

No-arbitrage Near-Cointegrated VAR(p) Term Structure Models, Term Premia and GDP Growth

Caroline JARDET ⁽¹⁾

Banque de France

Alain MONFORT ⁽²⁾

CREST, Banque de France
and Maastricht University

Fulvio PEGORARO ⁽³⁾

Banque de France, CREST and HEC Lausanne

First version : September, 2008. This version: June, 2011.

Abstract

No-arbitrage Near-Cointegrated VAR(p) Term Structure Models, Term Premia and GDP Growth

The recent macro-finance yield curve literature does not agree neither about term premia empirical properties nor about the importance or even the direction of its relationship with future economic activity. This paper proposes a two-step approach to handle both problems. First, in a VAR setting, we extract a reliable measure of the term premia by means of averaging estimators techniques aiming at optimally solving prediction problems when highly persistence processes are present and, thus, providing a so called Near-Cointegrated VAR(p) approach. Second, we analyze the dynamic response of the GDP to shocks on the term premia by using the New Information Response Function concept. First, we find that the NCVAR-based term premium measure is rather stable and contra-cyclical, with the expectation part accounting for most of the yield variability, which is consistent with the typical macroeconomic view. Second, we find that an increase of the long-term spread caused by a rise of a term premium induces two effects on future economic activity: the impact is negative for short horizons (less than one year), whereas it is positive for longer ones. Therefore, this result suggests that the above mentioned ambiguity could come from the fact that the sign of this relationship is changing over the period that follows the shock.

Keywords: Averaging Estimators, Persistence Problem, Near-Cointegration Analysis, No-Arbitrage Affine Term Structure Model, Term Premia, GDP Growth, New Information Response Functions.

JEL classification: C51, E43, E44, E47, G12.

¹Banque de France, Financial Economics Research Service [DGEI-DEMFI-RECFIN; E-mail: Caroline.JARDET@banque-france.fr].

²CREST, Laboratoire de Finance-Assurance [E-mail: monfort@ensae.fr], and Banque de France, Financial Economics Research Service [DGEI-DEMFI-RECFIN].

³Banque de France, Financial Economics Research Service [DGEI-DEMFI-RECFIN; E-mail: Fulvio.PEGORARO@banque-france.fr], CREST, Laboratoire de Finance-Assurance [E-mail: pegoraro@ensae.fr] and HEC Lausanne, Department of Finance and Insurance.

We received helpful comments and suggestions from David Backus, Michael Bauer, Mike Chernov, John Cochrane, Valentina Corradi, Pasquale Della Corte, Greg Duffee, Carlo Favero, Christian Gourieroux, Stéphane Grégoir, Lars Peter Hansen, Peter Hoerhdal, Scott Joslin, Sharon Kozicki, Nour Meddahi, Philippe Mueller, Monika Piazzesi, Mark Reesor, Jeff Russell, Mark Salmon, Ken Singleton, Eric T. Swanson, Fabio Trojani, Amir Yaron and sem-

1 Introduction

The recent macro-finance yield curve literature has focused on both the extraction of a reliable measure of term premia (on long-term bonds) and their relevant relationship with future economic activity. Indeed, both issues are critical for central banks. First, policymakers have to be able to identify accurately causes of fluctuations in long-term interest rates, notably whether they reflect changes in the expected path of the monetary policy rate (i.e. the short rate), or changes in risk compensations. For that purpose, they require a reliable decomposition of any long-term yield of interest into an expectation term and a term premium component. Second, whether or not changes in term premia affect future economic activity is also a key issue for policymakers, given the practical implications that this relationship has for the conduct of the monetary policy. More particularly, depending on the stimulating or shrinking effect of term premia on the growth rate of the Gross Domestic Product (GDP), a central bank can potentially modify its monetary policy intervention in order to achieve a given economic goal. However, despite a keen interest of academic research to these questions, empirical findings are still controversial.

As far as the extraction of the term premia is concerned, we observe relatively little consensus about their empirical properties. Indeed, while the typical macro wisdom requires a stable and contra-cyclical term premium with long-term yield movements mostly driven by the expectation component, recent papers by Kim and Orphanides (2005, 2007) and Beechey (2007) find, in sharp contrast, that the term premium measure has a large variability, the expectation part being rather stable. Other papers like Rudebusch and Wu (2008) and Bauer (2010) find, contradicting again the typical macroeconomic view, a term premium which is extremely stable and neutral with respect to the business cycle.

inar participants at the September 2008 Bank of Canada Conference on "Fixed Income Markets", October 2008 CREST Financial Econometrics Seminar, November 2008 Bank of France Conference on "Financial Markets and Real Activity", March 2009 University of Lugano (USI) Finance Seminar, April 2009 Warwick Finance Seminars, June 2009 Bank of France - Bundesbank Conference on "The Macroeconomy and Financial Systems in Normal Times and in Times of Stress", EFA 2009 Conference, ESEM 2009 Conference, Toulouse School of Economics Financial Econometrics Conference 2010, ASSA Meeting (Econometric Society) 2011 in Denver.

Regarding the link between term premia and future economic activity, some studies based on static regressions like those by Hamilton and Kim (2002), and Favero, Kaminska and Sodestrom (2005) find a positive relation between term premium and economic activity. In contrast, Ang, Piazzesi and Wei (2006), Rudebusch, Sack and Swanson (2007), and Rosenberg and Maurer (2007) find that the term premium has no predictive power for future GDP growth. Practitioners and private sector macroeconomic forecasters views suggest a relation of negative sign between term premium and economic activity. This negative relationship is usually explained by the fact that a decline of the term premium, maintaining relatively low and stable long rates, may stimulate aggregate demand and economic activity, and this explanation implies a more restrictive monetary policy to keep stable prices and the desired rate of growth. Because of the lack of agreement among the empirical findings, policy makers have no precise indication about the stimulating or shrinking effect of term premia on gross domestic product (GDP) growth [see Rudebusch, Sack and Swanson (2007), and the references there in, for more details].

This paper tries to propose a solution to both the optimal extraction of the term premium and its ambiguous relationship with economic activity by, first, extracting a reliable measure of long-term premia by means of a no-arbitrage Gaussian VAR(p) yield curve model and providing, then, a dynamic (instead of static, like in the above mentioned literature) analysis of the relationship between these two variables.

Coping with the first part of the problem asks for precise short rate forecasts over long horizons (by definition of term premia), and this requirement is particularly challenging once we realize the effects of the strong persistence of interest rates on the specification and estimation of VAR models. Indeed, on the one hand, the presence of "nearly non-stationary" processes in the VAR factor leads to impose cointegration relationships (unit roots) in the parametric specification (CVAR models) and, thus, associated forecasts at any horizon are very similar and close to the present value of the variable of interest. On the other hand, if we take into account the low power of cointegration/unit root tests against highly persistent alternatives, and we use an unconstrained VAR model, the ob-

tained forecasts tend to quickly converge to the unconditional mean of the variable. This sharp difference between CVAR-based and VAR-based long-horizon forecasts [see Cochrane and Piazzesi (2008)] automatically generates uncertainty about the reliability of the associated measures of the term premia. In addition, in finite sample, the magnitude of this difference is exacerbated by the well known "bias problem", that is the downward bias in the OLS estimator (of the unconstrained VAR model) induced, again, by the above mentioned huge serial dependence of yields [see Kim and Orphanides (2007) for a discussion of these problems]. In order to handle this problem, we have chosen what we name the Near-Cointegration approach: we estimate the stationary VAR dynamics (the data generating process) of our factor of interest (X_t) (say) by an averaging estimator *à la* B. Hansen (2010). More precisely, this methodology is based on averaging the estimates of a CVAR(p) and of the associated unconstrained VAR(p) model, and the averaging weight is obtained by minimizing the root mean squared forecast error of a variable of interest. Since our aim is to extract reliable measures of term premia on long-term bonds, this variable is chosen to be the function of future short rates appearing in the expectation part of the long-term yield of interest. This methodology seems to be particularly adapted to the term premia extraction given the promising forecast performances this kind of estimator has shown in Monte Carlo experiments as indicated by B. Hansen (2010) and confirmed by Jardet, Monfort and Pegoraro (2009a) in a successful comparison with bias-corrected estimators like Indirect Inference estimator, Bootstrap estimator, Kendall's estimator and Median-unbiased estimator ⁴. We thus specify and estimate a Near-Cointegrated VAR(p) yield curve model with stochastic market price of factor risk depending on present and lagged factor values. We find that our approach provides, in terms of root mean square forecast error, the best extraction of the 10-year term premium among the competing models. Coherently with the typical macroeconomic view and with empirically observed sustained countercyclical monetary policy actions, the NCVAR-based term premium measure is rather stable and contra-cyclical,

⁴It is important to point out that the averaging estimator strategy does not imply any parameter or model uncertainty of the investor [like, for instance, in L.P. Hansen and Sargent (2007, 2008)]. It is a statistical procedure adopted by the econometrician to propose estimation methods improving the out-of-sample forecast performances of the model.

the expectation part accounting for most of the yield variability. On the contrary, on the one hand, the less reliable OLS-based VAR decompositions are strongly affected by the persistence problem and assign a large portion of the long-term yield variability to the term premium [as in Beechey (2007)], thus unrealistically suggesting that future monetary policy rate is on average insensitive to the actual economic conditions. On the other hand, the CVAR model leads to systematically identify the measure of the term premium with the long-term spread, implying that the expected path of the future policy rate has no effect on the yield curve spread.

We handle the second part of the problem, namely the dynamic relationship between the term premia and economic activity, by applying a generalization of the notion of Impulse Response Function, that is, the concept of New Information Response Function introduced by Jardet, Monfort and Pegoraro (2009b). This approach allows us, in particular, to measure the dynamic effects, on any variable, of a new (unexpected) information at a given date, regarding any state variables, any yield to maturity or any linear filter of that variables. We provide, first, a dynamic analysis of the relationship between the long-term spread and future economic activity and then, we disentangle the effects of a rise of the spread entirely due to an increase of its expectation part, and a rise of the spread caused by an increase of the term premium only. Like in most studies proposed by the economic literature, we find that an increase of the spread implies a rise of the economic activity. We find similar results when the rise of the spread is generated by an increase of its expectation part. In contrast, an increase of the spread caused by a rise of the term premium induces two effects on future output growth: the impact is negative for short horizons (less than one year), whereas it is positive for longer ones. Our results suggest that, the ambiguity found in the literature regarding the sign of the relationship between the term premium and future activity, come from the fact that the sign of this relationship is changing over the period that follows the shock.

The paper is organized as follows. Section 2 gives a motivation for the use of the averaging estimator *à la* B. Hansen (2010), based on its prediction performances, while Section 3, after the description of the data, presents the Near-Cointegration methodology leading to a persistent but

stationary VAR(p) dynamics for the factor of interest $X_t = (r_t, S_t, g_t)'$, where r_t is a short rate, S_t a long-term spread and g_t is a one-period gross domestic product (GDP) growth rate. These variables are also considered in the pioneering paper of Ang, Piazzesi and Wei (2006) [APW (2006), hereafter] whose model constitutes a benchmark of our study. More precisely, this section stresses the persistence problem, presents a solution based on averaging estimators, and then specifies and estimates the Near-Cointegrated VAR(p) factor dynamics. Section 4 shows how the Near-Cointegrated model can be completed by a no-arbitrage affine term structure model, presents risk sensitivity parameter estimates, proposes a reliability check of our selected averaging parameter, focuses on the empirical performances of our model in terms of in-sample fit of the whole yield curve, and shows its ability to explain the observed violation of the expectation hypothesis over the maturity spectrum. Section 5 defines our preferred NCVAR-based measure of term premia, and compares it with those extracted by cointegrated and unconstrained VAR affine models. Section 6, using the general concept of New Information Response Function, studies the dynamic relationships between the spread, its components (expectation part and term premia) and the GDP growth. Section 7 concludes, while Jardet, Monfort and Pegoraro (2010) provides an online appendix with additional details, results and tables⁵ about state dynamics specification, in-sample fit of the yield curve and response functions.

2 Persistence, prediction and averaging estimators

2.1 Reliability of Term Premia Measurements

The first problem that this paper tries to tackle, in order to precisely study the relation between term premia and future economic activity, is the extraction of a reliable measure of such term premia, in particular long-horizon ones. Since a term premium is the difference between a yield and the prediction of a function of future short rates, the reliability of its measurement is the same

⁵These tables will be labeled by the lower case arabic *a.* letter and lower case roman numerals.

issue as the reliability of the prediction of the above mentioned function of future short rates.

It is well known that the short rate variable, as well as any yield, is very persistent and that standard unit root tests usually accept that it is non-stationary. However, it is difficult to admit non-stationarity because it would imply unrealistic asymptotic behaviors and it is generally considered that the short rate dynamics is stationary but close to non-stationarity or "near non-stationary". At this stage another problem occurs, namely the well know fact that the OLS or ML estimation methods highly underestimate the persistence because large biases appear in the finite sample behavior of these estimators [see e. g. Kendall (1954)]. So, important questions we will have to answer are the followings: Are prediction performances deteriorated by these biases ? Are bias correction methods appropriate for improving prediction performances ? Is there a better way to improve forecast performances ? We will provide answers to these questions in the next section.

2.2 Bias correction vs Averaging Estimators

In order to motivate the approach retained in this paper for the prediction of (a function of) persistent variables, let us consider a simple AR(1) model $y_t = \mu(1 - \rho) + \rho y_{t-1} + \varepsilon_t$, $t \in \{1, \dots, T\}$, where the ε_t 's are independently distributed as $N(0, \sigma^2)$, $y_0 = \mu$ and $T = 160$, which is a typical sample size in empirical studies based on quarterly data. It is easy to study the behavior of the OLS estimator $\hat{\rho}_T$ of ρ , by Monte Carlo methods and for various values of ρ (note that this behavior does not depend on μ and σ^2). In particular, using 50.000 simulations we can compute the bias of $\hat{\rho}_T$, namely $b_T(\rho) = E_\rho(\hat{\rho}_T) - \rho$. Figure 1 shows the bias function $b_T(\rho)$ for $\rho \in]0.4, 1[$, as well as the Kendall approximation $-(1 + 3\rho)/T$ (and a quadratic spline approximation $b_T^S(\rho)$ of $b_T(\rho)$ which will be useful below). It is clear from this figure that the bias is very large and, moreover, for ρ close to one this bias is much worse than its Kendall's approximation.

Since the main focus of this paper is the prediction of a function of future short rates, in order to extract measures of term premia, a natural question is to evaluate the property of bias-corrected estimators in terms of prediction. There are many methods to approximately correct

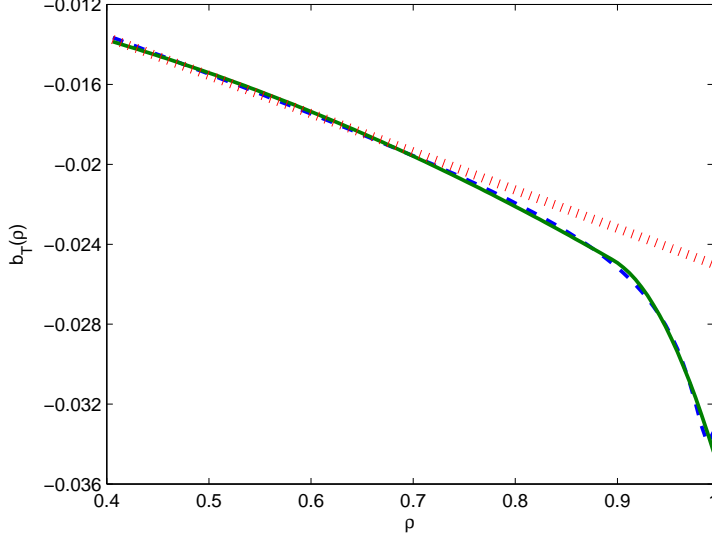


Figure 1:
Bias of the OLS estimator $\hat{\rho}_T$ (sample size $T = 160$): exact (green solid line), spline approximation (blue dashes) and Kendall's approximation (red dots).

for bias and here we retain the one which has been recognized as very efficient [see Gouriéroux, Monfort and Renault (1993), Gouriéroux and Monfort (1996), Gouriéroux, Touzi and Renault (2000) and Duffee and Stanton (2008)]: the indirect inference estimator defined by $\hat{\rho}_T^I = e_T^{-1}(\hat{\rho}_T)$, where $e_T(\rho) = E_\rho(\hat{\rho}_T)$ is approximated by $\rho + b_T^S(\rho)$. We also consider another kind of estimators, namely the class of "averaging estimators" proposed by B. Hansen (2010) and defined as a weighted average of the estimator of ρ in the non-stationary model, i.e. 1 in our case, and in the stationary one, i.e the OLS estimator $\hat{\rho}_T$:

$$\hat{\rho}_T^A(\lambda) = (1 - \lambda) + \lambda\hat{\rho}_T, \quad 0 \leq \lambda \leq 1. \quad (1)$$

We compare, in a Monte Carlo exercise, with 50.000 simulations and $\rho = 0.99$, the predictions of y_t , obtained from the above mentioned AR(1) process, when the autoregressive parameter is estimated by $\hat{\rho}_T^I$ and by $\hat{\rho}_T^A(\lambda)$. For each simulation, we compute a path of 180 observations, we calculate the OLS estimator of ρ (from the first $T = 160$ observations), we determine $\hat{\rho}_T^I$ and $\hat{\rho}_T^A(\lambda)$,

and then we generate the associated forecasts at horizon $q = 1$ and $q = 20$. The comparison among the forecast performances is based on the root mean squared forecast error (RMSFE), normalized by the one based on the true value of ρ . The results are given in Figure 2 ($q = 1$) and Figure 3 ($q = 20$).

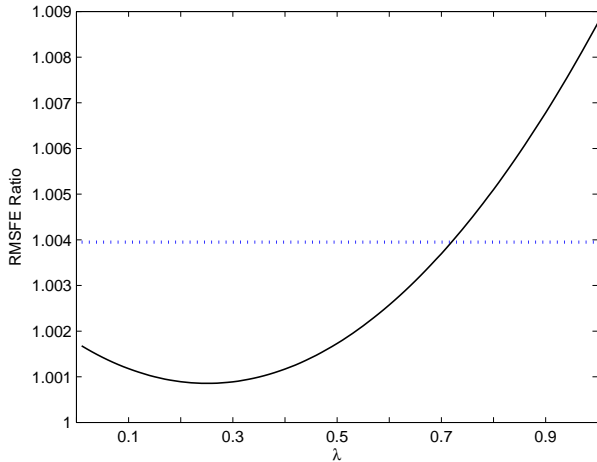


Figure 2:
RMSFE ratio with $\rho = 0.99$, $T = 160$, $q = 1$.
 $\hat{\rho}_T^A(\lambda)$ (blue solid curve), $\hat{\rho}_T^I$ (green dots).

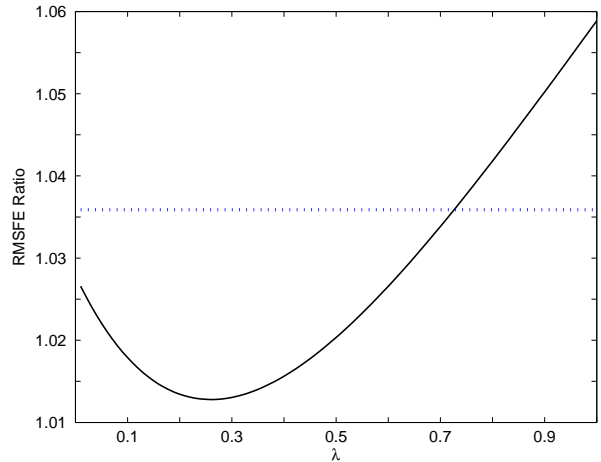


Figure 3:
RMSFE ratio with $\rho = 0.99$, $T = 160$, $q = 20$.
 $\hat{\rho}_T^A(\lambda)$ (blue solid curve), $\hat{\rho}_T^I$ (green dots).

The performances of the optimal averaging estimator, obtained with $\lambda \approx 0.25$ both for $q = 1$ and $q = 20$, is by far the best one; in the case $q = 20$, for instance, the percentage of increase of the RMSFE, compared with the one obtained from the true parameter value, is about three times smaller than for the indirect inference estimator and five times smaller than for the OLS estimator. Similar conclusions are obtained for different values of T and ρ and for bivariate models close to cointegration or "near-cointegrated" [see Jardet, Monfort and Pegoraro (2009a)]. These results clearly provide solid arguments in favor of the averaging estimator class, compared to the optimal bias correction method based on indirect inference, in terms of forecast performances and, thus, in terms of term premia extraction.

3 Near-Cointegration Analysis

3.1 Description of the Data

The data set that we consider in the empirical analysis contains 174 quarterly observations of U. S. zero-coupon bond yields, for maturities 1, 2, 3, 4, 8, 12, 16, 20, 24, 28, 32, 36 and 40 quarters, and U. S. real GDP, covering the period from 1964:Q1 to 2007:Q2. The yield data are obtained from Gurkaynak, Sack, and Wright (2007) [GSW (2007), hereafter] data base and from their estimated Svensson (1994) yield curve formula. In particular, given that GSW (2007) provide interest rate values at daily frequency, each observation in our sample is given by the daily value observed at the end of each quarter. The same data base is used by Rudebusch, Sack, and Swanson (2007) [RSS (2007), hereafter] in their study on the implications of changes in bond term premiums on economic activity. Observations about real GDP are seasonally adjusted, in billions of chained 2000 dollars, and taken from the FRED database (GDPC1)⁶. The short rate (r_t) and the long rate (R_t) are respectively given by the 1-quarter and 40-quarters yields.

3.2 Near-Cointegrated VAR(p) Dynamics

3.2.1 Handling the Persistence Problem for Term Premia Extraction

We first apply unit root tests and a cointegration analysis to the joint autoregressive dynamics of the short rate (r_t), the long rate (R_t) and the log-GDP (G_t), collected in a vector denoted by Y_t . This econometric procedure leads us to a vector error correction model, with two lags, for ΔY_t , that we can write as a Cointegrated VAR(3), or CVAR(3), for $X_t = (r_t, S_t, g_t)'$, the long-term spread $S_t = R_t - r_t$ being the only cointegrating relationship⁷. We thus obtained a constrained estimator, denoted by $\hat{\theta}_{T,cvar}$ [see table *a.viii*], of the true value of the parameter $\theta^{(o)} := (\nu^{(o)}, \Phi_1^{(o)}, \Phi_2^{(o)}, \Phi_3^{(o)})$

⁶See Jardet, Monfort and Pegoraro (2010) for further details.

⁷Details about VAR order selection, unit root tests and cointegration analysis are presented in Jardet, Monfort and Pegoraro(2010).

appearing in the data generating process (DGP):

$$X_t = \nu^{(o)} + \sum_{j=1}^3 \Phi_j^{(o)} X_{t-j} + \eta_t^{(o)}, \quad \eta_t^{(o)} \sim IIN(0, \Omega^{(o)}). \quad (2)$$

Another estimator of $\theta^{(o)}$ is obtained by the unconstrained OLS method applied to the stationary VAR(3) model, and it is denoted by $\hat{\theta}_{T,var}$ [see table *a.ix*].

The CVAR(3) specification suggested by the cointegration analysis has, on the one hand, the advantage to explain the autocorrelation in interest rates better than the unconstrained counterpart given by a VAR(3) model for X_t , but, on the other hand, has two important drawbacks. First, it assumes the non-stationarity of interest rates, while a wide literature on nonlinear models indicates that they are highly persistent but stationary [see, for instance, Gray (1996) and Ang and Bekaert (2002), and the references therein]. Second, interest rate forecasts over long horizons, coming from the alternative CVAR(3) and VAR(3) estimated dynamics, have unrealistic behaviors. For any given date in the sample period, we show these forecasts in figures 4 and 5, for the CVAR(3) and VAR(3) specifications, respectively, for a forecasting horizon q (say) rising from 1 to 40 quarters. In the CVAR(3) case (figure 4) the predictions remain close to the present value when q increases and in the VAR(3) case (figure 5) they revert to the unconditional mean. As a consequence, important differences will be found in the term premia extraction [see also Cochrane and Piazzesi (2008)].

As mentioned above, in this paper we adopt the class of averaging estimators, considered in Section 2 and proposed by B. Hansen (2010), in order to tackle the estimation problems induced by the interest rate persistence and affecting forecast performances. Hansen's results have been derived in a univariate and one-step-ahead framework and their generalization to a multivariate and multi-horizon setting raises difficult technical problems, in particular the multiplicity of the parameter paths leading to the constrained VAR at rates proportional to $1/T$. So we have decided to adopt a pragmatic approach and, extrapolating the Monte Carlo results of Section 2 and of Jardet, Monfort and Pegoraro (2009a), we have checked empirically whether the out-of-sample root mean squared

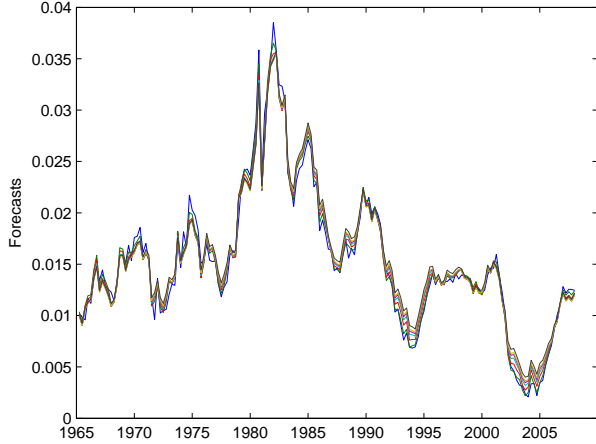


Figure 4:

q -step ahead short rate forecasts from the CVAR(3) model; $q = 1, 4, 8, 12, 16, 20, 40$ quarters.

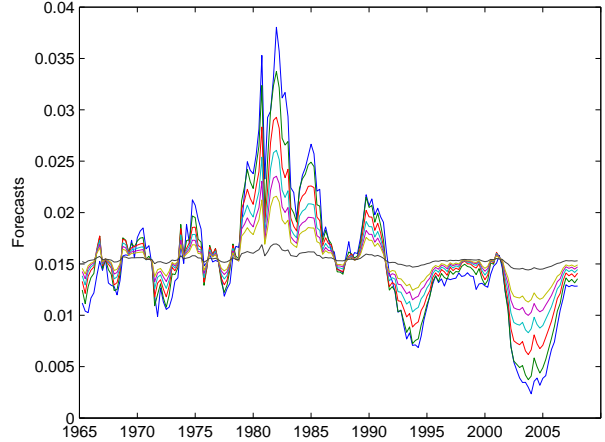


Figure 5:

q -step ahead short rate forecasts from the VAR(3) model; $q = 1, 4, 8, 12, 16, 20, 40$ quarters.

forecast errors, when forecasting some variable of interest at various horizons, are improved when using an averaging estimator based on the VAR(3) and CVAR(3) models. As explained below, our empirical findings thoroughly confirm Hansen's theoretical results.

3.2.2 Averaging Estimations and Extraction of Long-Term Short Rate Expectations

The averaging estimators $\hat{\theta}_T^{(nc)}(\lambda)$ (say) of $\theta^{(o)}$, of the Near-Cointegrated (stationary) VAR(3) dynamics of the state vector X_t , are obtained in the following way:

$$\hat{\theta}_T^{(nc)}(\lambda) = \lambda \hat{\theta}_{T,var} + (1 - \lambda) \hat{\theta}_{T,cvar}, \quad (3)$$

where $\lambda \in [0, 1]$ is a free parameter selected to minimize a criterion of interest (for each model, the conditional variance-covariance matrix is estimated from its residuals). Since our aim is to provide a reliable measure of the term premia on the 40-quarters long-term bond, we focus on minimizing the prediction error of the associated expectation part.

Given a yield with residual maturity h at date t , denoted by $R_t(h)$, we define its expectation term as $EX_t(h) = -\frac{1}{h} \log \tilde{B}_t(h)$ with $\tilde{B}_t(h) = E_t[\exp(-(r_t + r_{t+1} + \dots + r_{t+h-1}))]$. The associated term

premium is given by $TP_t(h) = R_t(h) - EX_t(h)$ (see Section 5.1 for a more detailed presentation). For a given maturity $h = 40$ quarters, the parameter $\lambda = \lambda(40)$ (say) is selected as the solution of the following problem:

$$\lambda^*(40) = \arg \min_{\lambda \in [0,1]} \sum_t [\tilde{B}_t^{obs}(40) - \tilde{B}_t(40, \lambda)]^2 \quad (4)$$

where, for each date t , $\tilde{B}_t^{obs}(40)$ is the observed realization of $\exp(-r_t - \dots - r_{t+39})$ while $\tilde{B}_t(40, \lambda)$ is the NCVAR(3) model's forecast of $\exp(-r_t - \dots - r_{t+39})$ using the parameter $\hat{\theta}_T^{(nc)}(\lambda)$ given by (3). The out-of-sample forecasts are performed during the period 1990:Q1 - 2007:Q2, using an expanding window for the estimation of models VAR(3) and CVAR(3). More precisely, we first compute the estimates $\hat{\theta}_{t,var}$ and $\hat{\theta}_{t,cvar}$ over the period 1964:Q1 to 1989:Q4 and we calculate $\tilde{B}_t(h, \lambda)$ with $t = 1989:Q4$. Then, at each later point in time t , we recompute $\hat{\theta}_{t,var}$ and $\hat{\theta}_{t,cvar}$ taking into account the new observation and, in doing so, we replicate the typical behavior of an investor that incorporates new information over time [see also Favero, Kaminska and Sodestrom (2006)].

The minimization (4) leads to $\lambda^*(40) = 0.2617$ with an associated RMSFE = 0.1012 smaller than those obtained by the CVAR(3), VAR(3) and VAR(1) models as well as the short rate AR(1) model. Indeed, in the case $\lambda = 0$ (CVAR(3) model) and $\lambda = 1$ (VAR(3) model) we obtain a RMSFE of 0.1155 and 0.1224, respectively, while the VAR(1) provides a RMSFE = 0.1406 and the AR(1) a RMSFE = 0.1411. This means that, in our sample period, the optimal extraction of the expectation part of the 10-year bond is obtained by a stationary VAR(3) model with estimator $\hat{\theta}_T^{(nc)}(\lambda^*)$ and in which the weight of the CVAR(3) model is three times larger than the one of the unconstrained VAR(3) model⁸. Thus, the estimated VAR(3) dynamics now has an autoregressive lag operator with the largest root equal to 0.987, instead of 0.93 when the OLS estimator is used. We will see in Section 4.3 that, when we select λ by minimizing the fitting error of the whole yield curve, we find $\lambda \approx 0.26$ again, thus reinforcing the reliability of our criterion.

⁸We have checked that λ remains robust to a change in the starting date of the out-of-sample exercise and to a rolling window exercise [see Table *a.xii* in Jardet, Montfort and Pegoraro (2010)].

	ν^*	Φ_1^*			Φ_2^*			Φ_3^*		
r_t	-0.0006	0.7735	0.1641	0.0367	-0.0806	-0.1865	0.0854	0.2896	-0.0287	0.0112
S_t	0.0009	0.0692	0.6552	-0.0229	0.1802	0.2742	-0.0747	-0.2417	-0.0012	-0.0081
g_t	0.0043	-0.0862	-0.3664	0.1972	-0.6097	0.4508	0.1927	0.6646	0.2890	-0.0140
	$\Omega^* \times 10^3$			Corr.		log-L	$ \psi $			
	0.0079	-0.0053	0.0051	ρ_{12}	-0.8424	2282.39	0.987			
	.	0.0050	-0.0013	ρ_{13}	0.2383	AIC	0.853			
	.	.	0.0577	ρ_{23}	-0.0787	-26.3787	0.602			
						SIC	0.560 ^(c)			
						-25.8827	0.319			
						FPE	0.189			
						6.78e-16	0.132 ^(c)			

Table 1: Parameter estimates of the true dynamics $X_t = \nu^{(o)} + \sum_{j=1}^3 \Phi_j^{(o)} X_{t-j} + \eta_t^{(o)}$, $X_t = (r_t, S_t, g_t)'$, $\eta_t^{(o)} \sim IIN(0, \Omega^{(o)})$, with estimator $\hat{\theta}_T^{(nc)}(\lambda)$ and $\lambda = \lambda^*(40) = 0.26$; ρ_{ij} denotes the empirical correlation between $(\eta_{it}^{(o)})$ and $(\eta_{jt}^{(o)})$; $|\psi|$ indicates the modulus of the roots of equation $|\Phi^*(\psi)| = 0$, with $\Phi^*(\psi) = (I_{3 \times 3} \psi^3 - \Phi_1^* \psi^2 - \Phi_2^* \psi - \Phi_3^*)$ denoting the characteristic polynomial; ^(c) indicates a pair of complex conjugate roots.

4 Near-Cointegrated Affine Term Structure Models

4.1 The Yield Curve Formula

In the previous sections we have specified and estimated the historical dynamics of the state variable X_t as a Near-Cointegrated Gaussian VAR(3) process with averaging parameter given by $\lambda^*(40) = 0.26$. The following step in our modelling procedure aims at deriving the associated arbitrage-free yield-to-maturity formula by specifying a positive stochastic discount factor (SDF) $M_{t,t+1}$ for each period $(t, t+1)$. More precisely, using the notation $\tilde{X}_t = (X_t', X_{t-1}', X_{t-2}')'$, we assume:

$$M_{t,t+1} = \exp \left[-r_t + \Gamma_t' \zeta_{t+1} - \frac{1}{2} \Gamma_t' \Gamma_t \right], \quad (5)$$

where the error term $\eta_t^{(o)}$ in (2) is written as $\eta_t^{(o)} = \Sigma^{(o)} \zeta_t$, with $\zeta_t \sim N(0, I_{3 \times 3})$, and $\Sigma^{(o)}$ is a lower triangular matrix such that $\Sigma^{(o)} \Sigma^{(o)'} = \Omega^{(o)}$; $\Gamma_t = \gamma_0 + \gamma \tilde{X}_t = \gamma_0 + \gamma_1 X_t + \gamma_2 X_{t-1} + \gamma_3 X_{t-2}$ is the affine (multiple lags) stochastic risk sensitivity vector; the constant term γ_0 is a (3×1) vector and $\gamma = [\gamma_1 : \gamma_2 : \gamma_3]$ is a (3×9) matrix. $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ are called risk sensitivity parameters.

Given that under the absence of arbitrage opportunity assumption (A.A.O.) the price $B_t(h)$ at date t of a zero-coupon bond (ZCB) maturing at $t+h$ is equal to $E_t[M_{t,t+1} \dots M_{t+h-1,t+h}]$, we have the following result.

Proposition: Let us assume that the factor X_t follows, under the historical probability, the Gaussian VAR(3) dynamics (2). Then, the yield with h periods to maturity at date t , denoted $R_t(h)$, is given by:

$$\begin{aligned} R_t(h) = -\frac{1}{h} \log B_t(h) &= -\frac{c'_h}{h} \tilde{X}_t - \frac{d_h}{h} \\ &= -\frac{c'_{1,h}}{h} X_t - \frac{c'_{2,h}}{h} X_{t-1} - \frac{c'_{3,h}}{h} X_{t-2} - \frac{d_h}{h}, \quad h \geq 1, \end{aligned} \tag{6}$$

where c_h and d_h satisfies, for $h \geq 1$, the recursive equations:

$$\begin{cases} c_h &= -\tilde{e}_1 + \Phi^{(o)'} c_{h-1} + (\Sigma^{(o)} \gamma)' c_{1,h-1}, \\ d_h &= c'_{1,h-1} (\nu^{(o)} + \Sigma^{(o)} \gamma_0) + \frac{1}{2} c'_{1,h-1} \Sigma^{(o)} \Sigma^{(o)'} c_{1,h-1} + d_{h-1}, \end{cases} \tag{7}$$

where :

$$\Phi^{(o)} = \begin{bmatrix} \Phi_1^{(o)} & \Phi_2^{(o)} & \Phi_3^{(o)} \\ I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \text{ is a } (9 \times 9) \text{ matrix,}$$

with initial conditions $c_0 = 0, d_0 = 0$ (or $c_1 = -\tilde{e}_1, d_1 = 0$), where \tilde{e}_1 is the (9×1) vector with all entries equal to 0 except the first one equal to 1, and where $c_{1,h}$ is the vector of the first 3 components of the 9-dimensional vector c_h (**Proof:** straightforward).

So, $R_t(h)$ is an affine function of the factor \tilde{X}_t , that is of the three most recent lagged values of the

3-dimensional factor X_{t+1} . Note that the risk-neutral dynamics of X_t is given by:

$$X_t = (\nu^{(o)} + \Sigma^{(o)}\gamma_0) + \sum_{j=1}^3 (\Phi_j^{(o)} + \Sigma^{(o)}\gamma_j) X_{t-j} + \xi_t^{(o)}, \quad \xi_t^{(o)} \stackrel{\mathcal{Q}}{\sim} IIN(0, \Omega^{(o)}).$$

Joslin, Priebsch and Singleton (2010) [JPS (2010), hereafter] also have recently specified their yield curve model as a particular Near-Cointegrated VAR(1) model. They handle the bias problem by imposing the largest eigenvalue of the historical autoregressive matrix to be equal to the (close to one) largest eigenvalue of the risk-neutral one. It is also important to stress that the presence of p lagged factor values in (6) does not allow to conclude that the macro factor X_t is spanned by the yield curve at the same date. Indeed, at date t we could build from (6) a system with nine observed yields \mathcal{R}_t (say) and nine unknowns (the components of \tilde{X}_t) that we could solve to obtain X_t , for instance, but the same procedure applied at dates $t+1$ and $t+2$ would give different values for X_t since the observed yields do not exactly satisfy equation (6). In order to correctly invert the yield curve formula, we have to introduce in (6) the lag operator L and once we write (with obvious notation) the system $\mathcal{R}_t = (\mathcal{C}_1 + \mathcal{C}_2 L + \mathcal{C}_3 L^2)X_t + \mathcal{D} = \mathcal{C}(L)X_t + \mathcal{D}$, we find by inversion of $\mathcal{C}(L)$ that the X_t is function of present and past values of the yield curve, and not just the current one like in the case where there is only one lag in the VAR.

4.2 Risk Sensitivity Parameter Estimates

The estimation of historical and risk sensitivity parameters follows a consistent two-step procedure, as adopted, among the others, by APW(2006), Monfort and Pegoraro (2007), and Garcia and Luger (2007). The estimation of risk sensitivity parameters $\theta_\gamma = (\gamma_0, \gamma_1, \gamma_2, \gamma_3)$ is obtained by constrained nonlinear least squares (CNLLS), using the observations on yields with maturities different from those used in the first step (that is, maturities ranging from 2-quarters to 36-quarters) and taking the historical parameters at their NCVAR(3) estimated values. A constraint is imposed in order to satisfy the no arbitrage restriction on the 10-years yield (the long rate). This Constrained NLLS

estimator is given by:

$$\left\{ \begin{array}{l} \hat{\theta}_\gamma = \text{Arg min}_{\theta_\gamma} S^2(\theta_\gamma) \\ S^2(\theta_\gamma) = \sum_t \sum_h (R_t^{obs}(h) - R_t(h))^2, \\ \text{s. t. } c_{40} = -40(1, 1, 0)', d_{40} = 0, \end{array} \right. \quad (8)$$

where, for each date t and maturity h , $R_t(h)$ is the theoretical yield determined by formula (6) in which the vector of parameters $\theta^{(o)}$ has been replaced by $\theta_T^{(mc)}(\lambda^*)$, and $R_t^{obs}(h)$ indicates the observed one. Risk sensitivity parameter estimates of the Near-Cointegrated VAR(3) factor-based term structure model are presented in Table 2⁹. It is interesting to point out that our NCVAR(3) model has under the risk-neutral probability a degree of persistence larger than under the historical one, since its largest root is 0.994 in the first case and 0.987 in the second one. So, our framework implies that, at the same time, the largest historical root is much bigger the one based on the unconstrained VAR(3) model (0.93), but it is smaller than the risk-neutral one [compare with JPS(2010)].

	γ_0	γ_1	γ_2	γ_3						
r_t	-0.234 [-4.912]	102.009 [6.615]	102.622 [5.374]	-13.795 [-4.721]	21.063 [0.881]	12.619 [0.460]	-21.957 [-7.432]	-97.484 [-7.202]	-13.044 [-0.823]	0.455 [0.187]
S_t	0.022 [0.331]	49.605 [1.812]	92.903 [2.707]	-1.080 [-0.182]	-98.565 [-2.227]	-81.329 [-1.601]	7.816 [1.391]	34.913 [1.595]	35.646 [1.344]	-3.002 [-0.687]
g_t	1.615 [1.460]	72.856 [0.165]	156.141 [0.284]	-44.240 [-0.503]	-253.550 [-0.361]	94.835 [0.119]	8.570 [0.099]	82.669 [0.240]	-422.016 [-1.019]	55.639 [0.865]

Table 2: Risk sensitivity parameter estimates for the NCVAR(3) factor-based term structure model. t -values are in brackets.

4.3 Cross-sectional reliability of the weighting parameter

The purpose of this section is to provide, first of all, a further argument of the reliability of the value $\lambda^* = \lambda^*(40) \approx 0.26$, besides those mentioned at the end of Section 3, and also to show the

⁹The risk sensitivity parameter estimates (obtained by CNLLS) of the CVAR(3), VAR(3) and VAR(1) factor-based term structure models are presented in Table *a.xiii*.

ability of our yield-to-maturity formula to explain the observed interest rates variability in terms of fitting performance.

The reliability check of the estimated averaging parameter is done in the following way: instead of using criterion (4) to fix λ , we fix it at the same time as the risk sensitivity parameters $\theta_\gamma = (\gamma_0, \gamma_1, \gamma_2, \gamma_3)$, by minimizing the fitting error of the whole yield curve, that is by using the CNLLS methodology (8). This value is found to be $\lambda^{**} = 0.256$, very close to $\lambda^*(40) \approx 0.26$.

	NCVAR(3) [$\lambda^{**} = 0.256$]	NCVAR(3) [$\lambda^* = 0.262$]	CVAR(3)	VAR(3)	VAR(1)
Mean	16.79	16.81	16.91	16.86	18.76
Median	12.11	12.22	12.89	12.55	16.02
Std. Dev.	13.80	13.88	14.02	13.96	15.23

Table 3: Annualized Absolute Pricing Errors (Basis Points).

As far as the in-sample fit is concerned, we compare in Table 3 the annualized absolute yield-to-maturity errors of our selected NCVAR(3) factor-based term structure model with the performances of the competing CVAR(3), VAR(3) and VAR(1) yield curve models. For each date t and each estimated model, we compute, over the maturities used to estimate the risk sensitivity parameters $\theta_\gamma = (\gamma_0, \gamma_1, \gamma_2, \gamma_3)$, the pricing error in the following way:

$$PE_t = \frac{\sum_h |R_t^{obs}(h) - R_t(h)|}{H}, \quad (9)$$

where $R_t^{obs}(h)$ and $R_t(h)$ are, respectively, the (annualized) observed and model-implied yields, and where H denotes the number of maturities used to estimate θ_γ . Given the time series PE_t , we calculate for each model the associated mean and standard deviation. We see from table 3 that the fitting based on the NCVAR(3) estimates is slightly better than the one based on the CVAR(3) and VAR(3) estimates and much better than the one based on the VAR(1) model [see table *a.xiv*] for further details].

We have also verified, working with Campbell-Shiller regressions, the ability of our approach to explain the observed violation of the Expectation Hypothesis theory over the maturity spectrum,

with VAR(1) and VAR(3) models failing for short maturities [see Jardet, Monfort and Pegoraro (2010), Table *a.xv*], for details]. We also have considered the case of a weighting parameter λ optimally selected on the basis of a criterion of interest like the forecast of yields over several horizons, and found that our model is again able to provide a reduction of the associated root mean squared forecast errors [see Jardet, Monfort and Pegoraro (2009c)].

5 The NCVAR-based Term Premia Measure

The first subsection briefly introduces the concept of term premium, while the second one presents a comparison of the NCVAR(3)-based term premia measures with the ones extracted by the CVAR(3), VAR(3) and VAR(1) models, as well as the Kim and Wright (2005) one (for the 10-year horizon).

5.1 Definition of the term premia

Let us consider $R_t(h)$ and r_t , that is, the yield of maturity h periods at time t , and the short rate. The former can be written as $R_t(h) = EX_t(h) + TP_t(h)$, where:

$$EX_t(h) = -\frac{1}{h} \log E_t \left\{ \exp \left[-\sum_{j=0}^{h-1} r_{t+j} \right] \right\} \quad (10)$$

denotes the expectation part of $R_t(h)$, and

$$TP_t(h) = R_t(h) - EX_t(h) \quad (11)$$

is, by definition, the h -period term premium. Note that, since $R_t(h) = -\frac{1}{h} \log E_t^{\mathbb{Q}} \left\{ \exp \left[-\sum_{j=0}^{h-1} r_{t+j} \right] \right\}$, \mathbb{Q} denoting the risk-neutral probability measure, the term premium thus defined is unbiased in the sense that it is equal to zero if risk-neutral and historical dynamics are identical. Also note that $EX_t(h)$ is easily computed using the recursive equations (7), with $\gamma_0 = 0$ and $\gamma = 0$ [see also Bernanke, Reinhart and Sack (2004)]. The spread of maturity h periods at time t , $S_t(h) = R_t(h) - r_t$

can then be written as $S_t(h) = EXS_t(h) + TP_t(h)$, where $EXS_t(h) = EX_t(h) - r_t$ is the expectation part of the h -period spread.

5.2 Comparison of term premia measures

On the basis of the results presented in Section 3.2.2, our NCVAR(3) estimation provides the best measure of the 10-year term premium, that is, the best decomposition of the long-term interest rate into expectation part $EX_t(40)$ and term premium $TP_t(40)$. The former can be interpreted as the expectation of the future path of the monetary policy rate, whereas the latter captures the compensation that investors require for bearing interest rate risk. We will show in what follows that our decomposition of the long rate provides an expectation term and a term premium component with empirical properties coherent with typical macro wisdom and with empirically observed sustained countercyclical monetary policy actions.

Let us start from a statistical comparison of our measure of $TP_t(40)$ with the one extracted by the CVAR(3), VAR(3) and VAR(1) specifications (see table 4 and figure 6).

	NCVAR(3)	CVAR(3)	VAR(3)	VAR(1)
Mean	1.35	1.43	1.25	1.19
Std. Dev.	1.08	1.09	1.62	1.47
Corr. with S_t	0.77	0.92	0.32	0.20

Table 4: Descriptive statistics of $TP_t(40)$ measures obtained from NCVAR(3), CVAR(3), VAR(3) and VAR(1) models. Corr. is the coefficient of correlation between $TP_t(40)$ and S_t .

As can be seen from table 4, our preferred measure of the term premium, based on the NCVAR(3) estimation, is more stable than the VAR(3) and VAR(1)-based ones. It is less correlated with the spread than the CVAR(3)-based measure, but more than the OLS-based ones. In other words, our preferred NCVAR(3) estimation procedure leads to a term premium that drives a great part of the spread variability, but not the quasi-totality, as it is the case for the CVAR(3) model in which $EXS_t(40)$ is close to zero (see figure 6). Indeed, when the short rate is considered as an I(1) (non stationary) process, the expectation part of the 10-year interest rate, $EX_t(40)$, is very close to the short rate and, thus, the expectation part of the spread, $EXS_t(40)$, is close to zero, making the

10-year spread almost equal to the term premium. From an economic point of view, this means that the expected path of future monetary policy rate has no effect on the spread or, equivalently, it means that it is almost completely identified with the actual short rate. Observe, in addition, that the CVAR(3) model provides a measure of the term premium rather close to zero at the beginning of the 80's (the so-called Volker period), and then substantially increasing over the following 20 years. These results seem economically unconvincing, being in contradiction with the implications of recent empirical studies [see Dai, Singleton and Yang (2007) and Ang, Bekaert and Wei (2008)], and highlight a limit of the CVAR approach for decomposing the long rate.

In contrast, when the OLS estimation procedure is applied to the VAR(1) and VAR(3) models, because of the bias problem, the forecast of the future sequence of short-term interest rates reverts too quickly to its unconditional mean, leading to a very stable $EX_t(40)$ component and, consequently, the long rate variability is transferred (simply by difference) to the term premium. This decomposition of the long rate thus implies, first, an expectation term that only slightly decreases (increases) when we enter into (we step out of) a recessionary period. Second, the VAR(3) and VAR(1)-based term premia measures move over the entire sample period in the same way as the long rate, thus suggesting, for instance, a systematic reduction of the risk level over the last 20 years of the sample (see figure 6). This result would suggest that future monetary policy rate is expected to be insensitive to actual economic conditions, including the current state of monetary policy, and that investors concerns for risk compensation do not vary so much with the business cycle, in particular in the second half of the sample. This is in contradiction with a claimed countercyclical monetary policy response, as well as the typical macro wisdom requiring a rather stable and contra-cyclical term premium and, thus, casts some doubt on the OLS-based decomposition.

Finally, from figure 6 we observe that our preferred NCVAR(3)-based decomposition provides a rather stable and contra-cyclical measure of the term premium, as suggested by macroeconomists believes, by theoretical works of Campbell and Cochrane (1999) and Wachter (2006), and by empirical studies of Harvey (1989) and Lustig, Roussanov and Verdelhan (2010), and an expectation

part that, at the same time, drives long-term yields variability and decreases (increases) when we enter into (we come out of) a recessionary periods, coherently with countercyclical monetary policy actions. In other words, our empirical findings seem much more plausible in light of the above mentioned features of monetary policy interventions as well as the macro wisdom and empirical studies.

Observe also that our decomposition, coherently with the literature [see Backus and Wright (2007), Cochrane and Piazzesi (2008) and the references therein], explains the conundrum episode (from June 2004 to June 2006) by a sharp decrease of the term premium that exactly offsets the rise of expectation part. During this period, as highlighted in figure 7, the CVAR(3) model clearly overestimates (underestimates) the magnitude of the term premium (the expectation part) which is almost identified with the long-term spread, while VAR(3) and VAR(1) models turn out to provide an expectation term almost equal to the long-rate and, consequently, a term premium around zero. From figure 8 we can easily verify that all these comments apply also to the decomposition of the 5-year interest rate and spread.

In figure 9 we consider a comparison with the $TP_t(40)$ measure retained by Rudebusch, Sack and Swanson (2007) as a representative one of the five they compare in their paper, namely the Kim and Wright (2005) measure (available only till July 1990). This graph shows that, between the last two recessions in the sample, our NCVAR(3) measure displays similar features, including peaks and troughs, but significant differences between these two measures are observed in the period 2001-2004 during which, the NCVAR(3) term premium is substantially higher than the Kim-Wright's one¹⁰. In particular, we note that the rise and the decrease of the term premium in the 2001 recession and in the 2004 conundrum episode, respectively, is more pronounced with the NCVAR(3) model. For comparison purposes we also present, in figure 9, the 10-year term premium obtained from the VAR(1) model *à la* Ang, Piazzesi and Wei (2006). In recent years, the VAR(1) measure tends to

¹⁰During this period, the values of the NCVAR(3)-based measure of the term premium are closer to those obtained with the Rudebusch and Wu (2008) measure. However, the latter seems to be neutral with respect to the business cycle [see Rudebusch, Sack and Swanson (2007) for further details].

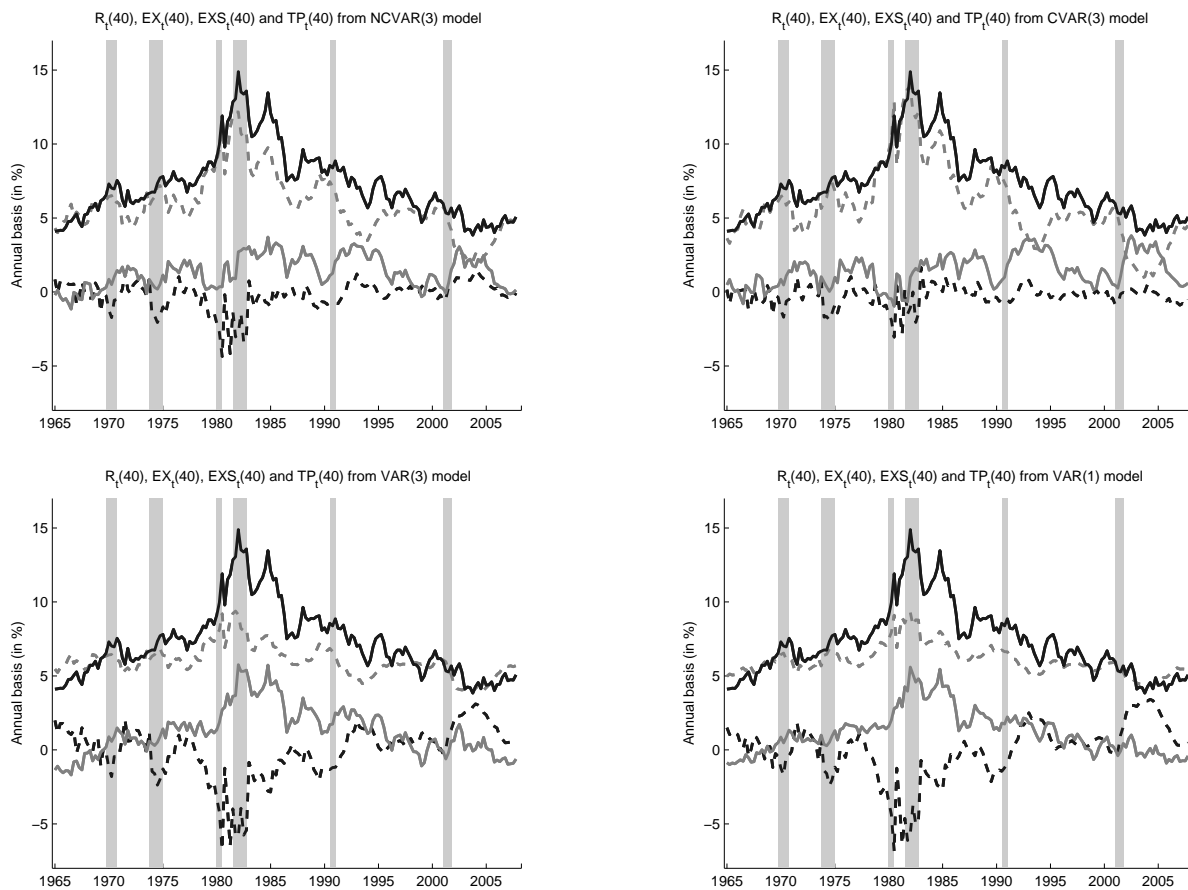


Figure 6:

Comparisons of model-based $EX_t(40)$ (grey dashed line), $TP_t(40)$ (grey solid line) and $EXS_t(40)$ (black dashed line) measures from NCVAR(3), CVAR(3), VAR(3) and VAR(1) models. The black solid line denotes $R_t(40)$, and shaded areas indicate recession periods (NBER).

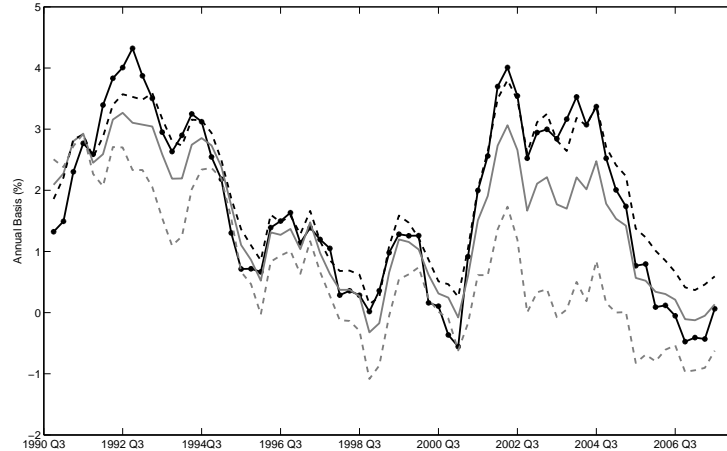


Figure 7: Comparisons of model-based $TP_t(40)$ measures with S_t (cross markers and black line). NCVAR(3) model (grey solid line); CVAR(3) model (black dotted line); VAR(3) model (grey dotted line).

severely underestimate the risk level compared with our preferred measure.

6 *NIRF* Analysis of the Term Premia

6.1 Motivating the *New Information Response Function (NIRF)* approach

There exists an extensive literature emphasizing the predictive power of the spread of interest rates on future GDP. However little is known about the specific contribution of each component of the spread, its expectation part and the term premium, in the explanation of this fact. This lack of agreement mainly concerns the specific effect of the term premium on future economic activity. While practitioners view tends to suggest a negative relationship between both variables, empirical findings, most of them based on static regressions, seem to indicate either the opposite effect, or a non significant relationship. Hamilton and Kim (2002), and Favero, Kaminska and Söderström (2005), for instance, tend to conclude to a positive and significant relationship between the term premium and future activity. In contrast, Ang, Piazzesi and Wei (2006), Rudebusch, Sack and Swanson (2007), and Rosenberg and Maurer (2007) do not find significant link between the level of

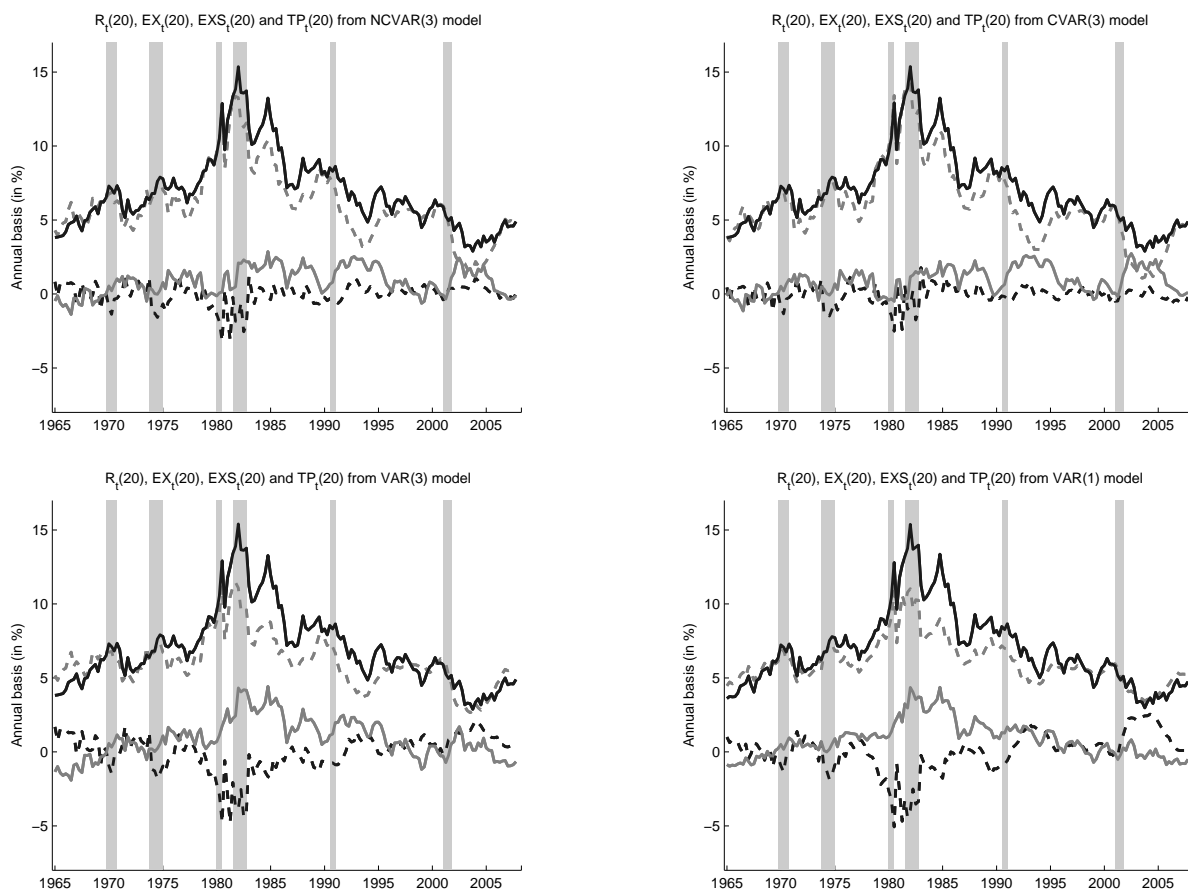


Figure 8:

Comparisons of model-based $EX_t(20)$ (grey dashed line), $TP_t(20)$ (grey solid line) and $EXS_t(20)$ (black dashed line) measures from NCVAR(3), CVAR(3), VAR(3) and VAR(1) models. The black solid line denotes $R_t(20)$, and shaded areas indicate recession periods (NBER).

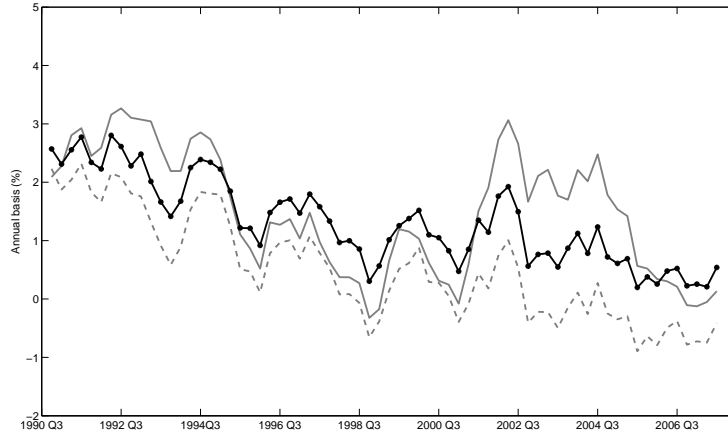


Figure 9: Comparing NCVAR(3)-based $TP_t(40)$ measure with Kim and Wright (2005). NCVAR(3) model (grey solid line); Kim and Wright (2005) model (cross markers and black line); VAR(1) model (grey dotted line).

the term premium and future output growth.

In this section, we try to shed light on this debate by proposing a dynamic analysis of the relationship between the term premium and economic activity. More precisely, we are interested in measuring the differential impact of a shock on the term premium on the real GDP. For that purpose we follow the *New Information Response Functions* approach (*NIRF* thereafter) proposed by Jardet, Monfort and Pegoraro (2009b). This approach seems to be particularly well suited to our question of interest. First, it allows a direct analysis of the effects of a new information on any unobservable variable that is a filtered variable of our state process $X_t = (r_t, S_t, g_t)'$, the term premium in our case. Second, constraints on this new information can be easily imposed. As we will show below, the latter point is particularly convenient to disentangle specific impact of a shock on the term premium from a shock on the expectation part of the spread.

In what follows, we consider that the dynamics of the 3-dimensional state process $X_t = (r_t, S_t, g_t)'$ is given by the Near-Cointegrated VAR(3) model described in the previous sections. Response functions obtained from our NCVAR(3) model are compared with those obtained from three competing models, VAR(3), CVAR(3) and the Ang-Piazzesi-Wei VAR(1) model. In doing so, we aim at analysing how our decomposition of the spread alter conclusions about the relationship between

term premium and economic activity.

The first subsection summarizes the methodology, the second one focuses on the definition of the shocks, while the last one presents the responses to a shock on the spread, and to a shock on its expectation part and its term premium component.

6.2 Definition of *New Information Response Functions*

Let us consider a n -dimensional VAR(p) process X_t , possibly non-stationary. We denote by η_t its innovation process. We want to measure the differential impact on X_t , $t = 1, \dots, T$, of a new information I_0 at date $t = 0$ (by convention). Typically, this new information will be the value h_0 taken by some function $h(\eta_0)$ of the innovation of the process at $t = 0$. More precisely, the *NIRF* is defined by:

$$NIRF(t) = E\left(X_t | I_0, X_{\underline{-p}}\right) - E\left(X_t | X_{\underline{-p}}\right), \quad t \geq 0,$$

where $X_{\underline{-p}} = (X'_{-1}, \dots, X'_{-p})'$. Exploiting the linearity of the model we can show that $NIRF(t) = D_t \delta$ with $\delta = E(\eta_0 | h(\eta_0) = h_0)$, and D_t is the t^{th} Markov matrix coefficient of the MA representation of X_t [see Jardet, Monfort and Pegoraro (2009b)].

This general concept of *NIRF* includes standard Impulse Response Functions like orthogonalized shocks, Uhlig (2005)'s impulse vectors, structural shocks or Pesaran and Shin (1998) "generalized" IRF. But the New Information Response Function is useful in a much more general context, in particular when considering shocks on filtered variables. More precisely, let us consider a m -dimensional process \bar{X}_t obtained by applying a linear filter on X_t , namely $\bar{X}_t = F(L)X_t$ where $F(L) = [F_1(L), \dots, F_n(L)]$ is a $(m \times n)$ matrix of polynomials in the lag operator. The innovation of \bar{X}_t at $t = 0$ is $\bar{\eta}_0 = F(0)\eta_0$. Therefore, if the new information at $t = 0$ is $\bar{h}(\bar{\eta}_0) = \bar{h}_0$, the *NIRF* is $NIRF(t) = D_t \delta$ with $\delta = E(\eta_0 | \bar{h}[F(0)\eta_0] = \bar{h}_0)$. Obviously, the new information may also be made of an information on both η_0 and $\bar{\eta}_0$: $h(\eta_0) = h_0$ and $\bar{h}(\bar{\eta}_0) = \bar{h}_0$, or $h(\eta_0) = h_0$ and $\bar{h}(F(0)\eta_0) = \bar{h}_0$. For our question of interest, \bar{X}_t corresponds to the term premium or the expectation part of the spread.

6.3 Definition of the shocks

Given the affine structure of our model, the expectation part of the spread $EXS_t(40)$ and the term premium $TP_t(40)$ are obtained by applying a linear filter to $X_t = (r_t, S_t, g_t)'$:

$$EXS_t(40) = F_{1,1}(L)r_t + F_{1,2}(L)S_t + F_{1,3}(L)g_t \quad (12)$$

$$TP_t(40) = F_{2,1}(L)r_t + F_{2,2}(L)S_t + F_{2,3}(L)g_t. \quad (13)$$

Hence, the innovation at $t = 0$ of $EXS_t(40)$ and $TP_t(40)$, denoted by $\bar{\eta}_{0,1}$ and $\bar{\eta}_{0,2}$ respectively, are:

$$\bar{\eta}_{0,1} = F_{1,1}(0)\eta_{0,1} + F_{1,2}(0)\eta_{0,2} + F_{1,3}(0)\eta_{0,3} \quad (14)$$

$$\bar{\eta}_{0,2} = F_{2,1}(0)\eta_{0,1} + F_{2,2}(0)\eta_{0,2} + F_{2,3}(0)\eta_{0,3}, \quad (15)$$

where $\eta_{0,1}$, $\eta_{0,2}$ and $\eta_{0,3}$ are the innovations at $t = 0$ of r_t , S_t and g_t respectively. In addition, by construction, we have $\eta_{0,2} = \bar{\eta}_{0,1} + \bar{\eta}_{0,2}$ ¹¹.

In what follows, we first define the shock on the term premium as a shock on a spread that is completely generated by a change of the term premium. More precisely, we are interested in the dynamic effects of a 1 percentage point increase in the spread that would be completely due to a 1 percentage point increase in its term premium component. In other words, the term premium shock is such that at date $t = 0$ (date of the shock) the innovation of the expectation part of the spread is zero. Given our notation, this implies the following constraints $\bar{\eta}_{0,1} = 0, \bar{\eta}_{0,2} = 1$. In addition, we have to remember that interest rates are observed at the end of the period (end-of-quarter observations), and they contain an information covering a following period corresponding to the residual maturity, whereas g_t is the growth rate of GDP between $t - 1$ and t , and contains an information on the two previous quarters. Therefore, a shock on the spread, or on one of its components, occurring at date t (end of the quarter), should have no effect on the growth rate of real GDP between $t - 1$ and t . Accordingly, we impose an additional restriction to ensure that the

¹¹This implies that the $F_{i,j}(0)$ verify $F_{1,1}(0) + F_{2,1}(0) = 0$, $F_{1,2}(0) + F_{2,2}(0) = 1$ and $F_{1,3}(0) + F_{2,3}(0) = 0$

growth rate of real GDP does not respond instantaneously to this kind of shocks. This implies the restriction $\eta_{0,3} = 0$.

Finally, a shock on the term premium at date t is fully characterized by the new information $I_0^{TP} = \{\bar{\eta}_{0,1} = 0, \bar{\eta}_{0,2} = 1, \eta_{0,3} = 0\}$. Given our definition of the *NIRF*, we have to determine the value of the vector $\delta^{TP} = E(\eta_0 | I_0) = E(\eta_0 | \bar{\eta}_{0,1} = 0, \bar{\eta}_{0,2} = 1, \eta_{0,3} = 0)$. Then, $\delta^{TP} = (\delta_1^{TP}, 1, 0)'$ with $\delta_1^{TP} = E(\eta_{0,1} | I_0^{TP}) = \frac{1-F_{2,2}(0)}{F_{2,1}(0)}$ immediately obtained from (15).

Conversely, a shock on the expectation part of the spread is defined as a shock on the spread that is entirely due to a move of its expectation part at the date of the shock ($t = 0$). More precisely, we focus on dynamic effects of a 1 percentage point increase in the spread that would be completely generated by a 1 percentage point increase in its expectation component. Given our notations, the new information now includes $\bar{\eta}_{0,1} = 1$ and $\bar{\eta}_{0,2} = 0$. For the reasons mentioned above, we also assume that this increase has no instantaneous effect on the real GDP, that is $\eta_{0,3} = 0$. Finally, the shock on the expectation part of the spread is fully characterized by the new information $I_0^{EXS} = \{\bar{\eta}_{0,1} = 1, \bar{\eta}_{0,2} = 0, \eta_{0,3} = 0\}$. From equation (14) we immediately obtain $\delta^{EXS} = (\delta_1^{EXS}, 1, 0)'$, where $\delta_1^{EXS} = E(\eta_{0,1} | I_0^{EXS}) = \frac{1-F_{1,2}(0)}{F_{1,1}(0)}$. For sake of comparison, we also report the response of GDP to a shock on the spread, say a 1 percentage increase in the spread, without disentangling which of its components is behind this shift. In this case, the new information is $I_0^S = \{\eta_{0,2} = 1, \eta_{0,3} = 0\}$. Hence, $\delta^S = E(\eta_0 | \eta_{0,2} = 1, \eta_{0,3} = 0) = (\delta_1^S, 1, 0)'$, where $\delta_1^S = E(\eta_{0,1} | \eta_{0,2} = 1, \eta_{0,3} = 0)$. In the gaussian case, δ_1^S is the coefficient of $\eta_{0,2}$ in the theoretical regression of $\eta_{0,1}$ on $\eta_{0,2}$ and $\eta_{0,3}$.

Note that, since $\delta^{TP} = (\delta_1^{TP}, 1, 0)$, $\delta^{EXS} = (\delta_1^{EXS}, 1, 0)$ and $\delta^S = (\delta_1^S, 1, 0)$, it is always possible to find a scalar μ such that $\delta^S = \mu\delta^{EXS} + (1 - \mu)\delta^{TP}$. Therefore, responses to a shock on the spread are linear combinations of responses to a shock on its expectation part and the term premium with weights summing to one¹².

¹²Let us denote by $NIRF^S(t)$, $NIRF^{TP}(t)$ and $NIRF^{EXS}(t)$ the New Information Response Functions at date t to shock on the spread, the term premium and the expectation part of the spread respectively. By definition of the *NIRF*, we have $NIRF^S(t) = D_t\delta^S$, $NIRF^{TP}(t) = D_t\delta^{TP}$, $NIRF^{EXS}(t) = D_t\delta^{EXS}$. With $\delta^S = \mu\delta^{EXS} + (1 - \mu)\delta^{TP}$, we immediately obtain: $NIRF^S(t) = \mu NIRF^{EXS}(t) + (1 - \mu)NIRF^{TP}(t)$.

6.4 Responses of the GDP to a shock on the spread or its components

Figures 10, 11 and 12 report responses of the real GDP to a 1 percentage point shock in the 10-year spread, the expectation part of the spread and the term premium, as defined previously¹³.

Regarding the shock on the spread (figure 10) we observe that an increase in the spread of 1 percentage point (that is 4 percentage point in annual basis) leads, after 20 quarters, to an increase in real GDP that ranges between 4% (that is an annual average growth rate equal to 0,8%) and 3% (average annual growth rate of 0,6%) depending on the model. This result confirms the well documented empirical finding of a positive relationship between the slope of the yield curve and future activity. In addition, responses of real GDP obtained with CVAR(3), VAR(3) and NCVAR(3) models are very similar. Responses obtained from a VAR(1) models are smaller, but display the same tendency.

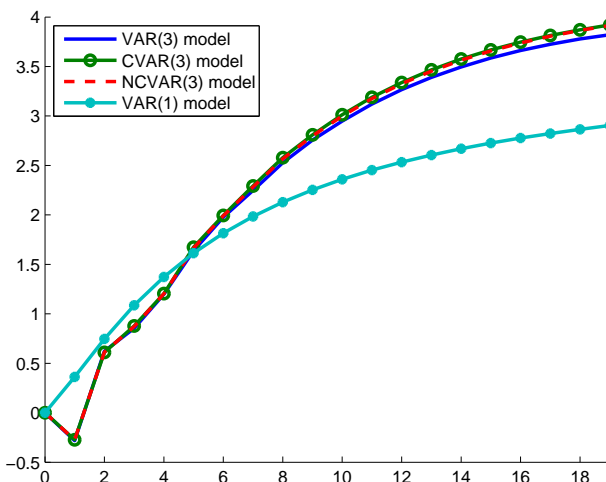


Figure 10: Response of GDP to a shock on the 10-year spread

We obtain the same kind of conclusions with a shock on the expectation part of the spread (figure 11): responses obtained with the four models are not very different. More precisely, a 1

¹³See Jardet, Monfort and Pegoraro (2009c) for the responses of yields of various maturities and responses of their corresponding term premia and expectation components to these shocks. In addition, in Jardet, Monfort and Pegoraro (2010), we also report responses of the real GDP to a 1 percentage shock in the h -year spread and its component, for $h = \{1y, 5y\}$.

percentage point increase in the spread that is entirely caused by a 1 percentage point increase in its expectation part leads to an increase in future real activity (with a range between 3% and 4% after 20 quarters). This result confirms the conventional interpretation of the predictive power of the spread which suggests that this shock could be interpreted as a monetary policy shock. According to this view, the rise of the expectation part of the spread is caused by an expansionary monetary policy, that is a decrease in the short term interest rate¹⁴ that improves the economic financing condition and eventually boosts economic activity.

Finally, figure 12 shows responses of real GDP to a 1 percentage point increase in the spread that is entirely due to a 1 percentage point increase in its term premium component (the term premium shock). Contrary to previous cases, shock on the term premium generates responses of real GDP that are very different depending on the considered model. More precisely, responses of real GDP obtained after a shock on the VAR(1) model are close to zero. This result confirms the empirical finding of Ang, Piazzesi and Wei (2006) which is based on a VAR(1) framework and on static regressions, and conclude to a no significant relationship between the term premium and future activity. Conversely, responses obtained from VAR(3), CVAR(3) and NCVAR(3) models indicate a negative relationship for short horizon (smaller than one year), whereas it is positive for longer horizon (after 20 quarters the increase in the real GDP ranges between 3% and 4,5%). A similar result has been recently shown in Joslin, Priebisch and Singleton (2010) by means of their Markovian of order one term structure model characterized by spanned and unspanned macro risks. They show that a shock in the "in-nine-for-one" forward term premium mainly affects industrial production growth by means of its unspanned component, while the spanned one is found to be unaffected. Our NCVAR(3) yield curve model, like the VAR(3) and CVAR(3) models, in which macro risk is spanned by present and past values of the term structure, provides an alternative

¹⁴See Jardet, Monfort and Pegoraro (2009c) for details regarding responses of the short rate and rates of various maturities to these shocks. More precisely, we observe that a shock on the expectation part of the spread leads to a decrease in the short term interest rate. In addition, we show that responses of various variables obtained after a shock on the short term interest rate, which could be interpreted as a monetary policy shock, are close to the ones obtained with a shock on the expectation part of the spread.

route to explain the effect of term premia shocks on future economic activity.

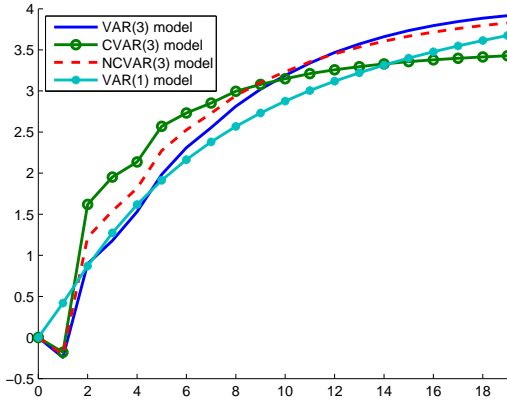


Figure 11:
Impulse response of GDP to a shock on the expectation component of the 10-year spread.

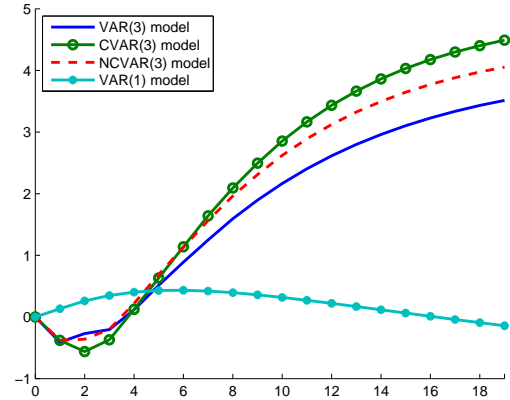


Figure 12:
Impulse response of GDP to a shock on the 10-year term premium.

To give an economic explanation to the response of the term premium shock is not straightforward because we need more ingredients to be able to interpret accurately the shock, such as for instance inflation, private investment or government spending. Notwithstanding, the shape of the response, a decrease followed by an increase in the GDP, provide us some insight about the nature of the shock. For instance, we can conjecture that the term premium shock could be compared with a shock on government spending that would be financed by an issue of long term bonds [see also Greenwood and Vayanos (2008)]. Such a policy can generate two opposite effects on activity. First, higher long term interest rates tends to reduce private investment, and have negative effect on real GDP. Second, public investment tends to boost activity. Our results suggest that the first effect dominates in the short run, explaining the decreasing trend of real GDP during the first year, and is progressively offset by the second effect, leading the real GDP to increase in the long run. Of course, at this stage of our analysis we can only venture some interpretation that one has to verify with a more accurate macroeconomic (structural) model [see, for instance, Rudebusch and Swanson (2008)]. In addition, we can think that the ambiguity found in empirical results based on static regressions of the term premium component on future activity, could stem from the changing

sign of this relationship over the period that follows the shock.

Finally, based on our "preferred" NCVAR(3) estimation, our results seem to speak in favor of a positive long run relationship between the term premium and future activity. In addition, this analysis highlights that the model used in measuring the term premium is a key element when one wants to gauge whether or not shifts in the term premium affect future activity. Notably, the number of lags seems to be critical. This point appears to be less important when one focuses on the effects of a shock on the spread or on its expectation part.

7 Conclusions and Further Developments

In this paper we have proposed potential solutions to the computation of the term premia and to their ambiguous relationship with future economic activity. This problem has been handled, first, by using an averaging estimator [*à la* B. Hansen (2010)] to draw reliable forecasts from a (Near-Cointegrated) VAR(3) model in order to extract reliable term premia measures. The term premia thus obtained provide a decomposition of the long yields which are quite different from those obtained by the VAR(3), CVAR(3) or VAR(1) models, especially in the recent years. Then, the general concept of *New Information Response Function (NIRF)*, introduced by Jarret, Monfort and Pegoraro (2009b), has been used to study the effects on future GDP growth induced by a shock on term premia. The two new features of our econometric approach, namely the averaging estimation and the number of lags, play an important role in the empirical findings, the first feature being central for the computation of the term premia and the second one for the evaluation of the responses to shocks.

From a macroeconomic point of view, the starting point of our analysis has been a 3-dimensional VAR model with GDP growth [as in APW (2006)] that guarantees at the same time a parsimonious parametrization of the model and ease of estimation. It is clear that it would be useful to extend the state vector in order to introduce other relevant information like the inflation rate [see Hordahl, Tristani and Vestin (2006, 2008), Rudebusch and Wu (2008), Bikbov and Chernov (2010)]. This

variable is another well known nearly non-stationary process that provides an additional source of persistence that one has to handle (in addition to the interest rate persistence) in order to provide reliable measures of inflation and real rate expectations (and associated real rate and inflation risk premia). The objective of an ongoing research work is to extract these measures by means of the Near-Cointegrated modelling and to consider a comparison with other approaches where yield and inflation persistence is taken into account by means of additional data sources like survey forecasts on interest rates and inflation rates [see Kim and Orphanides (2005) and Chernov and Mueller (2008)].

From an econometric point of view, it is also known that the persistence problem (and associated forecast performances) can be tackled in a way different from the above mentioned averaging estimator. For instance, it would be possible to try to extend to macro-finance models the switching regime approach which has been used successfully in pure finance models [see Bansal and Zhou (2002), Bansal, Tauchen and Zhou (2004), Dai, Singleton and Yang (2007) and Monfort and Pegoraro (2007)] and thus checking how persistence properties are transformed within each regime and verify if prediction performances improve [see Veronesi and Yared (2000), Evans (2003), Ang, Bekaert and Wei (2008) and Bikbov and Chernov (2008)]. Another possibility could be the shifting endpoint methodology of Kozicki and Tinsley (2001a, 2001b) and the generalization provided by Dewachter, Lyrio and Maes (2006). A comparison between these methodologies and the Near-Cointegrated one will be considered in future research.

REFERENCES

- Ang, A., Bekaert, G., (2002): "Short Rate Nonlinearities and Regime Switches", *Journal of Economic Dynamics and Control*, 26, 1243-1274.
- Ang, A., Bekaert, G., Wei, M. (2008): "Term Structure of Real Rates and Expected Inflation", *The Journal of Finance*, 63(2), 797-849.
- Ang, A., Piazzesi, M., Wei, M., (2006): "What does the Yield Curve tell us about GDP Growth", *Journal of Econometrics*, 131, 359-403.
- Backus, D., and Wright, J., (2007): "Cracking the Conundrum", Board of Governors of the Federal Reserve System, Working Paper 2007-46.
- Bansal, R., Zhou, H., (2002): "Term Structure of Interest Rates with Regime Shifts", *Journal of Finance*, 57, 1997-2043.
- Bansal, R., Tauchen, G., Zhou, H., (2004): "Regime Shifts, Risk Premiums in the Term Structure, and the Business Cycle", *Journal of Business and Economic Statistics*, 22(4), 396-409.
- Bauer, M., (2010): "Term Premia and the News", Federal Reserve Bank of San Francisco, Working Paper.
- Bernanke, B.S., Reinhart, V.R., and Sack, B. (2004): "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment", Board of Governors of the Federal Reserve System, Working Paper 2004-48.
- Beechey, M., (2007): "A Closer Look at the Sensitivity Puzzle: The Sensitivity of Expected Future Short Rates and Term Premia to Macroeconomic News", Board of Governors of the Federal Reserve System, Working Paper 2007-06.
- Bikbov, R., and Chernov, M., (2008): "Monetary Policy Regimes and the Term Structure of Interest Rates", London Business School, Working Paper.
- Bikbov, R., and Chernov, M., (2010): "No-Arbitrage Macroeconomic Determinants of the Yield Curve", London Business School, *Journal of Econometrics*, forthcoming.
- Campbell, J.Y., Cochrane, J. (1999): "By force of habit: A consumption-based explanation of aggregate stock market behavior", *Journal of Political Economy*, 107 (2), 205-251.
- Cochrane, J., and Piazzesi, M., (2008): "Decomposing the Yield Curve", Graduate School of Business, University of Chicago, Working Paper.
- Chernov, M., Mueller, P., (2008): "The Term Structure of Inflation Expectations", London Business School, Working Paper.
- Dai, Q., Singleton, K., Yang, W., (2007) : "Regime Shifts in a Dynamic Term Structure Model of U.S. Treasury Bond Yields", *Review of Financial Studies*, 20(5), 1669-1706.
- Duffee, G., Stanton, R., (2008): "Evidence on Simulation Inference for Near Unit-Root Processes with Implications for the Term Structure", *Journal of Financial Econometrics*, 6, 108-142.

- Dewachter, H., Lyrio, M., Maes, K., (2006): "A Joint Model for the Term Structure of Interest Rates and the Macroeconomy", *Journal of Applied Econometrics*, 21(4), 439-462.
- Evans, M., (2003): "Real Risk, Inflation Risk, and the Term Structure", *The Economic Journal*, 113, 345-389.
- Favero, C., Kaminska, I., Söderström, U., (2005): "The Predictive Power of the Yield Spread: Further Evidence and a Structural Interpretation", Working Paper, Università Bocconi.
- Garcia, R., Luger, R., (2007): "Risk Aversion, Intertemporal Substitution, and the Term Structure of Interest Rates", Working Paper, Université de Montreal.
- Gourieroux, C., Monfort, A. (1996): "Simulation based econometric methods", Oxford University Press.
- Gourieroux, C., Monfort, A., Renault, E., (1993): "Indirect Inference", *Journal of Applied Econometrics*, 8, S85-S118.
- Gourieroux, C., Renault, E., Touzi, N. (2000): "Calibration by simulation for small sample's bias correction", in *Simulation-Based Inference in Econometrics*, Mariano, Schuermann, Weeks eds, Cambridge University Press.
- Gray, S., (1996): "Modeling the Conditional Distribution of Interest Rates as a Regime Switching Process", *Journal of Financial Economics*, 42, 27-62.
- Greenwood, R., and Vayanos, D., (2008): "Bond Supply and Excess Bond Returns", Harvard Business School, working paper.
- Gürkaynak, R., Sack, B., Wright, J., (2007), "The U.S. Treasury Yield Curve: 1961 to the Present", *Journal of Monetary Economics*, 54, 2291-2304.
- Hamilton, J.D., Kim, D.H., (2002): "A re-examination of the predictability of the yield spread for real economic activity", *Journal of Money, Credit, and Banking*, 34, 340-360.
- Hansen, B., (2010): "Averaging Estimators for Autoregressions with a Near Unit Root", *Journal of Econometrics*, 158, 142-155.
- Hansen, L.P., and Sargent, T.J., (2007): "Robustness", Princeton University Press.
- Hansen, L.P., and Sargent, T.J., (2008): "Fragile Beliefs and the Price of Model Uncertainty", University of Chicago, working paper.
- Harvey, C.R. (1989): "Time-varying conditional covariances in tests of asset pricing models", *Journal of Financial Economics*, 24 (2), 289-317.
- Hordahl, P., Tristani, O., and Vestin, D., (2006): "A Joint Econometric Model of Macroeconomic and Term Structure Dynamics", *The Journal of Econometrics*, 131, 405 - 444.
- Hordahl, P., Tristani, O., and Vestin, D., (2008): "The Yield Curve and Macroeconomics Dynamics", *The Economic Journal*, 118, 1937 - 1970.
- Jardet, C., Monfort, A., and Pegoraro, F., (2009a): "Persistence, Bias, Prediction and Averaging Estimators", Banque de France Working Paper.

- Jardet, C., Monfort, A., and Pegoraro, F., (2009b): "New Information Response Functions", Banque de France Working Paper.
- Jardet, C., Monfort, A., and Pegoraro, F., (2009c): "No-Arbitrage Near-Cointegrated VAR(p) Term Structure Models, Term Premia and GDP Growth", Banque de France Working Paper.
- Jardet, C., Monfort, A., and Pegoraro, F., (2010): "Online Appendix for 'No-arbitrage Near-Cointegrated VAR(p) Term Structure Models, Term Premia and GDP Growth'", available at the website <http://www.crest.fr/pageperso/pegoraro/pegoraro.htm>.
- Joslin, S., Priebsch, M., and Singleton, K., (2010): "Risk Premiums in Dynamic Term Structure Models with Unspanned Macro Risks", Graduate School of Business, Stanford University, Working Paper.
- Kendall, M.G. (1954): "A note on bias in the estimation of autocorrelation", *Biometrika*, 41, 403-404.
- Kim, D.H., and Orphanides, A., (2005): "Term Structure Estimation with Survey Data on Interest Rate Forecasts", Board of Governors of the Federal Reserve System, Working Paper 2005-48.
- Kim, D.H., and Orphanides, A., (2007): "The Bond Market Term Premium: what is it, and how can we measure it?", Bank of International Settlement, Quarterly Review.
- Kim, D.H., and Wright, J., (2005): "An Arbitrage-Free Three-Factor Term Structure Model and the Recent Behavior of Long-Term Yields and Distant-Horizon Forward Rates", Board of Governors of the Federal Reserve System, Working Paper 2005-33.
- Kozicki, S., and Tinsley, P.A., (2001a): "Term Structure Views of Monetary Policy Under Alternative Models of Agent Expectations", *Journal of Economic Dynamics and Control*, 25, 149-184.
- Kozicki, S., and Tinsley, P.A., (2001b): "Shifting Endpoints in the Term Structure of Interest Rates", *Journal of Monetary Economics*, 47, 613-652.
- Lustig H. N., Roussanov, N.L., Verdelhan A. (2010): "Countercyclical Currency Risk Premia", NBER Working Papers 16427.
- Monfort, A., Pegoraro, F., (2007): "Switching VARMA Term Structure Models", *Journal of Financial Econometrics*, 5(1), 105-153.
- Pesaran, M.H., Shin, Y., (1998): "Generalized Impulse Response Analysis in Linear Multivariate Models", *Economic Letters*, 58(1), 17-29.
- Rosenberg, J.V., Maurer, S., (2007): "Signal or Noise? Implications of the Term Premium for Recession Forecasting", Federal Reserve Bank of New York Economic Policy Review, forthcoming.
- Rudebusch, G.D., Sack, B., Swanson, E., (2007): "Macroeconomic Implications of Changes in the Term Premium", Federal Reserve Bank of St. Louis Review, 89(4), 241-269.
- Rudebusch, G.D., Swanson, E., (2008): "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks", Federal Reserve Bank of San Francisco, working paper.
- Rudebusch, G.D., Wu, T. (2008): "A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy", *Economic Journal*, 118, 906-926.

Svensson, L.E.O., (1994): "Estimating and Interpreting Forward Rates: Sweden 1992-4", National Bureau of Economic Research Working Paper n. 4871.

Uhlig, H., (1994): "What Macroeconomists Should Know about Unit Roots: A Bayesian Perspective", *Econometric Theory*, 10(3/4), 645-671.

Wachter, J. A. (2006): "A consumption-based model of the term structure of interest rates", *The Journal of Financial Economics*, 79 (2), 365-399.

Veronesi, P., and Yared, F., (2000): "Short and Long Horizon Term and Inflation Risk Premia in the US Term Structure", Working Paper, University of Chicago - GSB.