WHY PLAY LOGICAL GAMES?

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We owe to Paul Lorenzen an extraordinarily rich intuitive idea, first presented in his papers 'Logik und Agon' [Lorenzen 1960] and 'Ein dialogisches Konstruktivitätskriterium' [Lorenzen 1961], the fruitfulness of which we are barely encompassing today. In the language of game theory, it is the idea of defining logical particles in terms of rules for non-collaborative, zero-sum games between two-persons, a proponent and an opponent, and to define truth in terms of the existence of a winning strategy for the proponent in such games. In a nutshell,[1] a 'dialogical' or 'Lorenzen game' is always played in alternate moves between an opponent *O* and a proponent *P*, with the proponent beginning by asserting a given statement, so games are always of the form *POPOPOPOP*, thus one can label moves, etc. Lorenzen distinguished between 'particle' and 'structural' rules for these games. Particle rules provide the meaning of the logical connectives: When *P* asserts '$\varphi \& \psi$', *O* chooses one of the conjuncts and *P* must defend it, so the game continues for that conjunct (the other is dropped); since *O* chooses, *P* has to have a defence of both conjuncts up her sleeve if she hopes to win. When *P* asserts '$\varphi \vee \psi$', then *O* asks that *P* chooses and defends one of the disjuncts; she can thus chose the disjunct for which she has a defence. For an implication '$\varphi \rightarrow \psi$', *O* will assert φ thus forcing *P* to defend ψ (in some versions, *P* is said also to have the possibility to attack φ). With '$\neg\varphi$', roles are exchanged, as the opponent now becomes the proponent of φ. For quantifiers, when *P* asserts '$\forall x\, A(x)$', *O* chooses

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[1] Alas, there are no textbooks for dialogical logic, but one may find useful introductions in [Felscher 1986], [Lorenz 1981] or [Rückert 2001]; I follow for the most part the latter presentation in this paper. A number of seminal papers are collected in [Lorenzen & Lorenz 1978]. As the point of this paper is purely philosophical, I shall remain at an informal level throughout.

an x and P must then show that it has the property A, and when P asserts '$\exists x\, A(x)$', then O asks that P exhibits an x that has the property A.

Structural rules concern the structure of the games, for example, the already-mentioned rule stating that players move alternatively or a rule forbidding delaying tactics. One important rule concerns atomic formulas: P can only assert an atomic formula if it had been previously asserted by O, so that the winning strategy for P would be independent of any information about atomic facts. This is known as the 'formal rule', which makes room for 'formal' as opposed to 'material' games, and for a definition of logical validity. In material games, this rule is replaced by a rule stating that true atomic propositions may be asserted, leading to a definition of validity as general truth. Lorenzen games always terminate in a finite number of steps and the winning rule states that the one who has no possible moves left has lost. Thus, if P has at least a move for any move chosen by O, then P will win and a formula is valid if and only if P has a (formal) winning strategy for that formula [Rückert 2001, 173-174].

Another proposal for 'logical games', as they may generally be called, was put forth by Jaakko Hintikka some years later [Hintikka 1968, 1973],[2] the heart of which being a game-semantic reading of the quantifiers first suggested by Leon Henkin [Henkin 1961].[3] One should recall that in a formula of the form

$$(1)\ \forall x\, \exists y\, A(x, y)$$

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[2] For an introduction to game-theoretic semantics, see [Hintikka & Sandu 1997].

[3] It was found out later on that C. S. Peirce, one of the inventors of quantification theory, was actually the first to propose a game-semantic interpretation of the quantifiers. See [Hilpinen 1982] and [Pietarinen 2006]. For an nice example of exegesis using game semantics, see [Pietarinen & Snellman 2006]. Incidentally, Henkin's paper appeared in the same volume as one of Lorenzen's first papers [Henkin 1961], [Lorenzen 1961]. This coincidence is not so fortuitous, as they were both at the Institute for Advanced Studies in the late 1950s, possibly getting influence from von Neumann and Morgenstern, both pioneers of game theory, as well as from Tarski.

the choice of y depends on the prior choice of x and that one may replace it with a second-order formula, involving what is known as a 'Skolem function':

$$(2)\ \exists F\ \forall x\ A(x, F(x))$$

Hintikka's suggestion was to follow Henkin in reading (1) in terms of a game between an opponent, variously named 'Nature', 'initial falsifier', or '∀belard', and a proponent, 'Myself', 'initial verifier' or '∃loise'. In (1), the opponent makes the first move and chooses an x, then the proponent must find for that x a y such that $A(x, y)$. If P can always find a y for every x that O throws at her, so to speak, then she wins the game, otherwise, she loses. Of course, if a function such as 'F' in (2) is available to the proponent, she only has to apply it to find a y for any x chosen by the opponent and thus win the game; this is why the existence of a Skolem function is equivalent to the existence of a winning strategy for the proponent. Furthermore, the existence of a winning strategy is necessary and sufficient for the sentence to be true. This reading of the quantifiers has been extended by Hintikka to 'branching' or 'Henkin quantifiers', first introduced by Henkin in the very same paper in which he introduced his game-semantic interpretation [Henkin 1961]. I shall come back to this briefly.

Hintikka's next move was to extend this reading of the quantifiers to conjunction and disjunction, by reading '$\varphi \& \psi$' as 'All the sentences φ and ψ…', where O chooses one of φ and ψ and the game proceeds accordingly, and '$\varphi \vee \psi$' as 'There is one of φ or ψ…', where P chooses instead. One can thus see that his reading of the quantifiers is at the heart of his game semantics. To obtain the complete set of rules, one must add a rule for negation as the mere exchange of roles (depending on the assumption that every game has a dual) and a rule for atomic sentences. (Hintikka's approach is model-theoretic: games will terminate in a finite number of steps with atoms, then one looks at the model, if the atom is valued as true, then P wins, otherwise, she loses.) There is, however, no rule for material implication '$\varphi \rightarrow \psi$', which is simply defined classically as '$\neg\varphi \vee \psi$'. Since negation is a sort of 'responsibility-shift', these further rules add no new 'semantic' idea to that contained in the clauses for the quantifiers, conjunction, and disjunction.

Hintikka has made numerous controversial claims for his game-theoretical semantics, e.g., that it is non-compositional, and for the Independence-Friendly Logic that covers this fragment of second-order logic, including an ambitious plea for re-thinking of the very nature of logic in *Principles of Mathematics Revisited* [Hintikka 1996]. I shall not discuss these further. One should note, however, that Lorenzen also made radicals claim concerning his games. He was a logical monist and his original intention was to provide philosophical foundations for intuitionistic logic, i.e., Heyting semantics. (More precisely, Lorenzen was expressly hoping to recover Beth's tableaux rules for intuitionistic logic [Barth & Krabbe 1982, 12-13].) I quote from his 1968 John Locke Lectures:

> Philosophically there is no reason to start with the historical fact that Heyting published a certain calculus or to look for an interpretation of that calculus. It is however, reasonable to start with material dialogues, to formalize this game, to look for admissible rules for winning-positions; this procedure leads us directly to an interpretation of the Gentzen calculus and then indirectly to an interpretation of the Heyting calculus. I would claim, therefore, that the dialogical approach justifies the logical intuitions of Brouwer and Heyting. [Lorenzen 1969, 39]

His claim was thus that his dialogical approach justifies Heyting semantics and not, conversely, that Heyting semantics justifies his choice of rules. This is indeed a rather ambitious claim (to which I shall come back). The equivalence theorem necessary between proofs in Gentzen's calculus for intuitionistic logic and strategies for winning dialogues was obtained only in 1985 by Walter Felscher [Felscher 1985], at the end of a long search for the right set of restrictions on structural rules for dialogues needed to obtain intuitionistic provability. Kuno Lorenz had in the meantime realized that a slight variation in one of the structural rules would give classical logic [Lorenz 1968] – yet another point to which I shall come back.

After a period of neglect, dialogical logic has enjoyed a revival recently, when Andreas Blass first proposed [Blass 1992] to use Lorenzen's ideas to provide a semantics for the then newly invented linear logic of Jean-Yves Girard [Girard 1987]. Blass's paper sparked numerous developments, with new competing semantics: Hyland-Ong games [Hyland 1997] [Hyland & Ong, 2000], Abramsky games [Abramsky & Jagadeesan, 1994] [Abramsky 1997, 2006], Japaridze games [Japaridze 1997], and even further logical developments, with Japardize's computability logic [Japaridze 2003], and Girard's 'ludics' [Girard 2001]. Game semantics allows one to provide semantics to a variety of logical systems and programming languages, and has thus emerged as a new paradigm within computer science. However, while computer scientists might have perfectly good reasons for turning to game semantics, the idea is not really picking up within philosophical circles. The obvious reason for this is that better-known paradigms, for example, truth-conditional semantics, have more firmly established pedigrees. Philosopher won't budge until they are shown that, in some sense, game semantics is a better alternative and they will only shrug their shoulders when pointed out that, e.g., it allows for the construction of syntax-independent, 'fully abstract' models for programming languages. Some prejudices definitely need to be overcome before game semantics is to displace its rivals in their minds. Some objections are devoid of any merit, such as the claim, often voiced, according to which dialogical games are needlessly complicated: Lorenzen games amount merely to reading proofs in Gentzen's natural deduction systems upside down [Lorenzen 1987, 81 & 96], and one can learn to do so probably as easily as one learns how to drive on the left side of the road, once one has learned to drive on the right side. From a philosophical point of view, however, more reticence need to be overcome, so the main task is to provide a coherent, believable story for seeing logic in terms of dynamic interaction between two players, in other words, one must give not a contrived but a natural answer to the question: why play logical games? The point of playing a game is to win, but what is a proponent doing when trying to win a logical game? What is the motivation for the opponent in these games? These questions have been raised recently by Wilfrid Hodges:

> In most applications of logical games, the central notion is that of a winning strategy for the [proponent]. Often these strategies (or their existence) turn out to be equivalent to something of logical importance that could have been defined without using games – for example a proof. But games are felt to give a better definition because they quite literally supply some motivation: [the proponent] is trying to win. This raises a question that is not of much interest mathematically, but it should concern philosophers who use logical games. If we want [the proponent's] motivation in a game *G* to have any explanatory value, then we need to understand what is achieved if [the proponent] does win. In particular we should be able to tell a realistic story of a situation in which some agent called [the proponent] is trying to do something intelligible, and doing it is the same thing as winning in the game. [Hodges 2004, § 2]

Hodges's question is thus a request for a description a realistic situation in which the proponent is trying to do something which is the same as winning in a logical game. An answer to it is rather important, as it is from this story that the 'particle' and 'structural' rules should emerge, so to speak. It is also not just the obvious prerequisite to any attempt at convincing sceptics about the value of game semantics, it strikes right at the heart of claims made on the behalf of logical games, such as Lorenzen's claim that his games justify Heyting semantics, and not vice-versa, or Hintikka's proposals to reform logic: if no good answer is forthcoming, these claims will simply fail to convince, as they have done so far. In the remainder of this paper, my task will be to assess available answers and to provide a new one.

At the moment, there are only two answers to Hodges' question in the literature, given by… Lorenzen and Hintikka, and Hodges has provided criticisms of both [Hodges 2001], [Hodges 2006]. He took a rather stern view, concluding some harsh comments on Hintikka by saying that "it is a little disappointing that nobody took the trouble to look for a better story"

[Hodges 2004, § 3]; Lorenzen does not fare better: "it turns out to be embarrassingly easy to make mincemeat out the fine details of Lorenzen's claims" [Hodges 2001, 22]. Hodges is at any rate not looking forward to be convinced: "each claim of this kind needs its own deconstruction" [Hodges 2001, 25]. As it is not possible fully to discuss both programmes here, I shall focus in what follows on Lorenzen games, keeping remarks on Hintikka games to a minimum. At all events, I have already given reasons to reject Hintikka's answer – especially concerning his use of Wittgenstein's notion of 'language-games' in that context –, and I shall not repeat them here [Marion 2006].

In the case of Hintikka, the problem of finding a convincing story is reduced to that of providing a story for the quantifiers, since, as I have shown, his only central semantic idea concerns the quantifiers. This is why Hintikka introduced his 'language-games' of 'seeking and finding' [Hintikka 1973, chap. 3]. It seems to me that the main objection to Hintikka's answer was already put forth by Neil Tennant in the 1970s [Tennant 1979]; I shall briefly rehearse it. As I pointed out, Hintikka extended his game-semantic reading to 'branching' or 'Henkin quantifiers'. When we wish to say that for all x there is a y and that for all z there is a w, such that $A(x, y, z, w)$, the usual notation is inappropriate, because we want the choice of y to depend on x and the choice of w to depend on z, but according to the usual conventions about scope, the expression

$$(3)\ \forall x \exists y \forall z \exists w\, A(x, y, z, w)$$

makes the choice of w depend not only on z but also on x. To express this, one needs 'branching' quantifiers, for which Hintikka devised a 'slash' notation:

$$(4)\ (\forall x)(\forall z)(\exists y/\forall z)(\exists w/\forall x)\, A(x, y, z, w)$$

The slash in '$\exists w/\forall x$' means that the choice of w is made independently of that of x. Here too, there is a corresponding second-order formula:

$$(5)\ \exists F\ \exists G\ \forall x\ \forall z\ \ A(x, F(x), z, G(z))$$

where functions 'F' and 'G' will provide the winning strategy for the proponent. Again, I must skip here discussing further claims by Hintikka, e.g., about non-compositionality for these games, so that Tarski's well-known truth definition would not provide a semantics, etc.[4] One should merely note that the proponent's winning strategy is provided here by a set of (Skolem) functions for which one can merely claim 'existence' in the classical, non-constructive sense of the term. This means, therefore, that the proponent cannot be said in such cases to know the set of functions that provide her with a winning strategy. In other words we are talking about playing a game for which the proponent has a winning strategy but could not really be said to be playing the game, because she has no available knowledge of the functions that she would need to apply in order to win the game, i.e., no knowledge of the strategy. To put it bluntly, she won without really knowing how she did it. Although Hintikka remains undeterred, e.g., at [Hintikka 1998, 171, n.34], it thus seems hardly to make sense, in light of Hodges' question, to speak of asserting a sentence which is true in reference to a game for which a winning strategy exists but cannot be known to the proponent. This is essentially the point made by Tennant, when he pointed out that "no person could apply these functions in a way that exhibits strategic intent" [Tennant 1979, 305].

This critique does not nullify Hintikka games in the least; if anything it shows that Hodges was right in finding Hintikka's original motivation unsatisfactory, but it leaves the door open to another, rather promising approach, which looks at these games as modelling cases where players have to cope with imperfect information, e.g., the proponent has to make a choice without knowledge of the opponents previous move.[5] This is the direction in which Hintikka games have already evolved, in particular in the work of Gabriel Sandu and Ahti-Veikko Pietarinen,[6] and Johan van Benthem has shown how it overlaps further with game theory and with recent developments in dynamic epistemic logic [van Benthem 2003, 2006]. There is thus a new answer to Hodges' question in gestation here, and this approach might provide a better understanding of these games and their applications.

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Lorenzen's answer to Hodges' question also has its own difficulties.[7] Like many logicians, Lorenzen felt dissatisfied with the usual Tarski-style semantical definitions, e.g, 'A&B is true if and only if A is true *and* is true' and 'A&B is false if and only if A is false *or* B is false', since these presuppose the availability of a metalinguistic 'and' and 'or' [Lorenzen 1987, 60 & 88]. As Jean-Yves Girard once put it: to understand Tarski, you need 'Mr. Metatarski', and so on [Girard 1999, sec. 23.]. He was thus looking around for a strict foundation and found what seems to be his key idea (and that of the Erlangen School, which spawned around his work), in the

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4 Hodges has given a compositional semantics in [Hodges 1997], but see also [Abramsky 2006]. For a discussion of this issue and further references, see [Hodges 2006].

5 There is a very brief suggestion to that effect in [Marion 2006, 268].

6 E.g., [Sandu & Pietarinen 2003].

7 The following remarks are not based on a exhaustive review of the literature, and I owe an apology to German readers for my excessive reliance on a handful of English translations. (For example, [Lorenzen 1987] is a translation of [Lorenzen 1968] and [Lorenzen 1974], along with some papers, including [Lorenzen 1982].) The key figure in Lorenzen's intellectual background is an obscure philosopher of science, Hugo Dingler, whose books are extremely rare in North America. As was certainly typical of German professors of that generation, Lorenzen founded a 'school', the Erlangen School, which played a considerable role within German academia around the 1970s, so there is an extensive literature to cover; alas, their books are also hard to find outside Germany, where Lorenzen and his epigones had not much of an audience. So, what follows must be taken *cum grano salis* and should be taken also as a plea: even if one is, as I am, not sympathetic to the Kantian overtones, one ought to investigate further this half-forgotten literature, it is a rich load. For a short overview and useful bibliography of the Erlangen school, see [Gethman & Sigwart 1994], one should also consult the papers collected in [Butts & Brown1989], but beware of the unsympathetic, biased overview in [Bubner 1981, 142-153].

philosophy of Hugo Dingler.[8] Dingler's ideas can be expressed in terms of Hans Albert's 'Münchhausen Trilemma' [Albert 1985, 18], according to which any attempt at a foundation is bound either to lead to an infinite regress, or to circularity, as one presupposes what one wishes to ground, or to end arbitrarily, "in the middle". Dingler, who had anticipated Albert's Trilemma, chose this last option [Dingler 1931, 21 & 1955, 97] [Albert 1985, 19n. & 41f.]. (Lorenzen follows him [Lorenzen 1987, 16].) Dingler asked that our scientific discourse be methodically reconstructed 'from the ground up', step by step – every step being only constructed on the basis of steps already carried out, to avoid circles [Dingler 1964, 26] – so that it could be open to rational discussion. But he rejected the sort of reductionist programmes that are now associated with analytic philosophy – of which, in his times, Carnap's *Aufbau*[9] was a typical example –, i.e., he rejected the idea that the vocabulary of physics can be reduced to an empirical base vocabulary, e.g., phenomenal or physicalist language. According to him, the buck stops at operations: "all sciences must have their ultimate basis in the theory of action" [Dingler 1931, 32]. He asked further that we look instead at a grounding 'in the middle', in our 'civil life', i.e., in an hypothetical state of scientific innocence.[10] To take the example of geometry, what we know about space is said to depend on operations performed within this *Lebensstandpunkt* [Dingler 1964, 42]. The same goes for measuring time (chronometry) and these operations will serve as the basis from which physics can be reconstructed. Dingler believed that he could thus show that the axioms of Euclidean geometry are the only *operationally* true ones, hence his life-

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[8] For Dingler's bibliography, see [Schroeder-Heister 1981].

[9] On the relations between Carnap and Dingler, see [Wolters 1985].

[10] One should note *en passant* that, although Dingler rejected attempts at a transcendental foundation, his programme is here related to Husserl's project in App. III to the *Crisis of European Sciences* on the origins of geometry[Husserl 1970], i.e., to Husserl's claims about the possibility (and the necessity in his times) of re-activating the evidences on which the first geometers build geometry – its 'proto-foundation' –, evidences whose validity trickles down the chain of logical inferences. On the relations between Husserl and Dingler, see [Wolters

long opposition to Einstein’s relativity theory, which requires the validity of a non-Euclidean geometry.[11]

Influenced here by Oskar Becker as well as Hugo Dingler, Lorenzen originally proposed a reconstruction of mathematics on an operational basis, i.e., as mechanical operations on strings of symbols in accordance to given rules [Lorenzen 1955]. For example, the basic arithmetical operation is counting, numerals being constructed by the operation:[12]

$$\begin{array}{r} | \\ n \rightarrow n| \end{array}$$

On this he grafted his innovative construal of logic in terms of games. Thus, as with Dingler, the rational reconstruction of logic will have as an arbitrary starting point the activities of a ‘prelogical’ life-world, from which one can eventually extract (after going through steps concerning predication, etc.) *the* ‘particle’ and ‘structural’ rules of the dialogical games for *the only operationally true logic*, intuitionistic logic. This is why Lorenzen argues that his rules are abstracted from what he called our ‘practical nonverbal activity’ (*die Praxis unseres sprachfreien Handelns*) or our ‘prelogical speech practice’ (*vorlogische Redepraxis*) – expressions which he obviously got from Dingler.[13] For example, Lorenzen writes:

> In the context of a specific practical activity any normal person can learn how to use sentences of, say, the form N [does] P or N [is] Q […] We learn this kind of sentence and the words that appear in them exemplarily. In this way we have a speech practice that is ‘justified’ within the context of practical activity. This is what Bühler called *empractic* justification. Only by participating in an

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1991]. The “constructive philosophy” of Lorenzen and of the Erlangen School has been characterized as “phenomenology after the linguistic turn” [Gethman & Siegwart 1994, 228].

11 Dingler mounted a failed challenge at the 86th *Naturforschersammlung* held at Bad Neuheim in 1920, which was, as it turns out, a turning point in German physics, as relativity theory was on that occasion finally adopted by the German physicists and Dingler became isolated; this is the beginning of numerous professional problems, that led eventually to his siding with Lenard’s *Deutsche Physik*, with all the obvious consequences. A similar but slightly different operational reconstruction of geometry and chronometry was carried out in the Erlangen school, see, e.g., Janich’s ‘proto-physics’ of time in [Janich 1985], or, for geometry the essays in [Lorenzen 1987, part vi].

12 Since there are only operations on signs, and no impredicative definitions, the ‘operative’ mathematics developed in [Lorenzen 1955] and, further, in [Lorenzen 1971] stands closest to Weyl’s predicativism.

13 There are of course many small but important differences between Dingler’s and Lorenzen’s programmes. See [Janich 1985, chap. 2] for a detailed presentation.

> activity do we acquire the speech "appropriate" to that activity. We learn by practice what it is to assert propositions or to context the affirmation or denial of propositions (e.g., by nodding or shaking one's head. We introduce a negator, ¬, where $\neg a$ is used to express that we are contesting a proposition *a*. [Lorenzen 1987, 83]

Lorenzen also presents his game rules as a 'normalisation' or 'regimentation' of conversational moves in the *Lebensstandpunkt* and Hodges objected to this that Lorenzen's notions of 'attack' and 'defense' cannot be lifted from a 'prelogical speech practice'. In support, he gave three arguments, looking at particle rules and trying to show that what the opponent does cannot be really construed as mere attack, sometimes it looks as he is doing a bit of helping.[14] The third argument has to do with the fact that in a dialogue, you can attack a claim either by arguing that it is not true or that it is useless for further deductions; Lorenzen is supposedly overlooking this second case, which Hodges illustrates by quoting from Strindberg's *Miss Julie*:

> Jean: If you take my advice, you'll go to bed.
> Julie: Do you think I'm going to be ordered about by you? [Hodges 2001, 24]

As a rejoinder, it is often claimed that Lorenzen's rule for '$\varphi \rightarrow \psi$', i.e., that *O* then asserts φ and *P* must defend ψ, is implicitly followed in most cases, but Hodges point is that there are *other* cases where another rule is followed, here that *O* merely refuses to draw any conclusions from '$\varphi \rightarrow \psi$'. I am not going to argue for or against this point,[15] as mere doubting is here sufficient for my case. What feature of our 'prelogical speech practice' forbids us to treat '$\varphi \rightarrow \psi$' classically as '$\neg\varphi \vee \psi$'? As I already mentioned earlier, the difference between classical and intuitionistic logic

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[14] There are suggestions that Lorenzen did not exactly see things the way Hodges portrays him, e.g., when Kamlah and him point out that players are "not discoursing against one another in order to carry their point, but rather with one another, so that in working together they may come up with true sentences", this being illustrated by the alleged move from the 'eristics' of the Sophists and the 'dialectics' of Socrates [Kamlah & Lorenzen 1984, 142]. This is a very interesting suggestion in itself but it is not clear if this is a correct representation of the difference between Socrates and the Sophists. At any rate, one should note that, assuming the distinction between the level of games and the level of strategies [Rückert 2001, 175-177], any collaboration should occur at the level of strategies, while the games should remain fully agonistic. (I owe this point to Helge Rückert, in conversation.)

[15] There is obviously more to say here, see note 24 below for a comment.

hangs in Lorenzen's games merely on the difference between two structural rules. These are [Rückert 2001, 168]:

Intuitionistic rule: Each player can either attack a (complex) formula asserted by his adversary or defend herself against the last attack that has not yet been answered.

Classical rule: Each player can either attack a (complex) formula asserted by his adversary or defend herself against any attack, including those already defended.

Indeed, with help of this last rule, one can prove easily '$\varphi \vee \neg\varphi$', but it is as easy to show that there is no winning strategy when the first rule is applied. Here are two games, the left-hand one uses the intuitionistic rule:

P: $\varphi \vee \neg\varphi$	*P*: $\varphi \vee \neg\varphi$
O: ?	*O*: ?
P: $\neg\varphi$	*P*: $\neg\varphi$
O: φ	*O*: φ
	P: φ

Note that in both games, *P* answers the challenge choosing $\neg\varphi$ as she cannot assert an atomic formula. On the left-hand game, *P* ends up with no move and loses, so there is no proof of '$\varphi \vee \neg\varphi$'. On the right-hand side, *P* is now allowed to reply again to the first challenge by stating φ, as it is already asserted by *O*, who has then no more moves and loses.

But how can this first rule be convincingly said to be anchored in the *Lebensstandpunkt*? If the difference between classical and intuitionistic logic is the prohibition of repeated attacks, where is a justification for this to be found? I would like simply to point out that even the very possibility of such objections shows at least this that one cannot so simply lift *one* set of rules from this hypothetical state of 'prelogical speech practice'. This much goes at least against Lorenzen's monism. (I shall come back to this point below.) But one could push this point further: it is not clear how can one define a *pre*logical – let alone pre*verbal*! – state where conversations take place from which one could extract not just one specific set of rules but any rules. The notion of prelogical conversations itself may very well be incoherent. The idea that logic is already included into the bargain, so to speak, with the ability to deploy any language or to controvert is tempting. But how one could cash it out in such foundational terms for logic is simply not obvious.

This is not an argument against Lorenzen games *per se* but against the story he provided as a motivation for them, i.e., against his answer to Hodges' question. I would like to give further emphasis to this critique by a short digression concerning Karl Bühler's notion of 'empractic' speech, alluded to by Lorenzen. Here is another passage:

> You can play ball without using words. In this prelinguistic activity we can 'empractically' – as Bühler called it – define the use of simple words. […] I trust you can easily imagine the sort of practical situations in which Leo would utter *imperative sentences* like the following:
> Throw!
> Throw ball!
> Mao! Throw ball!
> or *indicative sentences* like:
> Ball does fall
> not: Mao does throw
> […] The sentence forms that have been 'empractically' justified to this point can be extended further in various ways before we introduce logical operators. [Lorenzen 1987, 139-141]

In his 1934 book, *Theory of Language*, Bühler put forth an extended version of the 'context principle'. It is 'extended' because Bühler's notion of context (*Umfeld*) is not merely linguistic (as it would be for, say, Frege), it is also non-linguistic, in which case it is said to be either physical or behavioural; the latter is called 'empractic'. A typical case of empractic speech occurs when, sitting in a Viennese café, I see a waiter coming towards me and utter to him: '*einen schwarzen*', and he comes back a minute later with a black coffee [Bühler 1990, 178]. One must be careful in delineating Bühler's point here. In that passage, Bühler argues that in uttering '*einen schwarzen*' I do not mentally go through a sentence such as 'Please bring me a black coffee' – a point for which Wittgenstein is famous [Wittgenstein 1997, §§ 19-20] –,[16] that one can always construct such a sentence does not prove anything [Bühler 1990, 178]. He had already argued, with help of similar examples, that not all language signs are symbols, some are *signals* [Bühler 1990, 122] – in Wittgenstein's words, they are a different tool in the tool-box of language [Wittgenstein 1997, § 11] – and that such forms of speech are neither impoverished, nor incomplete [Bühler 1990, 122], this last being a point also made by Wittgenstein [Wittgenstein 1997, § 18]. How could this relate to rules for Lorenzen games? Let us, for the sake of the argument, grant these points made by Bühler (and Wittgenstein). One will notice that they aim at showing that *some* utterances in *some* contexts are not assertions of elliptic versions of declarative sentences – although they might look like it – but of an altogether different nature from assertions. But logical games are about evaluating assertions in terms of their logical form so there is strictly nothing that one would say about what is specific to 'empractic' forms of speech, which do not count as assertions, that could count as grounding rules for logical games in a prelogical state. Furthermore, even if one recognizes the validity of the Bühler-Wittgenstein point that, in uttering '*einen schwarzen*' I do not mentally go through a sentence such as 'Please bring me a black coffee', it remains that such uses of language can be seen as parasitic, because they rely on more fundamental ones such as the use of declarative sentences. Indeed it is not clear how one could claim that the use of '*einen schwarzen*' as 'signal' could stand on its own, without presupposing that a convention for it was already established with help of assertions. In these conditions, can the convention of shaking one's head really be said to be primary, i.e., can it be used as the *ground* for the logical meaning or use of '$\neg$'? Lorenzen's idea of founding his

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[16] Kevin Mulligan has shown in the most interesting paper on this topic [Mulligan 1997], that Bühler's theory is of great help to understand properly the language-game of builders at the beginning of *Philosophical Investigations* [Wittgenstein 1997, § 2].

game rules on an hypothetical prelogical or even preverbal state looks dangerously like an *hysteron proteron*. Of course, these remarks do not settle the debate – Lorenzen would probably dismiss the foregoing as *verbalen Nebel* [Lorenzen 1982, 29] – but, as Hodges said, each attempt at answering his question needs its own 'deconstruction' and Lorenzen's attempt at a 'foundation' in a *Lebensstandpunkt*, with help of Bühler, does not appear very promising.

As it turns out, however, Lorenzen did not claim to lift so straightforwardly his game rules for the prelogical, 'empractic' context of the *Lebensstandpunkt*. He claimed instead that in the prelogical one may learn 'rules of transition' which would legitimate in turn the introduction of the game rules:[17]

> Rules of transition [his example here is the Modus Ponens] in which we must affirm the conclusion if we have affirmed the premises are not logical rules. They are *prelogical*; they provide a set of practical linguistic activities, a set of linguistic practices, which, under rather complicated circumstances, justify the introduction of operators invented expressly for these linguistic practices, that is, logical operators. [Lorenzen 1987, 83]

The domain of these rules of transition, to which one needs merely to add the above rule for numerals in order to build mathematics (minus geometry) [Lorenzen 1987, 69-70], is called 'protologic' [Lorenzen 1987, 61 & 67]. With it, we reach what seems to be the heart of his position – I shall present it in very succinct terms –, which consist in seeing logical inference in dynamic terms, as an action or operation with linguistic signs according to a rigid schema [Lorenzen 1987, 61]. A calculus can therefore be described in terms of a finite number of 'atomic figures' and 'schematic operations', restricting ourselves to 'conditional imperative' of the sort 'If such and such figures are produced, then make the following figure'. As an example, Lorenzen proposes + and o as atomic figures and these rules, with a variable *a* for already constructed figures:

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[17] This point seems to have been missed in [Hodges 2001], which concentrates on [Lorenzen 1982], where it is not clearly explained.

(i) $a \rightarrow a$o
(ii) $a \rightarrow +a+$

From this one ask about the deducibility of a figure, e.g., +++ooo. One may further ask if adding new rules such as

(iii) $a \rightarrow +a$o$+$

would produce figures that could not be merely obtained from (i) and (ii). Lorenzen argues that this can only be demonstrated by showing how to eliminate all instances of the use of (iii) in favour of derivations using only (i)-(ii); he does this by showing how a proponent, Fritz, could always do so in deductions made by his opponent, Hans. (Although he does not use this expression, the addition of the new rule would thus produce a 'conservative extension'.) It is the theory of these elimination procedures which he wishes to call 'protologic' [Lorenzen 1987, 67], and for which he claims:

> From a philosophical perspective we should take particular note of the fact that certainty concerning such assertions [of the admissibility of rules] is only another expression for the certainty that we possess the ability to perform certain actions (here the elimination of a rule). We have achieved here a complete overview of the relation between action and knowledge – in any event, of the nature and manner of the interpretation of the two. [Lorenzen 1987, 66-67]

Now, according to (i) and (ii), we have

$a \rightarrow a$o
ao$\rightarrow +a$o$+$

Assuming transitivity, we get

$a \rightarrow +a$o$+$

which is (iii), and we now read '→' as logical implication. In other words, "if a calculus contains the rules $A \rightarrow B$ and $B \rightarrow C$, then $A \rightarrow C$ is admissible", so one can introduce as a principle

$$A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$$

on the basis of which one could justify the introduction of logical rules – presumably 'particle' rules [Lorenzen 1987, 67]. (Again, although Lorenzen does not speak in those terms, he is simply pointing out that a cut-elimination theorem is needed for the admissibility of new connectives.) He can thus finally arrive at his justification of intuitionistic logic :

> In fact, the usual logic can be operatively – that is on the basis of schematic operations – interpreted this way. With the exception of negation, everything is exactly as it is in the classical theory. For negation we have, in contrast, at first only intuitionistic logic – with which, however, we know that we can justify two-valued logic as at least a fiction. [Lorenzen 1987, 68]

One will have recognized here the parallel with the better known argument by Sir Michael Dummett and Dag Prawitz, based on the latter's 'inversion principle', which purports to show the more 'natural' character of intuitionistic logic because the elimination rule for double negation in classical logic does not respect this inversion principle [Dummett 1991, chap. 9], [Prawitz 1965, 1977]. My qualms here do not concern this argument *per se* – that would require writing another paper – although one should note that it is not clear how this would serve a justification of the intuitionistic over the classical rule, stated above, but Lorenzen's *philosophical* claim, in the last but one quotation, that assertions about the admissibility of rules are arrived at on the basis of a *prior* ability to perform actions characterized as 'elimination of a rule'. (In effect, Lorenzen is telling us something dubious like this: $\rightarrow$ is arrived at on the basis of an operational account of $\vdash$.) We are supposed to be here in the domain of 'protologic', i.e., our 'prelogical speech practice', but Lorenzen had accounted for the eliminability of rules in terms of a dialogue between Fritz and Hans. Whence this dialogue? Which rules are followed? Is winning a dialogue here a 'proof' of eliminability? I, for one, have no idea how the term 'empractic' would apply here, I can find no plausible explanation, and probably there isn't any because there is no such thing as antecedent 'prelogical' state from which one could learn to graft logical rules, because in order to get to learn the schematic operations, one already needs logic, the same way that in order to learn the rule to be applied when ordering a coffee in a

Viennese café, one already needs to master the use of assertions. I take it that Hodges meant something similar when he commented:

> [Lorenzen's] intention was to build up a logic from the notions involved in a *pre-logical* Redepraxis. Whatever these notions are, they surely don't include that of a proof. [Hodges 2001, 25]

Here, to reconstruct step by step cannot amount to giving the expected sort of 'foundations'. Seeing logical inference in dynamic terms as action or operation is a pregnant idea, but it does not have to stand and fall with this Dinglerian attempt at anchoring it in the *Lebensstandpunkt*.

Before leaving the issue, a brief remark about logical monism. I have given reasons for being sceptical of Lorenzen's monism, but this is, again, not an argument against his games, only against his philosophy. Indeed, recent work on Lorenzen games has moved into this direction: Shahid Rahman and his collaborators has shown (e.g., [Rahman & Keiff 2005]) that one could keep, on the one hand, the particle rules invariant and vary the structural rules, on the other, and obtain a formalisation of numerous known logics. This is known in the literature as the 'Dosen principle' [Rahman & Keiff 2005, sec. 1] [Rückert 2001, 177]. Alternatively, one may simply introduce new connectives, as one does in relevance or in linear logic; the principle here is sometimes known as 'Girard's principle' [Rahman & Keiff 2005, sec. 1]. The distinction between 'particle' and 'structural' rules thus allows one to generate new logics by systematic variation and combination of 'particle' and 'structural' rules. One can thus see that, *pace* Lorenzen, the dialogical approach provides a framework for logical pluralism.[18]

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As Hodges also said: "If games don't occupy quite the roles that Lorenzen and Hintikka have sometimes claimed for them, then it behoves us to try to find what roles they do occupy"

[Hodges 2001, 25]. Consequently, I shall now propose another answer (thus incurring the risk of a 'deconstruction') or at least suggest what a proper answer might look like, as I am to fall short of cashing it out in terms of a specific set of game rules. I shall get to my answer first by a brief historical detour, taking my cue from some other things Lorenzen said.

In his paper 'Logik and Agon' [Lorenzen 1960, 187], as well as in a number of other places,[19] Lorenzen referred *en passant* to the practice of refutation or 'dialectics' in Ancient Greece as both the original motivation for the development of logic and as a source for dialogical logic. This suggestion was not, as far as I know, followed by the scholarly investigation that it clearly deserves.[20] At all events, one should merely recall here a few facts. The Ancient Greeks had indeed developed a variety of sophisticated forms of question-answer dialogues in medical, legal, political, scientific and philosophical contexts. In philosophy, the Socratic method is well-known from Plato's dialogues, but 'dialectics' was already developed and used by Eleatic philosophers (e.g., Parmenides and Zeno) and the Sophists prior to Socrates. A set of rules was also described later by Aristotle in Book VIII of the *Topics*. In one particularly well-known variant, which fits Zeno's arguments, a designated proponent had to defend a given thesis φ, and the opponent's task was to lead the proponent to admit successively a number of claims ψ_1, ψ_2, …, ψ_i from which one could then force the proponent into an *elenchus*, i.e., to derive $\neg\varphi$ and contradict himself. The point is thus is to refute φ by showing that it leads to a contradiction (or to an absurdity, or to a plainly false statement in other variants). Assuming the principle of non-contradiction, one can devise an indirect proof: for any assertion φ, if propounding $\neg\varphi$ leads to a

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[18] Logical pluralism is already advocated in [Rahman & Keiff 2005] and [Rückert 2001]. For a recent plea for logical pluralism not from the viewpoint of game semantics, see [Beall & Restall 2005].
[19] E.g., [Kamlah & Lorenzen 1984, 142], [Lorenzen 1987, 78].

contradiction, then φ must be true. This method was used and the principle asserted (e.g. by Gorgias) well before Socrates and Plato.

Now, why would such disputes take place? There seem to be a natural answer for this, which is already stated in Aristotle's definition of 'dialectics' in *Topics*, I, § 10 as a dispute concerning assertions not known to be true or necessary – or as they were called, *hypotheses*. Assertions can be made that are directly verified or that are at least verifiable in principle. But in the case of metaphysical-cosmological truths or moral truths, as well as in mathematics, no such direct verification is even *in principle* possible. To take only one basic mathematical example, the observation that there are prime numbers spread throughout the natural number series as far as one can tell leads naturally to the question of their infinity, but the assertion 'there exists an infinity of prime numbers' is not verifiable by sifting through that infinite series, e.g., with help of the sieve of Eratosthenes. But one could ascertain it by use of an indirect proof, as Euclid famously did.[21] In the case of Eleatic philosophy, the situation was rendered even more acute by the proscription of appeals to verification by the senses (probably dictated by the wish to refute Heraclitus); thus, simply walking from one point to another could not count for an Eleatic as a refutation of Zeno's arguments against the reality of motion. (In Lorenzen's terms, Eleatic philosophers would only agree to play 'formal games'.) The point of playing these early forms of logical games was obviously to try and sort out good from bad assertions. If the proponent of φ was publicly driven into an *elenchus*, then φ would be dropped but if he successfully defended it, the result would not merely be that his skills would be admired by members of the audience, it

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[20] Indeed, numerous received views about the origins of logic are to be cast into doubt, and this should provide further grist to Lorenzen's mill. A programmatic presentation will be found in B. Castelnérac & M. Marion, '"Presocratic" philosophy and the Origins of Dialogical Logic', to appear.

would also entitle them to adopt φ for themselves.[22] So, for example, assertions such as 'the "one" is indivisible', became accepted as true, while the hypothesis that the diagonal of a square is commensurable with its side was found to lead to a contradiction and dropped.

There is a lot more to say here, but my point is merely to indicate that Greek dialectics already contains elements of an answer to Hodges' question. These elements can be systematized and given a more general foundation using the theory of assertion developed in the chapter on 'Assertion' in Sir Michael Dummett's *Frege. Philosophy of Language* [Dummett 1981, chap. 10] and in Robert Brandom's paper on 'Asserting' [Brandom 1983]. I shall not summarize these here, but simply extract what seems to me the central point of the Dummett-Brandom theory, within the context of this paper. The key idea is that we *act* on assertions and that for this very reason they better be not just true but be backed up with some justification. Of course, some are directly verifiable from the context but the majority of our assertions aren't. However, that does not mean that assertions should be made on no basis whatever, only that they might require justification. According to Dummett:

> we do not of course learn to make statements on no basis whatever, and, if we did, such utterances would not constitute assertions […], because there would not be such thing as acting on such statements. The process of learning to make assertions, and to understand those of others, involves learning what grounds, short of conclusive grounds, are regarded as justifying the making of an assertion, and learning also the procedure of asking for, and giving, the grounds on which an assertion is made. [Dummett 1981, 355]

Robert Brandom also views expressing claims as "bringing them into the game of giving and asking for reasons" [Brandom 2000, 57] and he has extended this analysis of assertions by introducing a distinction between the 'commitments' which a speaker takes on explicitly by

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[21] One should not forget here the wealth of arguments provided by Arpad Szabo in his controversial study *The Beginnings of Greek Mathematics*, devised to support the claim that Greek mathematics "grew out of the more ancient subject of dialectic" [Szabo 1978, 245].

[22] The democratic nature of these dialogues was first recognized by British radicals in the nineteenth-century, George Grote and Henry Sidgwick.

making an assertion or by assenting to someone else claim, and the commitments a speaker is 'entitled' to.[23] Thus, according to Brandom,

> In asserting a claim one not only authorizes further assertions, but commits oneself to vindicate the original claim, showing that one is entitled to make it. Failure to defend one's entitlement to an assertion voids its social significance as inferential warrant for further assertions. It is only assertions one is entitled to make that can serve to entitle others to its inferential consequences. Endorsement is empty unless the commitment can be defended. [Brandom 1983, 641][24]

Brandom on (Sellars on) Socratic method is also worth quoting in light of my above remarks about Greek dialectics:

> Socratic method is a way of bringing our practices under rational control by expressing them explicitly in a form in which they can be confronted with objections and alternatives, a form in which they can be exhibited as the conclusions of inferences seeking to justify them on the basis of premises advanced as reasons, and as premises in further inferences exploring the consequences of accepting them. […]. *Expressing* [claims] is bringing them into the game of giving and asking for reasons […] [Brandom 2000, 56-57]

The central point of the Dummett-Brandom theory of assertions can thus be stated as follows: an act of asserting a statement brings with it a commitment to defend the assertion, if challenged, so *to make an assertion is to make a move in a game, in which one is asked for and must provide grounds or reasons justifying the making of that assertion*. In other words, the 'game of asking for and giving reasons' is inscribed in the very nature of assertions. The notion of 'game' here can be given precise logical content in terms of the dialogue games first proposed by Lorenzen. My point is thus that *the Dummett-Brandom theory of assertions provides conceptual foundations for game semantics of the style first laid out by Lorenzen and, conversely, that game semantics can provide a logical precisification of this theory*.[25] At least the

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[23] There are other innovations, such as the introduction of perspectival commitment stores or 'deontic scoreboards', into which we need not go here.

[24] Incidentally, the claim that 'failure to defend one's entitlement to an assertion voids its social significance as inferential warrant for further assertions' could be used to explain away cases such as Julie's challenge, quoted in [Hodges 2001, 24]: 'Do you think I'm going to be ordered about by you?'. (See the above discussion.) She is simply issuing a challenge to Jean's entitlement to the conditional 'If you take my advice, you'll go to bed'.

[25] There are other reasons here why the 'Brandomian' might be interested in this game-theoretic alternative. First, one will have noticed that Brandom couches his argument for 'inferentialism' in terms taken from Gentzen and Dummett [Brandom 2000, chap. 1, sec. vii-xii]. But Brandom does not wish to follow Dummett in his use of the

avenue seems opened, as, to speak in Brandom's jargon, *it is as a matter of fact through such games that we make our reasons explicit*. Indeed, 'dialogical' semantics can be reformulated in terms of the 'game of giving and asking for reasons', so that 'to attack' becomes 'to ask for reasons' and 'to defend' becomes 'to give reasons'.[26] The point is to win the game, which is the same as succeeding in providing or 'making explicit' reasons for one's assertions. Here 'to make explicit' must mean 'to construct': by playing these games against each other, we entitle ourselves to truths by providing them with constructive winning strategies – a bit like playing chess games in order to find out which claims we are entitled to (and, by the same token, construct a justification).

Where would this leave us? The idea of providing an answer to Hodges' question along those lines seems so obvious, once stated, that one wonders why nobody came across it earlier. A quick search of the literature shows, however, that a one-time member of the Erlangen School, Friedrich Kambartel had already made a very similar proposal more than 25 years ago, by providing an account of the particle rules of Lorenzen in terms of games of assertions (here in a paper with Hans-Julius Schneider):[27]

> The need for assertions arises in situations where language competence is developed to the degree that action depends on correctly performed elementary statements, and where the participants do not agree on the correctness of such a performance. In this case one can either give up common orientation as provided by elementary statements, or one can try to overcome private opinions by

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requisit of 'harmony' in order to run the already-mentioned argument against classical negation. After all, Brandom is a proponent of logical pluralism, and a reformulation of his 'inferentialism' in game-theoretic terms, if possible, would allow him to avoid this difficulty. Furthermore, Lorenzen distinguishes between 'material' and 'formal' games, and he is, as far as I know, the only philosopher apart from Wilfrid Sellars (followed on this point by Brandom, e.g., in [Brandom 2000, chap. 2]) who believes that material implication is not just enthymematic. This point seems to me to be implicit in the fact that the point of material games is to justify implications by allowing the assertion of atomic formulas and the claim that "the material game has to be understood first […] With the formal game we are simulating material dialogues" [Lorenzen 1969, 35].

[26] Incidentally, one should notice here that in the Erlangen school attacks are also considered as 'rights' and defences as 'duties' [Lorenz 1981, 120]; we are thus not far from Brandom's normative vocabulary since equivalences obtain between 'right to attack' and 'asking for reasons' on the one hand and 'duty to defend' and 'providing reasons' on the other.

[27] See also [Kambartel 1979, 1981].

> reaching a new level of transsubjectivity, by *argumentation*. With argumentation we mean here, quite simply, all attempts to settle differences on the basis of previously or newly established agreements. […] Someone who now not only just states something, but *asserts* what he is stating, must be prepared to establish by argumentation a transsubjective agreement that his statement has been made correctly. Assertions are, on our everyday and scientific life, one of the language institutions, whereby we can rely on others in our orientations. […] Trivially the reliability of assertions is undermined if people make assertions without having the corresponding justifications at hand. [Kambartel & Schneider 1981, 169-170]

So, Kambartel formulated Lorenzen's particle rule in terms of attempting at reaching intersubjective agreement about the validity of assertions that are needed for common orientation [Kambartel 1979, 201-203; 1981, 406-4080, [Kambartel & Schneider 1981, sec. 7]. This corresponds essentially my proposal. However, would 'assertion games' simply look like Lorenzen games, with another spin? This is a question that will have to dealt with in another paper. But some brief tentative remarks are in order.

One should notice to begin with that Lorenzen games (as well as Hintikka games) deal merely with connectives that are also collectively known since Girard's introduction of linear logic [Girard 1997] as its 'additive' fragment: &, ∨, →, and ¬ (I skip the quantifier in what follows). Girard introduced 'multiplicative' conjonction Δ, disjunction ∇, and implication —o.[28] Should these also be accounted for? If they are not to be considered, one must give reasons other than the fact that one has the habit of living only with the additive fragment. I think, however, they should be accounted for, for the simple reason that additive connectives do not exhaust the possible meanings of the connectives; in other words, they do not cover all possible cases of playable assertions games.

To see this, let us look at, say, '$\varphi \vee \psi$'. Here, the game consists in *O* asking *P* to choose one of the disjuncts and the other is discarded. In other words, keeping the analogy with chess, *O*

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[28] I follow here in part the notation [Japardidze 1997], for no particular reason… The game-theoretic interpretation of negation already belongs to the multiplicative level [Abramsky 2006, 20], so I skip here the multiplicative counterpart of ¬.

asks *P* to choose which game they are to play. Now there are two natural ways to complete this picture of disjunction. First, one may imagine a case in which both games associated with the disjuncts are to be played, in a sequential manner, i.e., as in playing two chess games one after the other; it suffices for *P* to win '$\varphi \vee \psi$' that she wins only one of the two. Moreover, defences of the disjuncts as considered so far are independent from each other, there is no reason to limit ourselves to this, as one may also introduce multiplicative connectives Δ, ∇, —o, in order to account for cases when the games are played simultaneously and where there is a flow of information from one game to another. For example, with the new disjunction '$\varphi \nabla \psi$' (as *P* needs only to win one side to win the whole game), *P* may chose to play φ but is not obliged to discard ψ and may switch to it and back. Thus, one can, for example, capture the 'copycat' strategy,[29] which is so essential for computer programming: suppose that '$\neg\varphi \nabla \varphi$' is like *P* is playing two chess games against Deep Blue, so that Deep Blue plays white on the left-hand chess board while she plays white on the right-hand one. She could win by simply copying Deep Blue's first move, on the left-hand board, as her first move on the right-hand board and wait for Deep Blue's move, which she could then copy back on the left-hand board as her own reply to Deep Blue's first move on that board, and so on, thus insuring that she wins at least one of the two games. This is one of the reasons why, from a computational point of view, the multiplicative connectives are so important. But many philosophers will simply wish to part company here, saying that computer programming is of no concern to them. However, it makes little sense to deprive oneself from the multiplicative connectives since they capture game scenarios that, from a purely intuitive standpoint, make perfect sense. In terms of assertion games, where the point of playing is to construct a common justification, one cannot limit oneself to the additive fragment

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[29] See, e.g., [Abramsky 1997, 9] or [Japaridze 1997, 90-91].

(all the more so since, as shown in [Abramsky 2006], one may even generalize those games to cases with more than two players).

Furthermore, the problems with implication will not go away so easily: when '$\varphi \rightarrow \psi$' is simply defined as '$\neg\varphi \vee \psi$', as it is in the case of Hintikka games, and *P* chooses either $\neg\varphi$ or ψ, the other disjunct being discarded, so idea that information in φ must be carried over in the proof of ψ, so crucial in Heyting's semantics, is lost. This is not a problem for the classical logician, who does not care, but Lorenzen has introduced his rule for '$\varphi \rightarrow \psi$' to account for this. It is not clear, however, how faithful this is to Heyting, since there is no real information flow from the proof of φ to the proof of ψ. One should thus at least consider the rule for linear implication, '$\varphi \multimap \psi$', in effect '$\neg\varphi \nabla \psi$', where information provided in game/proof for φ can indeed be carried over by *P* in the game/proof of ψ. It seems 'natural' indeed that in a common process of constructing a justification for assertion '$\varphi \rightarrow \psi$', information given by *O* in the proof of φ can indeed be carried over by *P* in her proof of ψ. From a constructivist viewpoint, which is after all Lorenzen's, accounting for the multiplicative connectives might thus be desirable (at least the matter ought to be investigated further). Admittedly, these brief remarks do not amount to a fully-argued case, but, I hope, show that the idea that 'assertion games' must make room for multiplicative connectives cannot be so easily dismissed. If not, one should give real reasons (and not simple disguises for one's lack of interest).

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