

Dynamic Pricing with Loss-Averse Consumers and Peak-End Anchoring

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We study the dynamic pricing implications of a new, behaviorally motivated reference price mechanism based on the peak-end memory mode. This model suggests that consumers anchor on a reference price that is a weighted average of the lowest and most recent prices. Loss-averse consumers are more sensitive to perceived losses than gains relative to this reference price. We find that a range of constant pricing policies is optimal for the corresponding dynamic pricing problem. This range is wider the more consumers anchor on lowest prices, and it persists when buyers are loss neutral, in contrast with previous literature. In a transient regime, the optimal pricing policy is monotone and converges to a steady-state price, which is lower the more extreme and salient the low-price anchor is. Our results suggest that behavioral regularities, such as peak-end anchoring and loss aversion, limit the benefits of varying prices, and caution that the adverse effects of deep discounts on the firm's optimal prices and profits might be more enduring than previous models predict.

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1. Introduction

In April 2009, Apple increased the price of its best-selling iTunes tracks from \$0.99 to \$1.29, resulting in an abrupt and apparently more enduring than expected sales decline (*Wall Street Journal* 2010). While \$1.29 might well be a steep price to pay for a song, arguably, a trigger for the customer pushback lies beyond basic economics: Apple has trained consumers over time that a song, even a popular one, is worth \$0.99—no wonder they feel reluctant to pay 30 cents more. What if Apple, who had cleverly played the consumer anchoring card with the introduction of the iPhone, might have underestimated the long-run effect of the low iTunes price anchor on buyer behavior?

In repeat-purchase markets, consumers form price expectations, also known as reference prices. Prices are perceived as discounts or surcharges relative to these reference prices, and this perception affects demand and profitability. For example, while a price promotion might have a short-run positive impact on sales, the lowered price might result in the installation of a low price in consumers' memory, eroding price expectations and willingness to pay, and thereby negatively affecting profitability on the long run. This suggests that it is important for a firm to understand (1) how its pricing policy and history affects consumers' price expectations and purchase decisions, and (2) how to set prices over time to maximize profitability in this context.

The modeling literature on pricing with reference effects (see Popescu and Wu 2007 for a review) assumes that consumers' price expectations follow an exponentially smoothed adaptive expectations process (Mazumdar et al. 2005). In this fast-decaying memory process, consumers anchor on a weighted average of all past prices. Despite its prevalence in analytical pricing research, this model is not very well grounded in behavioral evidence (Rabin 1998).

We explore a different memory-based reference price model, based on the *peak-end rule* (Fredrickson and Kahneman 1993), which is supported by extensive research in psychology and provides a behaviorally compelling alternative to adaptive expectations. In the pricing context this model suggests that consumers remember the lowest (salient) and most recent prices, and the reference price is a weighted average of the two. Our goal is to operationalize the peak-end rule in a dynamic pricing context and understand how this consumer memory and anchoring process, combined with behavioral decision processes (e.g., loss aversion), influences the pricing strategies of the firm.

Substantial research in psychology suggests that memory, and hence remembered utility, follows a "snapshot model" (or "moment based approach"), whereby the overall evaluation of past experiences is based on only a few salient moments rather than a cumulative measure of all past experiences (Kahneman 2000). Moreover, evidence suggests that the most salient moments are the peak and

the end (i.e., the most extreme and most recent experiences), and memory-based judgment tasks are based on a combination (weighted average) of the two. This peak-end memory model, proposed by Fredrickson and Kahneman (1993), finds vast empirical support in the psychology literature, both for positive and negative experience contexts; see Fredrickson (2000) and Kahneman (2000) for reviews.

In the pricing context, we posit that the representative peak-end moments in reference price formation are associated with the lowest and latest price, i.e., the highest and the most recent transaction utility, consistent with the peak-end rule for positive experiences (see Nasiry 2010 for a model where consumers anchor on the highest and last prices). While an empirical investigation of the peak-end rule in the pricing context is still lacking, several studies find support for anchoring on most recent prices (e.g., Krishnamurthi et al. 1992, Chang et al. 1999) and extreme prices (Nwokoye 1975, Niedrich et al. 2001). Rajendran and Tellis (1994) find empirical support for a reference price that is a combination of the lowest price across brands and the brand's last price. Anchoring on low prices is also motivated by fairness considerations (Xia et al. 2004, Mazumdar et al. 2005).

In a review and analysis of various reference price conceptualizations, Lowengart (2002) suggests that memory-based behavioral reference prices, such as the one proposed here, are most appropriate for frequently purchased experienced-quality goods (e.g., food, fragrance, music). In such a context, he suggests that “marketers would probably benefit from constant price strategy, such as Every Day Low Prices, (EDLP), and should not use heavy price promotion tactics [...], to avoid establishing a low reference price in consumers minds [and] the loss effects that will appear after a price promotion” (p. 163). We investigate the validity of these insights in an analytical framework.

Our paper contributes to a growing body of behavioral operations literature, such as reviewed by Loch and Wu (2007). Related work that incorporate consumer learning and memory models in various operational contexts include Gaur and Park (2007, consumers learn fill rates), Liu and van Ryzin (2011, consumers learn about rationing risk), and Ovchinnikov and Milner (2011, consumers learn about the likelihood of last-minute sales). Specifically, our work builds on the literature on dynamic pricing with reference effects. Kopalle et al. (1996) and Fibich et al. (2003) show monotonicity and convergence of the optimal price paths under a piecewise linear demand model. Popescu and Wu (2007) extend these findings to general demand functions and reference effects. These papers all assume that consumer learning follows a fast-decaying, exponentially smoothed process. We investigate the robustness of their predictions under a different memory process, based on the peak-end rule. To the best of our knowledge, our paper is the first to model the peak-end rule in an operational setting.

From a methodological standpoint, the peak-end memory model makes the analysis of the firm's dynamic pricing problem substantially more involved. Our problem amounts to solving a dynamic program with nonsmooth reward and transition functions for which we develop a non-standard approach, interplaying relaxation and variational techniques.

Our results indicate that a constant pricing policy is optimal for a range of relatively low initial price expectations. This range of steady states is wider, the more loss-averse consumers are, and the more they are sensitive to the low-price anchor. Unlike with exponential smoothing, this range persists even when consumers are loss neutral (i.e., equally sensitive to discounts and surcharges), due to the asymmetric anchoring process. Overall, we find that behavioral asymmetries in anchoring and decision processes make constant pricing (and EDLP) more prevalent, partially confirming Lowengart's (2002) intuition cited earlier. Our results complement the literature supporting constant over dynamic pricing, e.g., when buyers are strategic (Su 2007), or myopic, in asymptotic regimes (Gallego and van Ryzin 1994), and when price adjustments are costly (Çelik et al. 2009).

Firms can profit from skimming or penetration strategies to gradually manage consumer price expectations when these are not in steady state. Behavioral regularities ensure that prices will converge on the long run to a steady state, which is lower than what firms ignoring reference effects would charge. These findings are consistent with existing reference price literature. Unlike exponential smoothing, however, initial low price expectations affect the value of the steady-state price, suggesting that the effect of low historical prices is more enduring than previously understood. Furthermore, the more salient the lowest historical price, the lower the optimal prices and profits. Back to the example of iTunes, our insights suggest that long-term effects on prices and profits are to be expected if the \$0.99 price-anchor is ingrained in consumer memory; these effects are best mitigated by gradually changing prices to manage expectations.

Overall, our results suggest that consumer memory and anchoring processes, in addition to decision processes, play an important role in determining how firms should manage prices.

2. The Model

This section describes how consumers make purchase decisions based on prices and reference prices, and how these decisions affect demand for a firm's product.

Mental accounting theory (Thaler 1985) posits that the utility from a purchase experience consists of two parts: acquisition and transaction utility. The former reflects the monetary value of the good, whereas the latter is the psychological value of the deal, determined by the gap $x = r - p$ between the mental reference price, r , and the actual price, p .

In a deterministic context, prospect theory (Tversky and Kahneman 1991) identifies key properties of the transaction utility, which are inherited by the aggregate reference dependent demand and validated empirically in the pricing context (Kalyanaram and Winer 1995). Specifically, demand increases in the magnitude of the reference gap, $x = r - p$ (reference dependence), and it is more sensitive to perceived surcharges than discounts of the same magnitude (loss aversion). To simplify exposition, we assume here that consumers' marginal sensitivity to perceived discounts (γ)—respectively, surcharges (λ)—is constant, i.e., reference effects are piecewise linear; our insights extend to nonlinear effects (see Nasiry 2010). These assumptions motivate the following demand model:

$$d(p, r) = d_0(p) - \lambda(p - r)^+ + \gamma(r - p)^+ \\ = \begin{cases} d_0(p) + \lambda(r - p), & \text{if } p \geq r; \\ d_0(p) + \gamma(r - p), & \text{if } p \leq r. \end{cases} \quad (1)$$

Loss aversion is captured by $\lambda \geq \gamma > 0$. We assume that the base demand, $d_0(p)$, is nonnegative, bounded, continuously differentiable, and decreasing in price, and the base profit $\pi_0(p) = p d_0(p)$ is nonmonotone and strictly concave. The firm's short-term profit is denoted $\pi(p, r) = p d(p, r)$. All our results extend for a nonzero marginal cost c .

The peak-end rule (Kahneman et al. 1993) suggests to model consumers' reference price, r_t , at any time $t > 1$, as a weighted average of the minimum price, m_{t-1} , and the most recent price, p_{t-1} :

$$r_t = \theta m_{t-1} + (1 - \theta)p_{t-1}, \quad m_{t-1} = \min(m_{t-2}, p_{t-1}), \quad (2)$$

where $\theta \in (0, 1]$ captures how much consumers anchor on the lowest price.

Given initial conditions m_0 and p_0 , the firm maximizes infinite horizon β -discounted revenues:

$$J(m_0, p_0) = \max_{p_t \in \mathbf{P}} \sum_{t=1}^{\infty} \beta^{t-1} \pi(p_t, \theta m_{t-1} + (1 - \theta)p_{t-1}),$$

where $m_t = \min(m_{t-1}, p_t)$,

where $\beta \in (0, 1)$. Without loss of generality, $m_0 \leq p_0$, and prices are confined to a bounded interval $\mathbf{P} = [0, \bar{p}]$, where $d_0(\bar{p}) = 0$ (to avoid trivial boundary solutions). The infinite horizon model implicitly assumes that lowest prices can be remembered indefinitely, a reasonable approximation when transaction frequency is high relative to the horizon length. Our main insights remain valid when modeling the possibility of forgetting or updating the minimum price (Nasiry 2010).

The Bellman equation for this problem is

$$J(m_{t-1}, p_{t-1}) = \max_{p_t \in \mathbf{P}} \left\{ \pi(p_t, \theta m_{t-1} + (1 - \theta)p_{t-1}) \right. \\ \left. + \beta J(\min(p_t, m_{t-1}), p_t) \right\}.$$

The value function, $J(m, p)$, is increasing in both arguments because transitions and profit per stage are increasing (cf. Stokey et al. 1989, Theorem 4.7). Intuitively, higher reference prices (i.e., memory of higher prices) enable the firm to extract higher profits.

Loss aversion ($\lambda \geq \gamma$) allows us to write the kinked profit $\pi(p, r)$ as the minimum of two smooth profit functions: $\pi_k(p, r) = [d_0(p) + k(r - p)]p = \pi_0(p) + k(r - p)p$, for $k \in \{\lambda, \gamma\}$. The following result, essential for our subsequent analysis, is proved in the online appendix. An electronic companion to this paper is available as part of the online version that can be found at <http://or.journal.informs.org/>.

LEMMA 1. *The short-term profit is*

$$\pi(p, r) = \min(\pi_\lambda(p, r), \pi_\gamma(p, r))$$

and it is supermodular in (p, r) .

Lemma 1 confirms the intuition that myopic firms, i.e., those focused on short-term profits ($\beta = 0$), should charge higher prices when consumers have higher price expectations (cf. Topkis 1998, Theorem 2.8.2). The Bellman equation can be written as follows:

$$J(m_{t-1}, p_{t-1}) = \max_{p_t \in \mathbf{P}} \left\{ \min(\pi_\lambda, \pi_\gamma)(p_t, r_t) \right. \\ \left. + \beta J(\min(p_t, m_{t-1}), p_t) \right\}; \\ r_t = \theta m_{t-1} + (1 - \theta)p_{t-1}. \quad (3)$$

The optimal pricing policy, denoted $p^*(m_{t-1}, p_{t-1})$, solves (3). For any (m_0, p_0) , the optimal price path $\{p_t\}_t$ is given by $p_t = p^*(m_{t-1}, p_{t-1})$, with $m_t = \min(m_{t-1}, p_t)$, $t \geq 1$; the corresponding optimal state path is $\{(m_t, p_t)\}_t$. We say that (m, p) is a steady state if $p^*(m, p) = p$; in other words, the corresponding optimal state path is constant.

3. Optimal Policy

In this section, we characterize the firm's optimal pricing policy, both on the long run (steady state) and in a transient regime. We first present the main result, then outline our approach. Section 4 further analyzes sensitivity to behavioral parameters and compares our results to the literature.

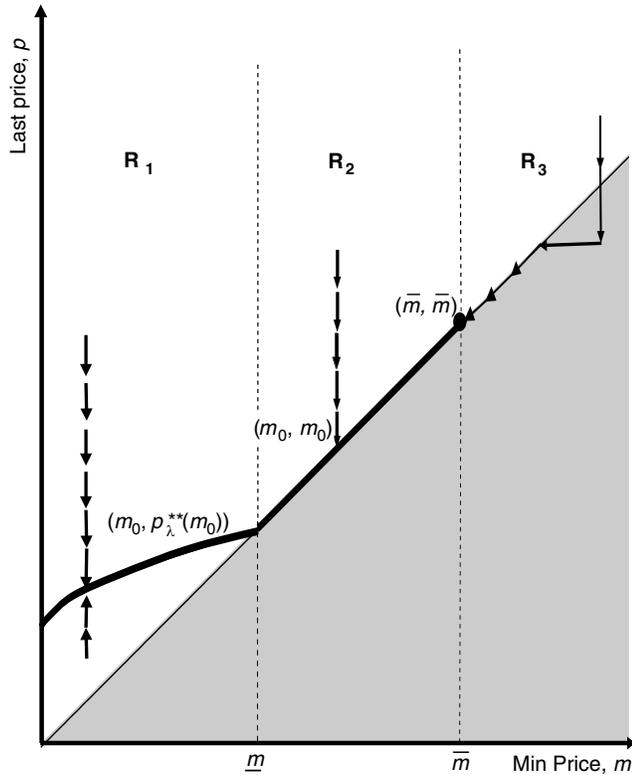
3.1. Main Result

Our main result, previewed in Figure 1, characterizes the set of steady states and the optimal transient policy of Problem (3). The analysis, outlined in the next section, leads us to distinguish three price-memory scenarios (low, medium, and high): $\mathbf{R}_1 = [0, \underline{m}]$, $\mathbf{R}_2 = [\underline{m}, \bar{m}]$, and $\mathbf{R}_3 = [\bar{m}, \bar{p}]$, where the thresholds $\underline{m} = \underline{m}(\lambda, \theta)$ and $\bar{m} = \bar{m}(\gamma)$ solve, respectively,

$$\pi'_0(p) - \lambda(1 - \beta(1 - \theta))p = 0, \quad (4)$$

$$\pi'_0(p) - \gamma(1 - \beta)p = 0. \quad (5)$$

Figure 1. Steady states (bold line) and generic optimal price paths (arrows) for each region of Problem (3).



In particular, $\underline{m} \leq \bar{m}$ by loss aversion ($\lambda \geq \gamma > 0$). We further define $p_\lambda^{**}(m)$ to solve

$$\pi'_0(p) - \lambda(2 - (1 - \theta)(1 + \beta))p + \lambda\theta m = 0. \tag{6}$$

Uniqueness of \underline{m} , \bar{m} , and $p_\lambda^{**}(m)$ follows because the left-hand sides of the above equations are strictly decreasing in p , by strict concavity of π_0 . We are now ready to state our main result.

PROPOSITION 1. *Given any (m_0, p_0) , the optimal price path of Problem (3) converges monotonically to a steady-state price, which is (a) $p_\lambda^{**}(m_0)$, if $m_0 \in \mathbf{R}_1$; (b) m_0 , if $m_0 \in \mathbf{R}_2$; and (c) \bar{m} , if $m_0 \in \mathbf{R}_3$. In particular, the optimal pricing policy, $p^*(m, p)$, is increasing in both m and p , and the set of steady states of Problem (3) is $\{(m, p_\lambda^{**}(m)) \mid m \in \mathbf{R}_1\} \cup \{(m, m) \mid m \in \mathbf{R}_2\}$.*

Proposition 1 indicates that it is not optimal for the firm to oscillate prices on the long run. The optimal pricing trajectory is either constant or converges monotonically to a steady-state price, which depends on the initial low-price anchor m_0 (but not on p_0). For sufficiently high $m_0 \geq \bar{m}$, the firm decreases prices down to a steady-state \bar{m} . $m_0 < \underline{m}$, the firm charges $p_\lambda^{**}(m_0)$, above m_0 , in steady state. In this case, the optimal price path is increasing if $p_0 < p_\lambda^{**}(m_0)$; otherwise, it is decreasing.

Proposition 1 characterizes a range of optimal constant pricing policies and shows how the firm can leverage consumer memory, anchoring, and reference effects to manage consumer price expectations when these are not in steady state. A firm may set off-equilibrium prices and expectations (e.g., iTunes, gasoline prices) if it incorrectly estimates demand or behavioral processes, or if it does not optimize prices in response to these effects, and in anticipation of future price changes.

To illustrate this point, consider a firm that ignores reference dependence and consumer-anchoring processes; so it offers a static price p^0 , which maximizes $\pi_0(p)$. Arguably, as a result of this policy, consumer expectations are anchored at $p^0 = m_0 = p_0$. A pertinent question is (a) whether the firm can do better by deviating from p^0 , and (b) how. Proposition 1 suggests that the answer is affirmative: (p^0, p^0) is not a steady state, because $m_0 = p^0 > \bar{m}$, by (5), so the firm can increase profits by decreasing prices gradually from p^0 down to \bar{m} . Our results suggest that a firm that ignores reference effects forgoes profit by overcharging customers (more on miscalibration in Nasiry 2010).

3.2. Solution Approach

In this section, we briefly sketch our approach for characterizing the optimal pricing policy of Problem (3). The proof of Proposition 1 is structured as follows. *Step I* characterizes the steady states. (1) We first identify a series of smooth upper-bound problems and characterize their steady states. (2) Then we match steady states of the original problem with those of these smooth relaxations and show that no other steady states exist. *Step II* characterizes the optimal transient policy. (1) We show that optimal price paths remain in the same region as the initial m_0 (cf. Figure 1). (2) Within each region, we argue that the optimal price path is monotone, so it converges to a steady state (in the corresponding region).

3.2.1. Step I: Steady States. The steady state analysis of Problem (3) requires a nonstandard approach, because both the short-term profit and the transition function (memory structure) are nonsmooth. Our analysis relies on matching steady states of Problem (3) with those of a series of smooth univariate upper-bound problems, for which standard methods can be applied.

Step I.1. For $\nu \in [0, 1]$, and $m \in \mathbf{P}$, consider the following smooth one-dimensional problem:

$$J_m^\nu(p_{t-1}) = \max_{p_t \in \mathbf{P}} \{(1 - \nu)\pi_\lambda(p_t, \theta m + (1 - \theta)p_{t-1}) + \nu\pi_\gamma(p_t, p_{t-1}) + \beta J_m^\nu(p_t)\}. \tag{7}$$

We next identify the steady states of the upper-bound problems J_m^ν .

LEMMA 2. (i) *For any $m \leq p$, we have $J(m, p) \leq J_m^\nu(p)$.*

(ii) For $\nu \in [0, 1]$ and $m \in \mathbf{P}$, Problem (7) admits a unique steady state, which solves

$$\begin{aligned} \pi'_0(p) - [\lambda(1-\nu)(2 - (1-\theta)(1+\beta)) + \nu\gamma(1-\beta)]p \\ + \lambda(1-\nu)\theta m = 0. \end{aligned} \quad (8)$$

(iii) For $m \in \mathbf{R}_2$ there exists $\nu \in [0, 1]$ so that m is a steady state of the corresponding Problem (7).

Lemma 2(ii) allows to identify the key quantities in Proposition 1, as follows. First, observe that $p_\lambda^{**}(m)$, given by (6), is the unique steady state of Problem (7) for $\nu = 0$; in particular $p_\lambda^{**}(\underline{m}) = \underline{m}$. The thresholds \underline{m} and \bar{m} , defined in (4) and (5), correspond to those values m for which the steady state of $J_m^{\nu=0}$, respectively $J_m^{\nu=1}$, equals m . Lemma 2(iii) shows that any $m \in [\underline{m}, \bar{m}]$ is a steady-state of J_m^ν for an appropriate ν . We next show that these are also steady states for our Problem (3).

Step I.2. We next identify the set of steady states of Problem (3) based on those of (7). First, we argue that, by approximating the value function J by J_m^ν , for appropriate values ν , the firm still charges optimal prices on the long run. Technically, this amounts to matching supergradients of the original problem with gradients of J_m^ν , for appropriate ν . Two types of steady states (m, p) are identified by Lemma 2(ii, iii), depending if $m < p$ or $m = p$, leading to Lemma 3(i, ii). This shows that the steady states of Problem (3) include those characterized in Proposition 1. Finally, Lemma 3(iii) shows that no other steady-states exist, concluding the steady-state analysis.

LEMMA 3. (i) For $m \in \mathbf{R}_1$, $(m, p_\lambda^{**}(m))$ is a steady state of Problem (3), where $p_\lambda^{**}(m)$ solves (6).

(ii) For $m \in \mathbf{R}_2$, (m, m) is a steady state of Problem (3).

(iii) The set of steady states of Problem (3) is $\{(m, p_\lambda^{**}(m)) \mid m \in \mathbf{R}_1\} \cup \{(m, m) \mid m \in \mathbf{R}_2\}$.

3.2.2. Step II: Transient Policy. Having characterized the steady-state policy of Problem (3), we next establish the transient properties of the optimal policy.

Step II.1. We first show that, starting at any initial state (m_0, p_0) , the optimal price path remains in the same region as m_0 . This will later allow us to analyze pricing paths region by region.

LEMMA 4. (i) If $m_0 \in \mathbf{R}_1 \cup \mathbf{R}_2$, then $p_t \geq m_0$ for all t . (ii) If $m_0 \in \mathbf{R}_3$, then $m_t \in \mathbf{R}_3$ for all t .

If $m_0 \leq \bar{m}$, the optimal price path stays above m_0 , i.e., the minimum price does not change over time. Otherwise, if $m_0 > \bar{m}$, the firm decreases prices over time, eventually below m_0 , but never below \bar{m} . In either case, the optimal state path remains in the same region as the initial state.

Step II.2. This final step relies on Lemma 1 to establish, region by region, that the optimal price paths of Problem (3) are monotone. Together with previous steps, this ensures that prices converge monotonically to a steady-

state price in the same region, specifically: (a) $p_\lambda^{**}(m_0)$, if $m_0 \in \mathbf{R}_1$, (b) m_0 , if $m_0 \in \mathbf{R}_2$, and (c) \bar{m} , if $m_0 \in \mathbf{R}_3$, concluding the proof of Proposition 1.

We first argue parts (a) and (b). For $m_0 \in \mathbf{R}_1 \cup \mathbf{R}_2$, $m_t = m_0$ by Lemma 4, so Problem (3) can be written (with m_0 as a parameter) as

$$\begin{aligned} J_{m_0}(p_{t-1}) = \max_{p_t \geq m_0} \{ \pi(p_t, r_t) + \beta J_{m_0}(p_t) \}, \\ \text{where } r_t = \theta m_0 + (1-\theta)p_{t-1}. \end{aligned} \quad (9)$$

That is, $J(m, p) = J_m(p)$ for $m \leq \bar{m}$, p . The optimal policy of Problem (9) is monotone by Lemma 1. Therefore, in our model $p_t^*(m_0, p_{t-1})$ is increasing in p_{t-1} , i.e., the optimal price path is monotone in a bounded interval, so it converges to a steady state in its region, as given by Lemma 3.

For Part (c), we first argue that prices must eventually fall below m_0 (but not below \bar{m} , by Lemma 4) at a certain time T . Until time T , a finite-horizon version of Problem (9) is solved, and the same structural results hold. After time T , we show that optimal prices $p_t = m_t$ solve the problem $\tilde{J}(p_{t-1}) = \max_{p_t \in \mathbf{P}} \{ \pi(p_t, p_{t-1}) + \beta \tilde{J}(p_t) \}$. Starting at $p_T = m_T \geq \bar{m}$, these prices decrease to \bar{m} , by Lemma 1. Details are in the online appendix (proof of Proposition 1).

4. Sensitivity Analysis and Comparison to the Literature

In this section, we characterize the sensitivity of the optimal policy with respect to behavioral parameters and discuss how our peak-end model predictions differ from the literature.

4.1. Sensitivity Results

Proposition 1 identified a threshold $\bar{m} = \bar{m}(\gamma)$, given by (5), such that for $m_0 \geq \bar{m}$ and, independent of how much weight θ consumers put on the minimum price, prices decrease to \bar{m} . Therefore, as long as initial price perception is sufficiently high, the peak-end features m_0 and θ do not affect the steady state. If, however, m_0 is low ($m_0 \leq \underline{m}$), both m_0 and θ affect the optimal long-run policy, $p_\lambda^{**}(m_0; \theta)$. The next result shows how behavioral parameters affect optimal policies and profits.

PROPOSITION 2. (a) $p^*(m, p; \theta)$ and $J(m, p; \theta)$ decrease in θ , $\theta \in (0, 1]$ and do not depend on θ for $m \geq \bar{m}$. (b) $p_\lambda^{**}(m; \theta)$ and $\underline{m} = \underline{m}(\lambda, \theta)$ decrease in θ , and $\bar{m}(\theta) \equiv \bar{m}$ is independent of θ . (c) $p_\lambda^{**}(m)$ and $\underline{m}(\lambda, \theta)$ are decreasing in λ ; $\bar{m}(\gamma)$ is decreasing in γ , and all are increasing in β .

Optimal prices, steady states, and profits decrease as consumers anchor more on lowest prices, as measured by θ ; these effects vanish for price expectations above the global threshold \bar{m} . In general, steady-state prices are lower, the more sensitive consumers are to deviations from the

reference price (λ, γ) . Finally, a more patient firm (higher β) charges higher steady-state prices.

The EDLP range $\mathbf{R}_2 = [\underline{m}(\lambda, \theta), \bar{m}(\gamma)]$ is defined so that the optimal policy is to maintain a constant (every day) low price equal to the initial low-price anchor $m_0 = p_0$; these are the steady states of the type (m, m) . The full range of steady-state prices $[p_\lambda^{**}(0; \theta), \bar{m}(\gamma)]$ is wider, including, for $m_0 < \underline{m}$, steady state prices above the low price anchor $p_\lambda^{**}(m_0; \theta) > m_0$. Proposition 2 shows that both the EDLP and steady-state ranges expand with θ (i.e., as consumers pay more attention to the minimum price) and with the degree of loss aversion λ/γ (i.e., as λ increases or γ decreases).

Our results showed when and how behavioral parameters $(\theta, \lambda, \gamma)$ impact the firm’s prices and profits, depending on initial consumer expectations. To decide which effects and parameters are relevant to assess empirically, we conclude by proposing the following parsimonious estimation procedure for determining the optimal long-run policy of the firm:

(1) Estimate consumers’ sensitivity to discounts (gains) γ and compute the global threshold $\bar{m} = \bar{m}(\gamma)$ based on (5). If historic prices exceed \bar{m} , then \bar{m} is the optimal long-term price.

(2) Otherwise, estimate consumer’s sensitivity to surcharges λ and compute $\underline{m}(0) = \underline{m}(\lambda, \theta = 0)$, based on (4). If the lowest historic price $m_0 \in [\underline{m}(0), \bar{m}]$, then m_0 is the optimal long-term price.

(3) Otherwise, if $m_0 < \underline{m}(0)$, estimate the anchoring parameter θ and calculate $\underline{m} = \underline{m}(\theta)$. The optimal long-term price is determined by Proposition 1, depending on how m_0 compares with \underline{m} .

The peak-end model and specific parameters appear to be particularly relevant when consumer price expectations are low. Back to the example of iTunes, our peak-end model predicts that Apple cannot profitably increase prices, unless $m_0 = 0.99 < \underline{m} = \underline{m}(\lambda, \theta)$. This means that both sensitivity to surcharges λ and the low-price anchoring parameter θ would have to be sufficiently small. The optimal long-run price, $p_\lambda^{**}(0.99; \theta) \geq 0.99$, depends on the initial anchor $m_0 = 0.99$ and on how salient it is (θ). This is in contrast with exponential smoothing predictions, which suggest that initial expectations do not affect the long-term price (Popescu and Wu 2007).

4.2. Peak-End vs. Exponential Smoothing Predictions

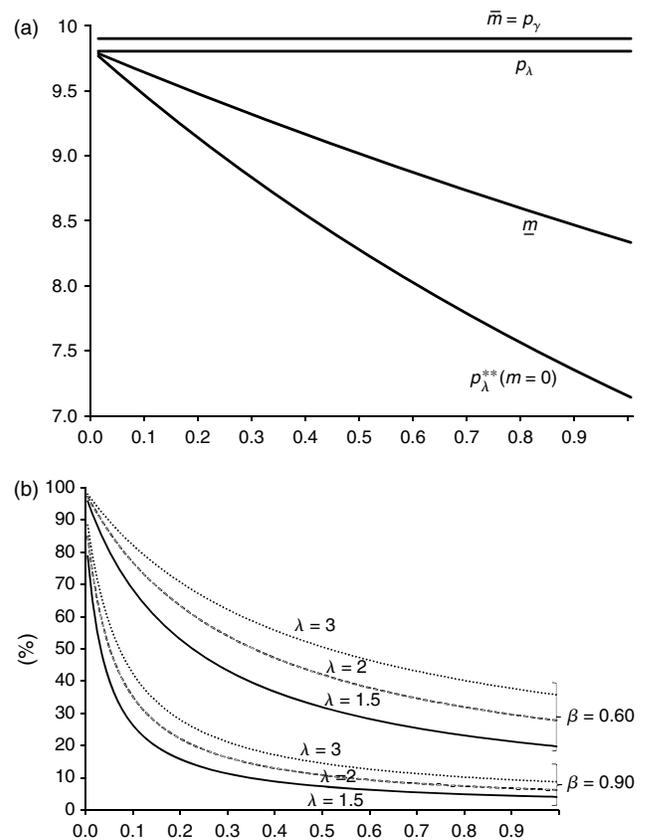
We conclude by comparing our model predictions with those under exponential smoothing. For a reference price model given by $r_t = \alpha r_{t-1} + (1 - \alpha)p_{t-1}$, $\alpha \in [0, 1)$, the corresponding range of steady-state prices is $[p_\lambda(\alpha), p_\gamma(\alpha)]$, where $\pi'_0(p_k(\alpha)) - k((1 - \beta)/(1 - \alpha\beta))p_k(\alpha) = 0$ for $k \in \{\lambda, \gamma\}$; if initial expectations are below (respectively, above) this range, prices converge monotonically to $p_\lambda(\alpha)$ (respectively, $p_\gamma(\alpha)$) (see, e.g., Corollary 2 in Popescu and

Wu 2007). In particular, for $\alpha = 0$, this model coincides with our peak-end model for $\theta = 0$, reflecting a case where consumers anchor only on the previous price. It is easy to see that $p_\gamma \triangleq p_\gamma(0) = \bar{m}(\gamma)$ and $p_\lambda \triangleq p_\lambda(0) = \underline{m}(0) = \underline{m}(\lambda, \theta = 0)$.

With loss-averse consumers, both models predict that a constant pricing policy is optimal for an intermediate range of initial expectations; this range expands as consumers are more loss-averse (i.e., as λ increases or γ decreases). The peak-end model, however, predicts a wider range of constant prices than under exponential smoothing. This range persists even when consumers are loss neutral, i.e., for $\lambda = \gamma$, $\bar{m}(\gamma) > \underline{m}(\lambda)$, unlike exponential smoothing, where $p_\lambda(\alpha) = p_\gamma(\alpha)$.

Figure 2 illustrates this comparison, suggesting that the discrepancy between peak-end and exponential smoothing predictions can be substantial. EDLP upper bounds for the two models coincide $\bar{m}(\gamma) = p_\gamma$, but lower bounds $\underline{m}(\lambda, \theta) \leq \underline{m}(\lambda, 0) = p_\lambda$ grow apart with θ (Figure 2(a)) λ and β (Figure 2(b)). The relative difference in EDLP ranges, $(p_\gamma - p_\lambda) / (\bar{m} - \underline{m}) \times 100\%$, is significant and surprisingly sensitive to (even small values of) the anchoring

Figure 2. (a) Steady states/EDLP range for exponential smoothing vs. peak-end models; $\beta = 0.90$, $\lambda = 2$. (b) Relative difference of EDLP ranges, $(p_\gamma - p_\lambda) / (\bar{m} - \underline{m}) \times 100\%$. In both figures $\gamma = 1$, $\alpha = 0$, $d(p) = 100 - 5p$.



parameter θ , more so than to loss aversion λ/γ ($=\lambda$ in Figure 2(b)). Specifically, for high enough discount factors ($\beta \geq 0.90$) and empirically observed values of the loss aversion index ($\lambda/\gamma \simeq 2$, cf. Ho and Zhang 2008), exponential smoothing predicts an EDLP range that is less than 10% of that under peak-end, for $\theta > 0.50$, and less than 50% for $\theta > 0.01$ (Figure 2(b)). In other words, the range of optimal constant pricing policies can easily double (downward) if consumers anchor, even slightly, on the lowest price.

Intuitively, there is relatively less opportunity value to manipulating prices under peak-end anchoring compared to exponential smoothing. On one hand, offering steep discounts can permanently erode demand in the future, as lowest prices remain salient in the memory anchoring process. On the other hand, the future benefit of increasing prices is short lived because these high prices affect only the reference price in the next period. This is unlike exponential smoothing, where the effect of all (even extreme) past prices lingers in memory but eventually vanishes.

To summarize, a key common insight from both reference price models is that behavioral asymmetries (loss aversion, peak-end anchoring) lead to more constant prices. However, the peak-end model predicts a significantly wider prevalence of EDLP than exponential smoothing. The distinction between peak-end and exponential smoothing is particularly relevant when initial price expectations are low. In this case, this lowest historical price might permanently erode the long-run steady state.

5. Conclusions

This paper operationalized a reference price model based on the peak-end rule (Fredrickson and Kahneman 1993) in a dynamic pricing context, where loss-averse consumers anchor on the lowest and most recent prices. Our results showed how peak-end anchoring processes interact with loss aversion to affect the structure of optimal pricing strategies. Behavioral regularities lead prices to converge monotonically over time, and induce a range of optimal constant pricing strategies, supporting EDLP. This range is wider the more loss averse consumers are, and the more they anchor on the lowest price. Overall, our results suggest that behavioral regularities, and in particular low price anchors, limit the benefits of varying prices.

6. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://or.journal.informs.org/>.

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