Hybrid Transform Coding for Channel State Information in MIMO-OFDM Systems

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Abstract—This paper is concerned with the efficient feedback of the channel state information (CSI) in multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems. A hybrid transform coding (HTC) scheme is proposed by exploiting the frequency and time correlation of CSI. An upper bound for the overhead-distortion performance of HTC is derived. Numerical results show that, compared with the available alternatives, the HTC scheme can achieve higher compression efficiency.

Keywords—feedback, channel state information (CSI), multiple-input multiple-output (MIMO), orthogonal frequency division multiplexing (OFDM), hybrid transform coding (HTC)

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1][2] and multiple-input multiple-output (MIMO) [3][4] are two promising techniques for the high speed transmission in wireless channels. These two techniques can provide fold capacity increase by exploiting, respectively, the frequency diversity and spatial diversity in frequency selective channels. They have been adopted in current and next generation wireless standards such as IEEE 802.16 [5] and Long-Term Evolution (LTE) [6].

The feedback channel state information (CSI) from user equipments (UEs) plays an important role in the downlink of MIMO-OFDM systems operating in frequency-division duplex (FDD) mode. With CSI at the base station (BS), the system performance can be significantly improved by adaptively customizing the transmitted waveforms to the channel, enabling channel-aware scheduling for multiple users, and so on. However, the overhead of the feedback CSI that guarantees an acceptable CSI accuracy at the BS is always a serious concern due to the limited bandwidth of feedback links. Hence feedback schemes with high compression efficiency are in demand.

In [7][8], Ericsson and DOCOMO propose a scalar compression technique by directly quantizing each element of the frequency-domain channel matrix at the UE. This technique, though incurs minor quantization errors, can reduce the overhead of CSI. However, it does not exploit the correlation of CSI in either the frequency or time domain and so the corresponding compression efficiency is not high. In [8], DOCOMO proposes a vector compression technique, where each row of the channel matrix is quantized by a pre-determined Grassmannian codebook known at both the UE and BS. Only the quantization index is sent to the BS for CSI reconstruction. This method can improve the compression efficiency but involves complicated quantization process. Qualcomm [9] proposes a compression technique based on multiple descriptions coding (MDC). In this technique, the CSI at different time instances is compressed by different codebooks with the same statistical properties. Compared to single codebook compression schemes with the same compression ratio, it can improve the feedback accuracy with a little more implementation cost. This method exploits the time-domain correlation of the channel but does not consider the frequency-domain correlation. Currently, how to improve the compression efficiency by exploiting the correlation in both the time and frequency domains at the same time still remains an open problem.

In this paper, we propose a novel compression scheme for the CSI of MIMO-OFDM systems. This scheme is referred to as hybrid transform coding (HTC). There are two compression options in HTC, i.e., direct coding and differential coding. The former exploits the correlation of CSI in the frequency domain by quantizing and coding in the time domain. The latter further exploits the correlation of CSI in the time domain by using the reconstructed CSI at one previous OFDM symbol as a reference for the CSI compression of the current OFDM symbol. Only the residual between the reference CSI and the current CSI is compressed by different codebooks with the same statistical properties. Compared to single codebook compression schemes with the same compression ratio, it can improve the feedback accuracy with a little more implementation cost. This method exploits the time-domain correlation of the channel but does not consider the frequency-domain correlation. Currently, how to improve the compression efficiency by exploiting the correlation in both the time and frequency domains at the same time still remains an open problem.

II. PRELIMINARIES

Consider an MIMO-OFDM system with K subcarriers, a BS with N antennas and a UE with M antennas. The overall downlink frequency-domain CSI at the s-th OFDM symbol is a three-dimensional matrix, i.e.,

\[ H^{(s)} = \begin{bmatrix}
H_{0,0}^{(s)} & H_{0,1}^{(s)} & \cdots & H_{0,N-1}^{(s)} \\
H_{1,0}^{(s)} & H_{1,1}^{(s)} & \cdots & H_{1,N-1}^{(s)} \\
\vdots & \vdots & \ddots & \vdots \\
H_{M-1,0}^{(s)} & H_{M-1,1}^{(s)} & \cdots & H_{M-1,N-1}^{(s)}
\end{bmatrix} \] (1)
where
\[ H^{(s)}_{m,n} = [H^{(s)}_{m,n,0}, H^{(s)}_{m,n,1}, \ldots, H^{(s)}_{m,n,K-1}] \]  
(2)
is a length-\( K \) column vector. In particular, \( H^{(s)}_{m,n,k} \) denotes the CSI of the link from the \( n \)-th transmit antenna to the \( m \)-th receive antenna on the \( k \)-th subcarrier. All \( \{ H^{(s)}_{m,n,k} \} \) are modeled as independent and identically distributed (i.i.d.) complex random vectors. The overall \( H^{(s)} \) is assumed perfectly known at the UE. Our task is to compress \( H^{(s)} \) in an efficient way with minor distortion.

In MIMO-OFDM systems, the frequency-domain CSI \( H^{(s)}_{m,n} \) for each antenna link is transformed from the time-domain CSI, which is defined by
\[ h^{(s)}_{m,n} = [h^{(s)}_{m,n,0}, h^{(s)}_{m,n,1}, \ldots, h^{(s)}_{m,n,K-1}], \]  
(3)
using discrete Fourier transform (DFT). Here all the \( K \) entries (which are referred to as ‘taps’ conventionally) in (3) are modeled to be equally spaced in the time domain with gap \( \Delta_t \) (which is known at both the UE and BS). Traditionally, the first tap is always assumed with no delay. Hence the entry \( h^{(s)}_{m,n,k} \) denotes the channel coefficient of the \( k \)-th tap with time delay \( (k-1)\Delta_t \). Conventionally, the time-domain CSI is always modeled to contain only a small number (denoted by \( L \) below) of non-zero taps that follow identical and independently complex Gaussian distribution with zero mean and unit variance.

To exploit the correlation of CSI in the frequency domain, a straightforward approach of compressing \( H^{(s)} \) is to sample \( K^* (K^* < K) \) complex entries of each \( H^{(s)}_{m,n} \) directly. The following theorem gives a lower bound for the minimum value of \( K^* \) that guarantees perfect CSI reconstruction. This theorem is the key of the compression method developed in this paper, whose proof is given in the Appendix.

**Theorem 1:** A necessary condition to perfectly reconstruct the CSI from direct frequency-domain sampling is
\[ K^* \geq 2L. \]  
(4)

Theorem 1 indicates that, to guarantee perfect reconstruction for the CSI of each antenna link, the number of variables that need compression in the frequency domain is at least twice of that in the time domain. Comparatively, the CSI concentrates on fewer coefficients in the time domain and can then be compressed more efficiently. This characteristic has been utilized in source coding widely to reduce the statistical correlation [10] and is adopted in this paper to exploit the frequency-domain correlation of CSI.

Note that besides the \( L \) non-zero tap coefficients, the time-domain CSI of each antenna link, i.e., \( h^{(s)}_{m,n} \), also includes the delay indexes of these taps (denoted by \( \{ n^{(i)} \}, i = 0, 1, 2, \ldots, L-1 \} \). Hence the compression of each \( h^{(s)}_{m,n} \) involves \( L \) complex numbers and \( L \) integer numbers. The latter can be represented by a finite number of bits without distortion. Furthermore, when a certain degree of distortion is acceptable at the BS, we can select a part of most significant taps (e.g., \( L^* \) taps with \( L^* < L \) or compression. Here \( L^* \) can be configured by the BS in semi-persistent manners. Hence the overhead of CSI in the time domain can be even less. This is a good tradeoff between overhead and fidelity.

In this paper, the exploitation of the time-domain correlation for CSI is also considered. With the assumption that the CSI is fed back every \( \Delta \) OFDM symbols, this time-domain correlation can be modeled as [11]
\[ h^{(s)}_{m,n,\rho(i)} = \rho \cdot h^{(s-\Delta)}_{m,n,\rho(i)} + \sqrt{1-\rho^2} \cdot w^{(s)}_{m,n,\rho(i)}, \forall m,n,l \]  
(5)
where \( \rho \in [0,1] \) and the terms \( \{ h^{(s)}_{m,n,\rho(i)} \} \) have the same distribution as \( \{ h^{(s)}_{m,n,\rho(i)} \} \). Note that the values of \( \{ n^{(i)} \} \) may also change from symbol to symbol but the total number of non-zero taps is assumed fixed at \( L \).

From (5) we can observe that, when \( \rho \) is close to 1, the CSI for adjacent OFDM symbols (either in the time or frequency domain) is similar to each other and their difference always has very small dynamic range compared with the original coefficients. This indicates that, instead of quantizing the large-scaled channel coefficient directly, we can quantize this difference using fine quantization steps to reduce reconstruction error. Hence the compression efficiency is further increased.

### III. HYBRID TRANSFORM CODING

Based on the discussion in Section II, we develop a new compression method in this section. This method is referred to as hybrid transform coding (HTC). In HTC, the CSI \( H^{(s)} \) on \( MN \) links will be processed individually. The corresponding work flow is shown in Fig. 1. HTC provides two options for compression, i.e., direct coding and differential coding. These two options will be discussed in detail below.

#### A. Direct Coding

At the UE, we first perform \( K \)-point DFT on each \( H^{(s)}_{m,n} \) to obtain \( h^{(s)}_{m,n} \). Then \( L^* (L^* \leq L) \) most significant taps in \( h^{(s)}_{m,n} \) are selected and their real and imaginary parts are separately quantized by a common \( B \)-bit codebook \( Q^0 = \{ Q_0, Q_1, \ldots, Q_{2^B-1} \} \) that is pre-designed and known at both the UE and the BS. The corresponding delay indexes \( \{ n^{(i)} \} \) are directly represented by a finite-length binary string. Finally, both the quantized indexes of the selected

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1 For example, we can adopt a defaulted \( L^* \) at both the BS and UE initially. This defaulted value is adjustable by the BS according to the distortion of reconstructed CSI. The updated value of \( L^* \) is then informed to the UE via downlink signaling.
\{ \tilde{h}^{(s)}_{m,n,l} \} \) and the binary representations of the corresponding \( n^{(0)} \) are fed back to the BS.

The CSI reconstruction procedure at the BS is as follows. Upon receiving the feedback information for each antenna link from the UE, the BS first de-quantizes the tap coefficients \( \{ \tilde{h}^{(s)}_{m,n,l} \} \) based on the codebook \( Q \). These tap coefficients, together with the recovered delay information \( n^{(0)} \), are used to form the reconstructed time-domain CSI (denoted by \( \tilde{h}^{(s)}_{m,n} \)). Finally, inverse DFT (IDFT) is performed on \( \tilde{h}^{(s)}_{m,n} \) to obtain the corresponding frequency-domain CSI (denoted by \( \tilde{H}^{(s)}_{m,n} \)).

Note that in (6) we use the reconstructed CSI \( \tilde{h}^{(s)}_{m,n,\Delta l} \), instead of the real CSI \( h^{(s)}_{m,n,\Delta l} \), because the latter is unavailable at the BS and using the former as reference can prevent quantization error propagation. This treatment is borrowed from source coding such as H.263 and H.264.

Afterwards, we only compress \( \Delta h^{(s)}_{m,n} \) in a similar way as that in direct coding. The only difference is that the real and imaginary parts of each coefficient selected from \( \Delta h^{(s)}_{m,n} \) are quantized by another fine codebook \( \mathcal{B}^{Q} \). This processing is performed on \( \mathcal{B}^{Q} \).

The re-construction process related to differential coding at the BS is also similar to that for direct coding. Let \( \Delta \tilde{h}^{(s)}_{m,n} \) denote the reconstructed result of \( \Delta h^{(s)}_{m,n} \). After obtaining \( \Delta \tilde{h}^{(s)}_{m,n} \) via de-quantization, we add \( \Delta \tilde{h}^{(s)}_{m,n} \) to \( \tilde{h}^{(s)}_{m,n,\Delta l} \) and obtain the reconstructed time-domain CSI \( \hat{h}^{(s)}_{m,n} \).

Then the corresponding frequency-domain CSI, \( \hat{H}^{(s)}_{m,n} \), can be easily obtained by IDFT.

C. A Brief Summary

The overall process of HTC is summarized as follows. At the UE, we use a buffer to store the most recently available \( \tilde{h}^{(s)}_{m,n} \), which is initialized, e.g., by all zeros. Every time when the CSI is available, the UE processes both direct coding and differential coding and then compares their differences. The one with less distortion is finally adopted and the newly obtained \( \tilde{h}^{(s)}_{m,n} \) is used to update the buffer as the reference of future CSI compression. Consequently, each feedback instance includes three parts, i.e., the coding type index indicating the compression option used, the quantization result and the corresponding delay information. The inverse operations at the BS have been introduced in the previous two sub-sections and so we omitted the details here.

The overhead of quantization result is determined by the number of selected coefficients \( L' \) and the payload of codebook \( B \). Without loss of generality, we assume \( n^{(0)} < n^{(1)} < \ldots < n^{(L'+1)} \). Note that in practical systems, cyclic prefix (CP) is always adopted to avoid interference among adjacent OFDM symbols, indicating that the length of CP, denoted by \( n_{\text{max}} \), is a natural upper bound for the maximum time delay of all non-zero taps. Hence we have \( n^{(l)} < n_{\text{max}} \) for all \( l \). Based on the observation that \( n^{(l)} \in [n^{(l)-1}, {n_{\text{max}}-l}] \), we can conclude that the number of bits used to represent each \( n^{(l)} \) should be \( \lceil \log_2(n_{\text{max}}-L'+l-n^{(l)}) \rceil \) where \( \lceil \cdot \rceil \) denotes the ceiling function. Hence the average number of bits to represent each \( n^{(l)} \) should be

\[
\bar{n}_{\text{ave}} = E_{n^{(l)}} \left\{ \frac{1}{L'} \sum_{s=0}^{L'-1} \log_2(n_{\text{max}}-L'+l-n^{(l)}) \right\} \tag{7}
\]
Hence we have

\[
MN \cdot (1 + L^* n_{\text{ave}} + L' \cdot 2B).
\]

(8)

Compared with the direct frequency-domain compression methods, HTC involves additive DFT/IDFT operations at the UE and the BS. They are inherent operations in OFDM systems and have fast implementation algorithms with very low computational cost. Hence the implementation complexity of HTC is similar to the existing CSI compression methods.

IV. PERFORMANCE ANALYSIS

In this section, we derive an upper bound for the overhead-distortion performance of HTC.

A. Relative Mean Square Error (MSE)

In this paper, the distortion measure for HTC is selected as the relative mean square error (MSE) defined below.

\[
D = \frac{E(\|H^{(s)} - \tilde{H}^{(s)}\|_F^2)}{E(\|H^{(s)}\|_F^2)} = \frac{E(\sum_{m,n}\|H_{m,n}^{(s)} - \tilde{H}_{m,n}^{(s)}\|_F^2)}{E(\sum_{m,n}\|H_{m,n}^{(s)}\|_F^2)},
\]

(9)

where \(\tilde{H}^{(s)} = \{\tilde{H}_{m,n}^{(s)}\}_{M \times N}\) is the reconstructed frequency-domain CSI, and \(\|\cdot\|_F\) denotes the Fresenius norm.

The above definition of relative MSE can be simplified by the following steps.

Firstly, recall that all \(\{H_{m,n}^{(s)}\}\) are i.i.d. and so are all \(\{\tilde{H}_{m,n}^{(s)}\}\). Then we can rewrite (9) as

\[
D = \frac{E(MN\|H_{m,n}^{(s)} - \tilde{H}_{m,n}^{(s)}\|_F^2)}{E(MN\|H_{m,n}^{(s)}\|_F^2)} = \frac{E(\sum_{m,n}\|H_{m,n}^{(s)} - \tilde{H}_{m,n}^{(s)}\|_F^2)}{E(\sum_{m,n}\|H_{m,n}^{(s)}\|_F^2)}, \quad \forall m,n.
\]

(10)

For notation simplicity, we let \(m = n = 0\) from now on without loss of generality.

Secondly, according to Parseval’s theorem, the energy of a signal is kept unchanged before and after DFT/IDFT. Hence we have

\[
D = \frac{E(\|H_{0,0}^{(s)} - \tilde{H}_{0,0}^{(s)}\|_F^2)}{E(\|H_{0,0}^{(s)}\|_F^2)} = \frac{E(\sum_{i=0}^{L-1}\|h_{0,0,0}^{(s)} - \tilde{h}_{0,0,0}^{(s)}\|_F^2)}{E(\sum_{i=0}^{L-1}\|h_{0,0,0}^{(s)}\|_F^2)}
\]

(11)

where the last equation holds because \(h_{0,0,0}^{(s)}\) (and \(\tilde{h}_{0,0,0}^{(s)}\)) only contains \(L\) non-zero taps.

Finally, since the real and imaginary parts of each \(h_{0,0,0}^{(s)}\) are identically distributed and processed, their corresponding quantization errors have the same statistical properties (e.g., mean and variance). Hence (11) further reduces to

\[
D = \frac{E\left(\sum_{i=0}^{L-1}\text{Re}(h_{0,0,0}^{(s)}) - \text{Re}(\tilde{h}_{0,0,0}^{(s)})\right)^2}{E\left(\sum_{i=0}^{L-1}\|h_{0,0,0}^{(s)}\|_F^2\right)} = \frac{2\sum_{i=0}^{L-1}E(r_{0,0}^{(s)} - \tilde{r}_{0,0}^{(s)})^2}{L\sum_{i=0}^{L-1}\|h_{0,0,0}^{(s)}\|_F^2} \leq \frac{2L\alpha}{L}E(r_{0,0}^{(s)} - \tilde{r}_{0,0}^{(s)})^2 + 2\sum_{i=0}^{L-1}E(r_{0,0}^{(s)})^2
\]

(12)

where \(\text{Re}(\cdot)\) denotes the real part of its argument. \(r_{0,0}^{(s)}\) and \(\tilde{r}_{0,0}^{(s)}\) denote \(\text{Re}(h_{0,0,0}^{(s)})\) and \(\text{Re}(\tilde{h}_{0,0,0}^{(s)})\), respectively.

The relative MSE defined in (12) will be used to evaluate the overhead-distortion performance of HTC in the rest of this section. For simplicity, here we consider the simplest uniform quantization in both \(Q^0\) and \(Q^1\). The resultant relative MSE serves as an upper bound for that of the real system since the quantization books \(Q^0\) and \(Q^1\) can be optimized, e.g., by the Lloyd algorithm.

B. Direct Coding

Without loss of generality, here we assume the first \(L^*\) non-zero taps are selected for compression. Then we have \(\tilde{r}_{0,0}^{(s)} = 0\) for all \(l \leq l \leq L-1\) and (12) can be rewritten as

\[
D = \frac{2L\alpha}{L}E(r_{0,0}^{(s)} - \tilde{r}_{0,0}^{(s)})^2 + 2\sum_{i=0}^{L-1}E(r_{0,0}^{(s)})^2
\]

\[
\leq \frac{2L^*}{L}E(r_{0,0}^{(s)} - \tilde{r}_{0,0}^{(s)})^2 + 2\frac{(L - L^*)}{L}E(r_{0,0}^{(s)})^2
\]

\[
= 2\alpha\cdot E(r_{0,0}^{(s)} - \tilde{r}_{0,0}^{(s)})^2 + 1 - \alpha
\]

(13)

where \(\alpha = L^*/L\). Note that in (13) the inequality holds because the right hand-side of “\(\leq\)” represents the distortion when \(L^*\) non-zero taps are randomly selected, which is larger than that when \(L^*\) most significant taps are selected, as proposed in HTC.

Denote by \(d_0\) the quantization step of the uniform codebook \(Q^0\). The quantization levels in \(Q^0\) can then be represented as \{\pm d_0/2, \pm 3d_0/2, \ldots, \pm (2^B-1)d_0/2\}. Hence we have

\[
(r_{0,0}^{(s)} - \tilde{r}_{0,0}^{(s)})^2 \leq (d_0)^2/4, \quad |r_{0,0}^{(s)}| \leq (d_0 - (2^B-1)d_0/2)^2, \quad |r_{0,0}^{(s)}| > 2^{B-1}d_0.
\]

(14)

Taking average over the distribution of \(r_{0,0}^{(s)}\) on both sides of (14), we can obtain

\[
E(r_{0,0}^{(s)} - \tilde{r}_{0,0}^{(s)})^2 \leq 2\int_0^{d_0/4} f(t) dt + 2\int_{2^{B-1}d_0/2}^{\infty} f(t) dt \leq 2^{2B}d_0^2
\]

(15)

where \(f(t) = \pi^{-1/2} e^{-t^2}\) is the probability density function of a Gaussian random variable \(t\) with \(t \sim N(0, 1/2)\). With \(f(t)\) replaced by its definition and after some derivations, (15) can be further written as
\[
E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 \leq \frac{d_i^2}{4} + \frac{(2 - 2^a) \cdot d_i}{2\sqrt{\pi}} e^{-\frac{q_{i}^2 + \delta_i^2}{4}} \\
+ \left[ 2^a \cdot (2 - 2^a) \cdot d_i^2 + 1/2 \right] \cdot \text{erfc}(2^a \cdot d_i) \quad (16)
\]

where \(\text{erfc}(\cdot)\) is the complementary error function.

From (13) and (16), we can obtain the following upper bound for the overhead-distortion performance of direct coding in HTC.

\[
D < a \cdot \frac{d_i^2}{2} + \frac{(2 - 2^a) \cdot d_i}{2\sqrt{\pi}} e^{-\frac{q_{i}^2 + \delta_i^2}{4}} \\
+ \left[ (2^a - 2^a - 2^a) \cdot d_i^2 + 1 \right] \cdot \text{erfc}(2^a \cdot d_i) + 1 \cdot a . \quad (17)
\]

### C. Differential Coding

In differential coding, we still assume the residual of the first \(L'\) taps are selected for compression. Then we have \(\bar{r}_{q_{i}}^{(s)} = r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)}\) for all \(L' \leq l \leq L-1\) and (12) can be rewritten as

\[
D = \frac{2}{L} \sum_{l=0}^{L-1} E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 + \frac{2}{L} \sum_{l=0}^{L-1} E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 \\
\leq \frac{2}{L} \sum_{l=0}^{L-1} E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 \\
+ \frac{2}{L} \left( \sum_{l=0}^{L-1} E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 + \sum_{l=0}^{L-1} E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 \right) \\
= \frac{2}{L} \sum_{l=0}^{L-1} E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 + \frac{2}{L} \sum_{l=0}^{L-1} E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 \\
\leq 2E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 + 2(1-a)E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2. \quad (18)
\]

Note that in (18), the first inequality follows the in-equation \((a + b)^2 \leq a^2 + b^2\). The second equality holds because \((r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2\) has the same statistical properties as \((r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2\). The second inequality follows the same reason in (13). The two terms in (18) will be derived separately below.

For the first term, we assume the quantization step of the uniform codebook \(Q\) (denoted by \(d_i\)) satisfies \(d_i = (2^a-1) \cdot d_i\). This is a realizable uniform codebook and the resultant distortion performance can be used as an upper bound for that of HTC with optimized codebooks. Based on this assumption, we can see that \(Q\) is a nested quantizer of \(Q\). The former simply quantizes one step of the latter using fine steps. As a consequence, when \(\bar{r}_{q_{i}}^{(s)}\) is obtained via direct coding, \(\bar{r}_{q_{i}}^{(s)}\) can be regarded as the quantization result of an equivalent uniform codebook with quantization levels \{\pm d_i/2, \pm 3d_i/2, \pm 5d_i/2, \ldots, \pm q_{\text{max}}d_i/2\} where \(q_{\text{max}} = 2^a - 2^a + 1\). Hence by replacing \(d_i\) and \((2^a-1)\) with \(d_i\) and \(q_{\text{max}}\) in (16), respectively, we can obtain

\[
E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 \leq \frac{d_i^2}{4} + \frac{(q_{\text{max}} - 1) \cdot d_i}{2\sqrt{\pi}} e^{-\frac{(q_{\text{max}} + 1) \cdot d_i^2}{4}} \\
+ \left( \frac{d_i^2}{4} + \frac{1}{2} \cdot \text{erfc}(q_{\text{max}} + 1) \cdot d_i^2 \right). \quad (19)
\]

For the second term in (18), from the time-domain correlation model (5), we have

\[
E(r_{q_{i}}^{(s)} - \bar{r}_{q_{i}}^{(s)})^2 = E\left(\rho \cdot r_{q_{i}}^{(s)} + \sqrt{1 - \rho^2} \cdot r_{w}^{(s)} - \bar{r}_{q_{i}}^{(s)}\right)^2 \\
= E(\rho - 1) \cdot r_{w}^{(s)} \cdot \rho - 2(1-a)(1-\rho). \quad (20)
\]

Combining (18), (19) and (20), we can obtain the upper bound of the overhead-distortion performance of differential coding in HTC as

\[
D \leq \frac{d_i^2}{2} + \frac{(q_{\text{max}} - 1) \cdot d_i}{2\sqrt{\pi}} e^{-\frac{(q_{\text{max}} + 1) \cdot d_i^2}{4}} \\
+ \left( \frac{d_i^2}{4} + \frac{1}{2} \cdot \text{erfc}(q_{\text{max}} + 1) \cdot d_i^2 \right) + 2(1-a)(1-\rho). \quad (21)
\]

### V. NUMERICAL RESULTS AND DISCUSSION

In this section, we show some numerical results for HTC based on the Extended Vehicular A (EVA) [13] channel model that is widely used in 3GPP (3rd Generation Partnership Project) for performance evaluation. We consider a MIMO-OFDM system with \(M = N = 2\). The system bandwidth is set to be 20MHz and the relative speed of the UE is set at 3 km/h. \(\Delta_t = 140\) (i.e., 10 millisecond in LTE), \(K = 2048\), \(n_{\text{max}} = 160\) (i.e. the largest normal CP length in LTE) and \(\rho = 0.9\). The codebooks \(Q\) and \(Q\) are generated using the Lloyd algorithm [12].

![Fig. 2. The overhead-distortion performance of HTC with different parameter settings (L', B) marked on the dashed lines. The overhead is calculated based on (8).](image-url)

Fig. 2 shows the overhead-distortion performance of HTC with different \(L'\) and \(B\) under above system setting.
The corresponding upper bounds\(^3\) derived in Section IV are also plotted for reference. From the upper bound results, we can see that using uniform codebooks can already achieve a low relative MSE (i.e., around -8dB), which is low enough to support channel-aware scheduling at the BS. When the codebooks are optimized by the Lloyd algorithm, the HTC performance can be significantly improved, e.g., by 5–8dB. When \(L^* = 3\) and \(B^* = 3\), we can make a good tradeoff between the fidelity and overhead.

We also make a comparison between HTC (with \(B = 3\) and \(L^* = 3\)) and the method proposed by Ericsson [7] in EVA under the same system setting. The results are listed in Table 1, from which we can clearly see that, when the overhead is 27.2kbits/s, Ericsson’s method can achieve the best fidelity. However HTC can achieve much better fidelity (-15.9dB vs. -9.1dB) with only less than 2/3 overhead comparing to Ericsson’s method.

### VI. CONCLUSIONS

In this paper, we present a novel compression method, i.e., HTC, for the CSI of MIMO-OFDM systems. HTC has the capability of exploiting the correlation of CSI in both the time and frequency domains. Experimental results demonstrate that high compression efficiency can be achieved by HTC in EVA.

It is worthy to mention that HTC can achieve similar results in Extended Pedestrian A (EPA) and Extended Typical Urban (ETU) [13]. The related experimental results are not shown in this paper due to space limitation.

### APPENDIX: PROOF OF THEOREM I

Treating \(H^{(s)}_{m,n}\) as a time-domain signal and according to Nyquist sampling theorem, we can see that the sampling frequency (denoted by \(f_s\)) should be at least twice the highest frequency contained in \(H^{(s)}_{m,n}\) (denoted by \(f_c\)) so as to avoid distortion, i.e.,

\[
f_s = \frac{K}{T_s} \geq 2f_c,
\]

where \(T_s\) denotes the periodicity of one OFDM symbol.

By DFT, the signal \(H^{(s)}_{m,n}\) can be transformed into \(h^{(s)}_{m,n}\) as shown in (3). Since we have treated \(H^{(s)}_{m,n}\) as a time-domain signal, \(h^{(s)}_{m,n}\) can then be regarded as the frequency-domain spectrum of \(H^{(s)}_{m,n}\). Without loss of generality, we assume

\[0 < n^{(0)} < n^{(1)} < \ldots < n^{(L-1)} \leq n_{\max} \leq K.
\]

Then we have \(n^{(L-1)} \geq L\). By definition, the corresponding highest frequency \(f_c\) can be represented as

\[f_c = n^{(L-1)}/T_s.
\]

From (22) and (23), we obtain

\[K \geq 2n^{(L-1)} \geq 2L.
\]

This completes the proof. \(\Box\)

### REFERENCES


\(^3\) The upper bounds in Fig. 2 are for differential coding only. Those for direct coding are not plotted because when \(\rho = 0.9\), direct coding always performs worse than differential coding and almost all CSI is compressed by differential coding in the numerical experiments.