking must now continue to check all possibilities. This corresponds to
the peak in the search cost found at 20 edges in Fig. 2. At this point we say that the
problem is critically constrained in that there are just enough constraints to eliminate the
solutions. Beyond this point, additional edges continue to prune the intermediate states
in the tree which results in a decreased search cost. This discussion also suggests why
the maximum search cost is seen just as the last solutions disappear.

This general observation, relating search behavior to the number of consistent states
of various sizes, forms the basis of approximate theories of this transition behavior
[56]. In fact this analysis suggests a series of transitions. First, when there are very few
constraints, we can expect so many solutions that often one will be found with little or no
backtrack, giving a search cost linear in the size of the problem. As more constraints are
added, there will be a transition to exponentially large cost, with the rate of exponential
growth increasing as more constraints are added, eventually reaching a peak for the
critically constrained problems. Beyond this point, the cost grows exponentially, but
more slowly. Eventually, for very highly constrained problems, there will be sufficient
pruning to make the cost again grow only polynomially. This description applies on
average, while the worst case with a given number of constraints can be much harder.

We should note this analysis does not describe, even qualitatively, all the observed
behaviors associated with these searches; in particular, the existence of rare but extremely
hard searches within the underconstrained regime for many common backtrack search
methods [13, 25]. Instead this requires a more careful consideration of the variation in
the number of the consistent states actually searched to find the first solution. In the
context of Fig. 3, the rare hard cases are due to searches that include a large number of
the states indicated in gray.

A peak in the search cost for critically constrained problems is also seen with more
sophisticated backtracking strategies as well as for local search methods such as heuristic
repair [40], simulated annealing [32] and GSAT [48]. As with the backtracking search
described above, small problem examples can also give some intuitive understanding
of why these local methods find problems near the transition to be hard [18]. In this
case it is due to changes in the proportion of complete assignments that have local
improvements.

2.5. Why are phase transitions important?

The phase transitions in computational problems that have been discovered to date
have revealed surprising regularities across many different problems and many different
algorithms. Such universal behavior was quite unexpected and has the potential to
illuminate the nature of computational problems in fresh and fruitful ways. In particular,
the location of the phase transition point might provide a systematic basis for
selecting the type of algorithm to use on a given problem. Moreover, having made the
analogy between physical phase transitions and computational phase transitions, one is
left to wonder whether other phenomena from statistical physics might be manifest in
computational systems.

The information exchange need not all be one way however. Whereas the study of
physical systems is limited by the laws of physics, in computer science we can posit
arbitrarily strong coupling between any variables we please. This motivates the study of
phase transition phenomena in higher dimensions than has hitherto been the case and
could result in new advances in other fields.

3. Contributions

The papers collected in this special issue extend the work on phase transitions in
combinatorial search in a number of directions. These include refinements of the loca-
tion of the transition for 3-SAT by Selman, Mitchell and Levesque and Crawford and
Auton, an examination of hard instances among generally easy underconstrained prob-
lems by Gent and Walsh, and an investigation of random binary constraint problems
by Prosser. Previous results are generalized to other problem ensembles for SAT by
Mitchell and Levesque and graph coloring by Hogg, which also describes an additional
parameter measuring local constraint clustering. Smith and Dyer also discuss the im-
portance of additional parameters by examining the dependence of the transition on the
graph structure of binary constraint problems.

This issue also presents a variety of related phenomena. For instance, Freeman de-
scribes the behavior of the generated search trees. Schrag and Crawford present the
behavior of the prime implicates, thus providing an aspect of problem structure, other
than solubility, that is independent of the particular search method used. Most recent
work has been mainly on constraint satisfaction problems, but similar phenomena are
seen in optimization, described by Zhang and Korf, and Bylander’s study of planning
problems.

An important new result is the use of scaling methods from the statistical mechanics
theory of phase transitions for the case of search problems. This work, by Selman
and Kirkpatrick, shows quantitatively how the transition behavior, formally requiring an
infinitely large system, can also describe large but finite search problems.

Finally, this issue includes two papers on exploiting knowledge of phase transitions to
improve search: for optimization, by Zhang and Pemberton, and constraint satisfac-
tion, by Clearwater and Hogg.

4. Open issues and prospects

The observations reported in this issue have revealed intriguing regularities in the
structure of certain NP-complete problems. However they have also spawned many new
questions.

In particular, the empirical studies all plot some average or median performance
measure against simple structural parameters. Although these plots reveal clear easy-
hard-easy patterns they are nevertheless still associated with extremely high variances.
In practical terms this means that a random problem instance generated in the suppos-
edly “hard” region may not actually be that hard to solve. This suggests that the current
parameters used to specify structure in problem ensembles are too crude. Perhaps other,
more sophisticated measures of problem structure will prove to have greater discrimina-
tory power. One way to proceed would be to seek out better theoretical understanding of what is responsible for making some problem instances so much harder than others, as discussed in this issue by Hogg and Schrag and Crawford. Another would be to use machine learning techniques [26,34,41] to try to discover some useful combination of structural parameters based on the ability to predict the search cost of a set of examples.

A second open issue concerns the range of problems and characteristics over which phase transition behavior is exhibited. To date most of the research has been directed at studies of $k$-SAT, graph coloring and the traveling salesman problem. In these cases the performance measure of interest has been some measure of computational search cost. However, it would be interesting to learn whether there are any phase transitions, for example, in the quality of the optima found by optimization algorithms [11]. An additional question along these lines is how other signatures of phase transitions manifest themselves in search problems. In many physical phase transitions, the transition marks a change from a disordered to ordered structure, described by some so-called order parameters taking on nonzero values. The ordered phase is characterized by long range correlations among the system's components. Thus we can ask if there are analogies of these characteristics for search problems too.

Another exciting question to explore concerns the range of algorithms, for a particular type of problem, over which phase transition phenomena persist. For example, to date all the algorithms, other than generate-and-test, that have been examined have exhibited a characteristic easy-hard-easy pattern centered at a fixed transition point, including recent observations with hypothetical quantum computers [20]. Indeed this is what makes the phase transition phenomenon so interesting! But other aspects of the phase transition phenomenon, such as the reported anomalously hard cases in easy regions, do appear to be more sensitive to algorithm selection. In particular, in a recent paper by Baker [1] dependency-directed backtracking is shown to cure, or at least greatly ameliorate, the occurrence of hard cases in easy regions. This provides yet more evidence that there may be a systematic difference between hard problems in the easy region and hard problems in the hard region [24]. It also points to the use of phase transition results to select different algorithms for different problems based on simple structural properties.

Other applications of the phase transition results concern their relevance for independent or cooperative parallel search [23]. It appears that the nature of information exchanged and the frequency of exchange should be adapted depending on problem structure. But these ideas still need to be developed in detail. Even within the context of a single local search algorithm there is probably useful information embodied in the problem structure to determine an optimal restart time for algorithms such as heuristic repair [40] or GSAT [48], perhaps in conjunction with a statistical sampling of the search space [7]. However, the large variance associated with the current transitions gives only limited improvement as a heuristic for backtracking search methods where initial poor choices can have a major affect on the search cost [19].

The area of phase transitions in computation offers a rich environment for the development of a principled empirical basis for AI. However, we must be careful. We should not focus exclusively on problems simply because they are easy to articulate. The phase transition results have been criticized for merely being artifacts of the particular problem ensembles researchers have used. This seems unlikely due to the robustness
of the behavior in a number of different problem ensembles as well as the success of statistical mechanics in describing complex physical systems based on the behavior of idealized models. However, the critics have alerted us to the need to build random problem generators that can be tuned to fit the unique characteristics of real world problems such as job-shop and telescope scheduling, classroom timetabling and some numerical computations [16,28,37]. If the statistical insights afforded by phase transition analyses are to be of practical use we will have to evaluate them using problem ensembles that reflect the realities of real-world computation.

The papers contained in this special issue embody the current state of understanding of these phenomena and the deep connections they reveal between structural complexity of problems (measuring how they are represented) and computational complexity of algorithms.

References


