MODIFICATIONS OF THE “CENTRAL-METHOD” TO CONSTRUCT STEINER TRIPLE SYSTEMS

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0. Introduction

0.1. Steiner triple systems

Let \( V \) with \( |V| = v \) be a finite set and \( B \) a set of 3-subsets of \( V \). The elements of \( V \) are called points, those of \( B \) lines. If any 2-subset of \( V \) is contained in exactly one line, then the pair \( (V, B) \) is called a Steiner triple system of order \( v \), in short \( \text{STS}(v) \). Each point lies on exactly \( r = \frac{1}{2}(v - 1) \) lines and we have \( |B| = b = \frac{1}{3}v(v - 1) \). The condition \( v = 7, 9 + 6n, n \in \mathbb{N}_0 \), is necessary and sufficient for the existence of \( \text{STS}(v) \) (the trivial cases \( v = 1, v = 3 \) are excluded). The set of these “admissible” numbers, of these “Steiner numbers” is denoted by \( \text{STS} \).

0.2. Ovals in \( \text{STS}(v) \)

A non-empty subset \( O \subset V \) in a \( \text{STS}(v) \) is called an oval if each point of \( O \) lies on exactly one tangent and each other line of the \( \text{STS}(v) \) has at most two points in common with \( O \). A line is called a tangent if it meets \( O \) in exactly one point. If there are exactly two intersection points or if there is no intersection point then we have a secant or a passant respectively. The points of \( O \) are called on-points, the points of the tangents which are not on-points are called ex-points and the remaining points in-points. With respect to an oval \( O \) there are exactly \( r \) tangents, \( \frac{1}{2}r(r - 1) \) secants, \( \frac{1}{2}r(r - 1) \) passants and we have \( |O| = r \). The number of tangents through an ex-point is even iff \( r \) is even.

0.3. Special ovals in \( \text{STS}(v) \)

An oval \( O_K \) is called a knot oval if all tangents have exactly one point \( Z \) in common. \( Z \) is called the knot of the oval. Each ex-point different from \( Z \) lies on exactly one tangent and there are no in-points. It is known that there exist systems \( \text{STS}(v) \) with a knot oval if and only if \( v \in \text{HSTS} := \{7, 15 + 12n, n \in \mathbb{N}_0 \} \). \[2\]. Sometimes the set \( H = O_K \cup \{Z\} \) is called a hyperoval. The complement of \( H \) together with the passants of \( O_K \) forms a subsystem \( \text{STS}(r) \). It is possible to prove...