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Performance analysis of free-space optical communication systems over atmospheric turbulence channels

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Abstract: Turbulence fading is one of the main impairments affecting the operation of free-space optical (FSO) communication systems. The authors study the performance of FSO communication systems, also known as wireless optical communication systems, over log-normal and gamma–gamma atmospheric turbulence-induced fading channels. These fading models describe the atmospheric turbulence because of its very good agreement with experimental measurement data. Closed-form expressions for the average (ergodic) capacity and the outage probability are derived for both statistical models. Another contribution of this work is a study of how the performance metrics are affected by the atmospheric conditions and other parameters such as the length of the link and the receiver's aperture diameter. The derived analytical expressions are verified by various numerical examples and can be used as an alternative to time-consuming Monte-Carlo simulations.

1 Introduction

Free-space optics, also known as wireless optical, is a potentially high-capacity and cost-effective communication technique, which has been receiving attention and commercial interest [1–3]. Furthermore, FSO communications are not subject to frequency spectrum regulations and are not jamming from external sources. However, the performance of FSO communication systems is highly susceptible to adverse atmospheric conditions caused by the variations in the refractive index because of inhomogeneities in temperature and pressure changes. As a result, the optical signal intensity rapidly fluctuates, known as scintillation, degrading the system performance particularly for ranges greater than 1 km [4–10]. Hence, it is reasonable to expect that FSO channels appear to have randomly time-varying characteristics, and thus, channel capacity becomes a time-dependent random variable (RV). Consequently, metrics such as average capacity, representing the average best for error-free transmission, and the outage probability can be considered as particularly useful in evaluating the wireless optical channel performance [7, 10–18].

Various statistical models have been proposed over the years to describe the optical channel characteristics with respect to the atmospheric turbulence strength [3]. Specifically, log-normal and gamma–gamma distributions have been found to be suitable for modelling optical channels for weak-to-moderate and moderate-to-strong turbulence channels, respectively, since they provide a good agreement between theoretical and experimental data [4, 6, 8, 9, 18]. In [8], Uysal *et al.* have evaluated the error performance of coded FSO links over gamma–gamma turbulence fading. Additionally, the bit-error-rate (BER) performance of an FSO heterodyne communication system in the presence of gamma–gamma turbulence fading has been studied in [5], whereas the overall performance of a relayed FSO system has been examined in [10]. In [14], Li and Uysal have studied the ergodic capacity of log-normal turbulence channels. Furthermore, Zhu and Khan have studied the error rate performance of FSO links over a log-normal turbulence model in [19]. However, to the best of the authors' knowledge, there are no closed-form expressions for the evaluation of the average capacity and the outage probability of FSO communications for both fading models.

In this work, we study the performance of FSO channels by investigating their outage probability and the average (ergodic) capacity, respectively. Thus, we derive closed-form expressions for the outage probability and the average capacity of optical links over atmospheric turbulence-induced fading channels modelled by the log-normal and the gamma–gamma distribution with respect to the turbulence strength, as well as the influence of other important systems' parameters, such as optical link length and the receiver's aperture diameter.

The rest of the paper is organized as follows: in Section 2, we introduce the FSO channel model. In Section 3, we present the mathematical expressions for the estimation of the outage probability. Next, in Section 4, we derive closed-form expressions for the average channel capacity of the log-normal and the gamma–gamma modelled FSO channels. In Section 5, we present numerical results as derived for each of the two models and we examine the capabilities of FSO systems as a function of their parameters. The final conclusions are presented in Section 6.

2 Channel model

We consider a point-to-point FSO communication system using intensity modulation/direct detection (IM/DD). The laser beam propagates along a horizontal path through a turbulence channel with additive white Gaussian noise (AWGN). The channel is assumed to be memoryless, stationary and ergodic, with independent and identically distributed (i.i.d.) intensity fading statistics. We also consider that the channel state information (CSI) is available at both the transmitter and the receiver. In this case, the statistical channel model is given by [7, 18–20]

$$y = sx + n = \eta I x + n \tag{1}$$

where y is the signal at the receiver, $s = \eta I$ is the instantaneous intensity gain, η is the effective photo-current conversion ratio of the receiver, I is the normalized irradiance, x is the modulated signal that takes values 0 or 1 and n is the AWGN with zero mean and variance $N_0/2$. For weak-to-moderate atmospheric turbulence conditions, the turbulence-induced fading is assumed to be a random process that follows the log-normal distribution [3], whereas for moderate-to-strong turbulence conditions a gamma–gamma distribution is used [3, 10].

2.1 The log-normal turbulence model

The probability density function (PDF) of the log-normal model is given by [3]

$$f_I(I) = \frac{1}{I\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(I) + \sigma^2/2)^2}{2\sigma^2}\right) \tag{2}$$

where σ is the standard deviation of the log-normal distribution, which depends on the channel's characteristics as given by [9, 17]

$$\sigma^2 = \exp\left[\frac{0.49\delta^2}{(1 + 0.18d^2 + 0.56\delta^{12/5})^{7/6}} + \frac{0.51\delta^2}{(1 + 0.9d^2 + 0.62d^2\delta^{12/5})^{5/6}}\right] - 1 \tag{3}$$

where $d = \sqrt{kD^2/4L}$, $k = 2\pi/\lambda$ is the optical wave number, L is the length of the optical link and D is the receiver's aperture diameter. The parameter δ^2 is the Rytov variance given by

$$\delta^2 = 1.23 C_n^2 k^{7/6} L^{11/6} \tag{4}$$

with C_n^2 being the altitude-dependent turbulence strength and varying from 10^{-17} to 10^{-13} m$^{-2/3}$ according to the atmospheric turbulence conditions [2, 8].

The cumulative distribution function (CDF) for the log-normal distribution model is obtained by integrating (2), yielding

$$F_I(I) = \frac{1}{2}\mathrm{erfc}\left(-\frac{\ln(I) + \sigma^2/2}{\sqrt{2}\sigma}\right) \tag{5}$$

where erfc is the complementary error function defined in [21].

By defining the instantaneous electrical signal-to-noise ratio (SNR) as $\mu = (\eta I)^2/N_0 = s^2/N_0$, the average electrical SNR will be given by $\bar{\mu} = (\eta E[I])^2/N_0$ as previously defined in [19], where $E[.]$ denotes the expectation. Then, considering that $E[I] = 1$ since I is normalized to unity, and after a power transformation of the RV I in (2), the electrical SNR PDF can be written as

$$f_\mu(\mu) = \frac{1}{2\mu\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(\mu/\bar{\mu}) + \sigma^2)^2}{8\sigma^2}\right) \tag{6}$$

By integrating (6), the CDF with respect to μ can be obtained as

$$F_\mu(\mu) = \frac{1}{2}\mathrm{erfc}\left(\frac{\ln(\bar{\mu}/\mu) - \sigma^2}{2\sqrt{2}\sigma}\right) \tag{7}$$

Hence, (6) and (7) present the PDF and the CDF of the electrical SNR of the log-normal turbulence channel.

2.2 The gamma–gamma turbulence model

The PDF of the gamma–gamma model is given by [3]

$$f_I(I) = \frac{2(ab)^{(a+b)/2}}{\Gamma(a)\Gamma(b)} I^{\{(a+b)/2\}-1} K_{a-b}\left(2\sqrt{abI}\right) \tag{8}$$

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where $K_\nu(.)$ is the modified Bessel function of the second kind of order ν, $\Gamma(.)$ is the gamma function and a, b can be directly related to atmospheric conditions through the expressions below [8, 18]

$$a = \left[\exp\left(\frac{0.49\delta^2}{\left(1 + 0.18d^2 + 0.56\delta^{12/5}\right)^{7/6}}\right) - 1\right]^{-1} \tag{9}$$

and

$$b = \left[\exp\left(\frac{0.51\delta^2}{\left(1 + 0.9d^2 + 0.62d^2\delta^{12/5}\right)^{5/6}}\right) - 1\right]^{-1} \tag{10}$$

where the parameters δ and d have been defined above.

By integrating (6), the CDF of the gamma–gamma distribution with respect to I can be derived as [5, 10]

$$F_I(I) = \frac{(abI)^{(a+b)/2}}{\Gamma(a)\Gamma(b)} G_{1,3}^{2,1}\left(abI\left|\begin{matrix} 1 - \frac{a+b}{2} \\ \frac{a-b}{2}, \frac{b-a}{2}, -\frac{a+b}{2}\end{matrix}\right.\right) \tag{11}$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer G-function [22].

The PDF of the instantaneous electrical SNR, μ, is obtained from (6) with a power transformation of I as

$$f_\mu(\mu) = \frac{(ab)^{a+b/2}}{\Gamma(a)\Gamma(b)} \frac{\mu^{\{(a+b)/4\}-1}}{\bar{\mu}^{(a+b)/4}} K_{a-b}\left(2\sqrt{ab\sqrt{\frac{\mu}{\bar{\mu}}}}\right) \tag{12}$$

By expressing the Bessel function $K_\nu(.)$ in terms of Meijer G-function

$$\left(K_\nu(z) = \frac{1}{2} G_{0,2}^{2,0}\left(z\left|\begin{matrix} - \\ \frac{\nu}{2}, -\frac{\nu}{2}\end{matrix}\right.\right)\right)$$

and integrating (12) by using [22, eq. (21)], the CDF of the electrical SNR of the gamma–gamma turbulence fading can be easily derived as

$$F_\mu(\mu) = \frac{(ab)^{(a+b)/2}}{\Gamma(a)\Gamma(b)} \left(\frac{\mu}{\bar{\mu}}\right)^{(a+b)/4} \times G_{1,3}^{2,1}\left(ab\sqrt{\frac{\mu}{\bar{\mu}}}\left|\begin{matrix} 1 - \frac{a+b}{2} \\ \frac{a-b}{2}, \frac{b-a}{2}, -\frac{a+b}{2}\end{matrix}\right.\right) \tag{13}$$

Hence, following (12) and (13), the PDF and CDF of the gamma–gamma distribution with respect to μ can readily be evaluated as a function of the atmospheric turbulence strength and the parameters of the optical link.

3 Outage probability

Following the above, we extract closed-form expressions for the evaluation of the outage probability of an FSO link. We consider this as a particularly important metric for the design of such a system as it represents the probability that the instantaneous SNR falls below a critical threshold, μ_{th}, which corresponds to the receiver's sensitivity limit. Thus, the outage probability for weak-to-moderate turbulence strength is obtained from the log-normal distribution model (7), given by

$$P_{\text{out}} = \Pr(\mu \le \mu_{\text{th}}) = F_\mu(\mu_{\text{th}}) = \frac{1}{2}\operatorname{erfc}\left(\frac{\ln(\bar{\mu}/\mu_{\text{th}}) - \sigma^2}{2\sqrt{2}\sigma}\right) \tag{14}$$

whereas for moderate-to-strong turbulence strength the above probability arises from the gamma–gamma distribution model (13) and is given by

$$P_{\text{out}} = \Pr(\mu \le \mu_{\text{th}}) = F_\mu(\mu_{\text{th}}) = \frac{(ab)^{(a+b)/2}}{\Gamma(a)\Gamma(b)}\left(\frac{\mu_{\text{th}}}{\bar{\mu}}\right)^{(a+b)/4} \times G_{1,3}^{2,1}\left(ab\sqrt{\frac{\mu_{\text{th}}}{\bar{\mu}}}\left|\begin{matrix} 1 - \frac{a+b}{2} \\ \frac{a-b}{2}, \frac{b-a}{2}, -\frac{a+b}{2}\end{matrix}\right.\right) \tag{15}$$

4 Average channel capacity

The average (ergodic) capacity, $\langle C\rangle$, represents the practically achievable capacity of an FSO channel with atmospheric turbulence-induced fading and it is a quite crucial metric for evaluating the link performance [7, 11–14]. Considering that perfect channel-state information (CSI) is available at both the receiver and the transmitter of an FSO communication system, the average achievable capacity can be defined as

$$\langle C\rangle = \int_0^\infty B\log_2\left(1 + \frac{(\eta I)^2}{N_0}\right) f_I(I)\,\mathrm{d}I \tag{16}$$

where B is the signal transmission bandwidth and $f_I(I)$ the PDF of the corresponding distribution model.

4.1 Log-normal turbulence fading

For weak-to-moderate atmospheric turbulence conditions, the average capacity of the FSO system under consideration can be obtained by applying (2)–(16), yielding

$$\langle C\rangle = \frac{B}{2\sigma\sqrt{2\pi}\ln(2)}\int_0^{+\infty}\frac{\ln(1+\mu)}{\mu}\exp\left[-\frac{(\ln(\mu) - A)^2}{8\sigma^2}\right]\mathrm{d}\mu \tag{17}$$

where $A = \ln(\bar{\mu}) - \sigma^2$ and taking into account [15, 16, 21], we obtain the following summation, for the estimation of the

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average capacity

$$\langle C\rangle = BC_0\left\{\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\left[\operatorname{erfcx}\left(\sqrt{2}\sigma k+\frac{A}{2\sqrt{2}\sigma}\right)\right.\right.$$
$$\left.+\operatorname{erfcx}\left(\sqrt{2}\sigma k-\frac{A}{2\sqrt{2}\sigma}\right)\right]+\frac{4\sigma}{\sqrt{2\pi}}$$
$$\left.+A\exp\left(\frac{A^2}{8\sigma^2}\right)\operatorname{erfc}\left(-\frac{A}{2\sqrt{2}\sigma}\right)\right\} \tag{18}$$

where

$$C_0=\frac{\exp(-A^2/8\sigma^2)}{2\ln(2)}$$

In order to evaluate (18), the infinite sum over k must be calculated. However, evaluating numerically the result of (17) and (18) and taking into account a very accurate approximation for the evaluation of the average capacity [17], we obtain

$$\langle\tilde{C}\rangle = BC_0\left\{\sum_{k=1}^{8}a_k\left[\operatorname{erfcx}\left(\sqrt{2}\sigma k+\frac{A}{2\sqrt{2}\sigma}\right)\right.\right.$$
$$\left.+\operatorname{erfcx}\left(\sqrt{2}\sigma k-\frac{A}{2\sqrt{2}\sigma}\right)\right]+\frac{4\sigma}{\sqrt{2\pi}}$$
$$\left.+A\exp\left(\frac{A^2}{8\sigma^2}\right)\operatorname{erfc}\left(-\frac{A}{2\sqrt{2}\sigma}\right)\right\} \tag{19}$$

The relative estimation error $\left|\langle\tilde{C}\rangle-\langle C\rangle\right|/\langle C\rangle$ is found to be of the order of 10^{-9} for most of the cases that we study below. Therefore, we can conclude that (19) is practically a perfectly accurate closed-form expression for the evaluation of the average capacity of a log-normal-modelled FSO channel, and thus, $\langle C\rangle \cong \langle\tilde{C}\rangle$.

4.2 Gamma–gamma turbulence fading

For medium-to-strong atmospheric turbulence conditions and by applying (8) to (16), the average capacity can be written as

$$\langle C\rangle=\frac{B(ab/\bar{\mu})^{(a+b)/2}}{\Gamma(a)\Gamma(b)\ln(2)}\int_0^{+\infty}\ln(1+\mu)\mu^{\{(a+b)/4\}-1}$$
$$\times K_{a-b}\left(2\sqrt{ab\sqrt{\frac{\mu}{\bar{\mu}}}}\right)\mathrm{d}\mu \tag{20}$$

By expressing $K_\nu(.)$ as the Meijer G-function and also taking into account that

$$\ln(1+z)=G_{2,2}^{1,2}\left(z\left|\begin{matrix}1,1\\1,0\end{matrix}\right.\right)$$

and integrating [22], the following closed-form mathematical expression is obtained

$$\langle C\rangle=\frac{BA_0}{\bar{\mu}^{(a+b)/4}}$$
$$\times G_{2,6}^{6,1}\left(\frac{(ab)^2}{16\bar{\mu}}\left|\begin{matrix}-\frac{a+b}{4},\ -\frac{a+b}{4}+1\\ \frac{a-b}{4},\ \frac{a-b+2}{4},\ \frac{b-a}{4},\ \frac{b-a+2}{4},\\ -\frac{a+b}{4},\ -\frac{a+b}{4}\end{matrix}\right.\right) \tag{21}$$

where

$$A_0=\frac{(ab)^{(a+b)/2}}{4\pi\Gamma(a)\Gamma(b)\ln(2)}$$

5 Numerical results

Using the above-derived closed forms for the outage probability, (14) and (15), and the average capacity, (19) and (21), we now investigate the reliability and the performance of an FSO link using practical system parameters. The parameters under consideration that affect the system efficiency are the length L of the FSO link and the receiver's aperture diameter, D, for different values of the atmospheric turbulence strength, C_n^2. For typical FSO systems, the value of L varies between 500 and 8000 m, and D, between $D = 0.01$ and 0.2 m. In the analysis below we consider two typical values of L ($L = 1000$ m, short length FSO system and $L = 7000$ m, long length FSO system), and two values of D ($D = 0.02$ and 0.08 m) taking into account three different atmospheric turbulence conditions, weak ($C_n^2 = 8.4\times10^{-15}$ m$^{-2/3}$), moderate ($C_n^2 = 1.7\times10^{-14}$ m$^{-2/3}$) and strong ($C_n^2 = 5.0\times10^{-14}$ m$^{-2/3}$), respectively.

For weak-to-moderate conditions, we use the log-normal model, whereas for strong conditions, the gamma–gamma model. For the selection of the turbulence strength values, we took into account that for an FSO link near the ground the value of the turbulence strength can be taken to be approximately 1.7×10^{-14} m$^{-2/3}$ during the daytime and 8.4×10^{-15} m$^{-2/3}$ at night [2, 7]. In this study, we consider $\lambda = 1.55$ μm, which is a typical wavelength for an FSO communication system [2]. Similar results can be obtained for any other wavelength, using (14), (15) and (19), (21).

In Fig. 1, we present the outage probability of the FSO link, as a function of the normalized average electrical SNR, $\bar{\mu}/\mu_{th}$, for weak, moderate and strong turbulence strength. In this case the length of the link is $L = 1000$ m and the aperture diameter of the receiver is, $D = 0.08$ m. It is obvious that the atmospheric conditions affect the reliability of the system but, even for strong turbulence, $C_n^2 = 5.0\times10^{-14}$ m$^{-2/3}$, and small values of the normalized electrical average SNR – but above 8 dB – its performance is still acceptable although the

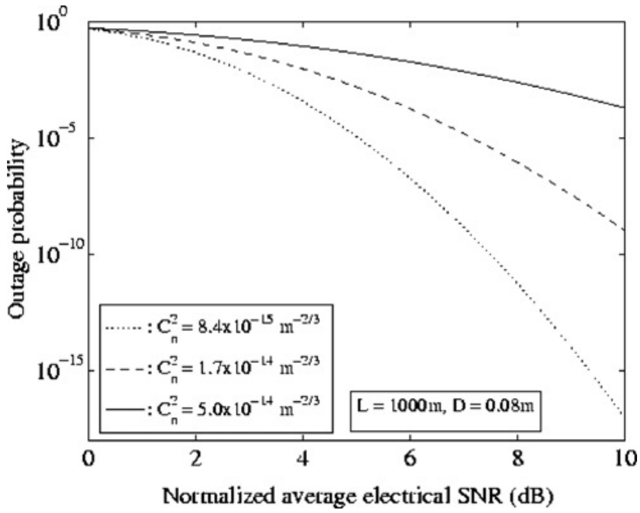

Figure 1 *Outage probability against the normalised average electrical SNR, $\bar{\mu}/\mu_{th}$, for weak-to-strong turbulence strength, C_n^2, for L = 1000 m and D = 0.08 m*

outage probability is smaller than 10^{-3}. The average channel capacity as a function of the average electrical SNR, $\bar{\mu}$, for different values of the strength of the atmospheric turbulence, for the above configuration, is presented in Fig. 2. Furthermore, in this figure, we plot the capacity of an AWGN channel. It is clear that the performance of the system is slightly affected by the atmospheric conditions, since even for strong turbulence, the average capacity decreases only 1.3%, when the turbulence strength increases from $C_n^2 = 8.4 \times 10^{-15}$ to $=5.0 \times 10^{-14}\,\mathrm{m}^{-2/3}$, for $\bar{\mu} = 20\,\mathrm{dB}$.

By increasing the length of the link, the influence of the atmospheric conditions becomes stronger. This is obvious in Figs. 3 and 4, where we evaluate the average capacity and the outage probability, for the same parameters as in Figs. 1 and 2, respectively, but for $L = 7000$ m. As shown in Fig. 3, for large values of C_n^2 the reliability of the system decreases even for large values of the normalized average SNR. More specifically, for $C_n^2 = 5.0 \times 10^{-14}\,\mathrm{m}^{-2/3}$ and $\bar{\mu}/\mu_{\mathrm{th}} = 10\,\mathrm{dB}$, the outage probability is of order of 10^{-1} whereas in the previous case (i.e. Fig. 1) it was of the order of 10^{-3}. Similar results are obtained in Fig. 4, where, even for weak turbulence, $C_n^2 = 8.4 \times 10^{-15}\,\mathrm{m}^{-2/3}$, the average capacity is $\langle C \rangle/B = 5.3$ b/s/Hz, for $\bar{\mu} = 20$dB, whereas for the case with the shorter link (i.e. Fig. 2) the respective capacity was 6.7 b/s/Hz, i.e. a decrease of 21%.

Another very significant parameter of an FSO communication system is the aperture diameter of the receiver, D. In order to study the influence of D in the

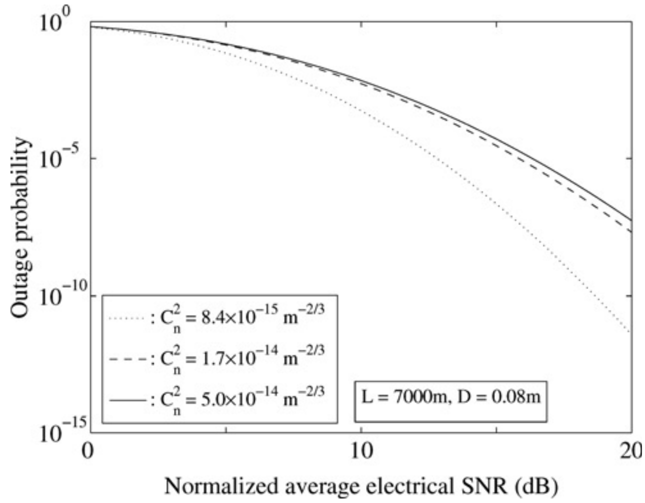

Figure 3 *Outage probability against the normalised average electrical SNR, $\bar{\mu}/\mu_{th}$, for weak-to-strong turbulence strength, C_n^2, for L = 7000 m and D = 0.08 m*

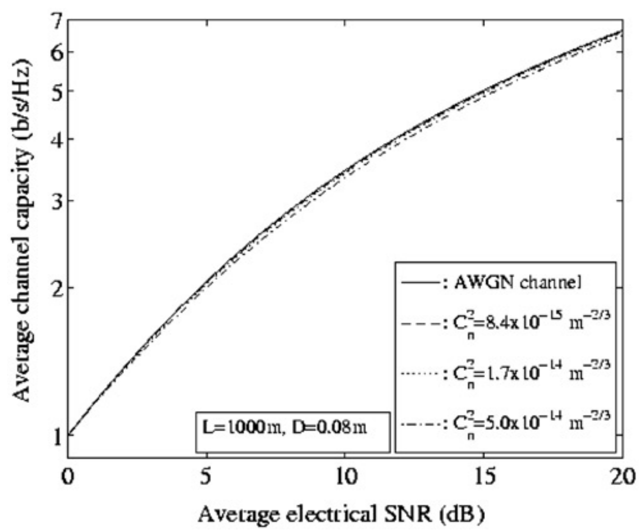

Figure 2 *Average capacity, $\langle C \rangle/B$, against the average electrical SNR, $\bar{\mu}$, for weak-to-strong turbulence strength, C_n^2, L = 1000 m and D = 0.08 m*

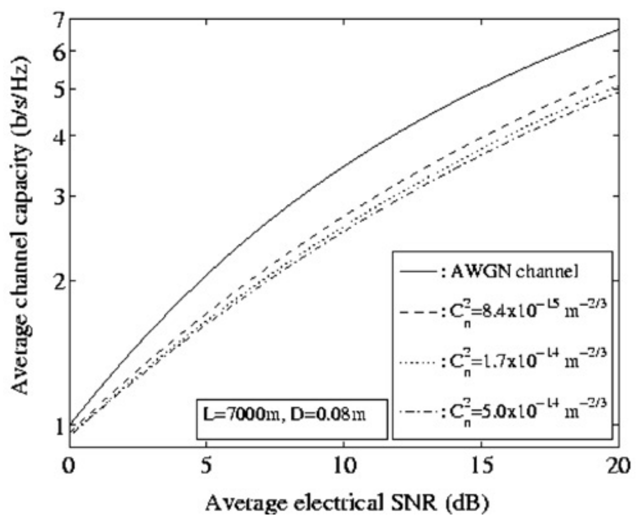

Figure 4 *Average capacity, $\langle C \rangle/B$, against the average electrical SNR, $\bar{\mu}$, for weak-to-strong turbulence strength, C_n^2, L = 7000 m and D = 0.08 m*

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capabilities of the system, we evaluate the outage probability and the average capacity of the system for $L = 1000$ m and $D = 0.02$ m. The results that are obtained are plotted in Figs. 5 and 6. Comparing the results of Fig. 1 with those of Fig. 5 we conclude that the outage probability of the system is fairly larger in the case of small D. For example, in Fig. 1 for strong turbulence conditions, $C_n^2 = 5.0 \times 10^{-14}\ \mathrm{m}^{-2/3}$ and $\bar{\mu}/\mu_{\mathrm{th}} = 10$dB, the outage probability was 1.1×10^{-4}, whereas in Fig. 5 the equivalent result was 4.0×10^{-2}. Similar results are obtained for the average capacity. Thus, comparing Figs. 2 and 6, we observe that for $C_n^2 = 5.0 \times 10^{-14}\ \mathrm{m}^{-2/3}$ and $\bar{\mu} = 20$ dB the average capacity is 6.1 b/s/Hz for small D, whereas for $D = 0.08$ m in Fig. 2, the average capacity is 6.6 b/s/Hz.

These conclusions are more unambiguous in Figs. 7 and 8 where we evaluate the average capacity and the outage probability of an FSO system with $D = 0.02$ m and $L = 7000$ m. Comparing the results of this case with the above ones, we conclude that the combination of small D and large L for strong turbulence conditions practically results in the interruption of the link. Thus, designing a system for such cases must aim to have acceptable values for the system's reliability and performance metrics.

Finally, in Fig. 9 the estimation error of the average capacity, $|\langle \tilde{C}\rangle - \langle C\rangle|/\langle C\rangle$, against the average electrical SNR, $\bar{\mu}$, is evaluated for one of the above-mentioned cases (i.e. $C_n^2 = 8.4 \times 10^{-15}\ \mathrm{m}^{-2/3}$, $L = 1000$ m and $D = 0.02$ m)

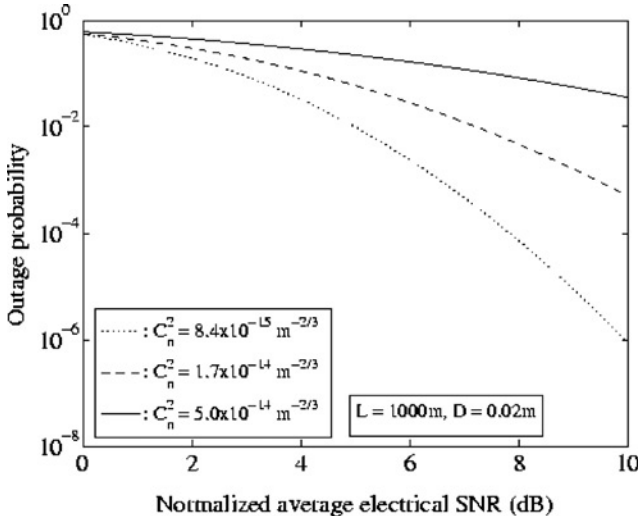

Figure 5 *Outage probability against the normalised average electrical SNR, $\bar{\mu}/\mu_{th}$, for weak-to-strong turbulence strength, C_n^2, for L = 1000 m and D = 0.02 m*

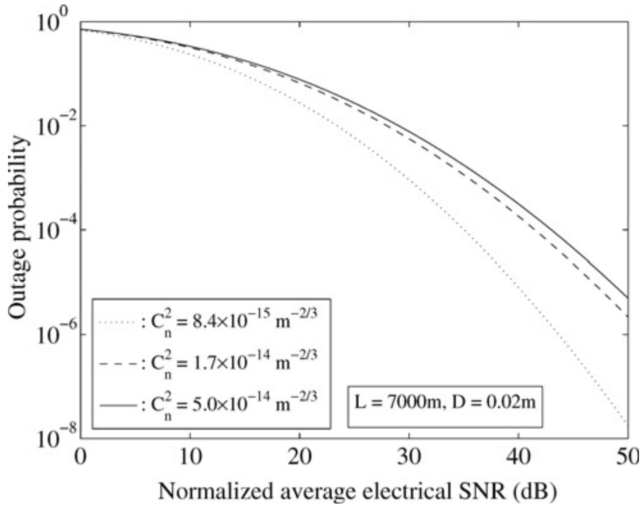

Figure 7 *Outage probability against the normalised average electrical SNR, $\bar{\mu}/\mu_{th}$, for weak-to-strong turbulence strength, C_n^2, for L = 7000 m and D = 0.02 m*

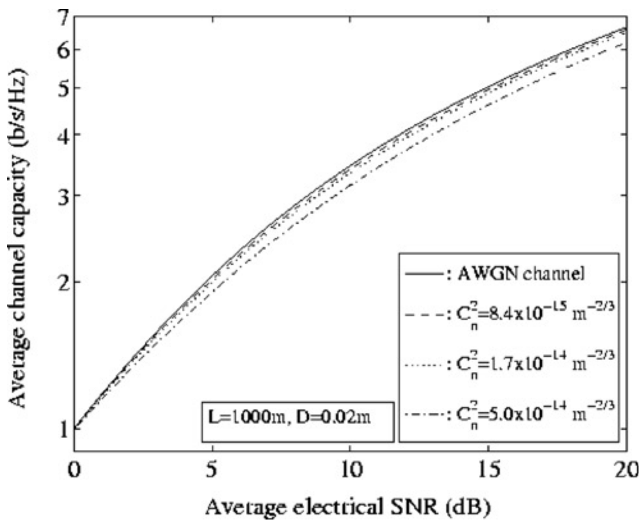

Figure 6 *Average capacity, $\langle C\rangle/B$, against the average electrical SNR, $\bar{\mu}$, for weak-to-strong turbulence strength, C_n^2, L = 1000 m and D = 0.02 m*

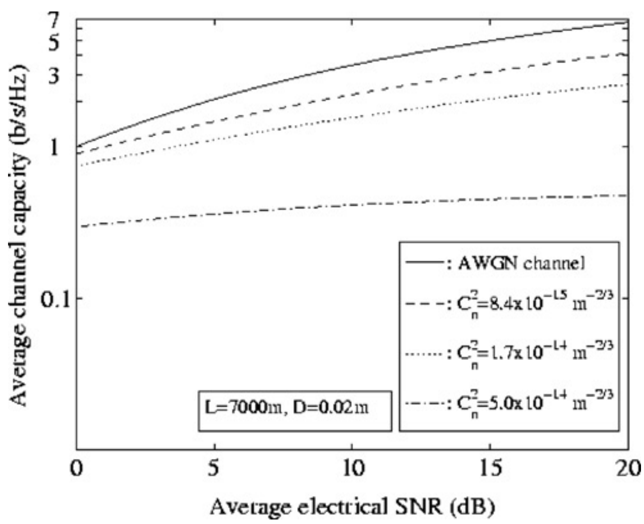

Figure 8 *Average capacity, $\langle C\rangle/B$, against the average electrical SNR, $\bar{\mu}$, for weak-to-strong turbulence strength, C_n^2, L = 7000 m and D = 0.02 m*

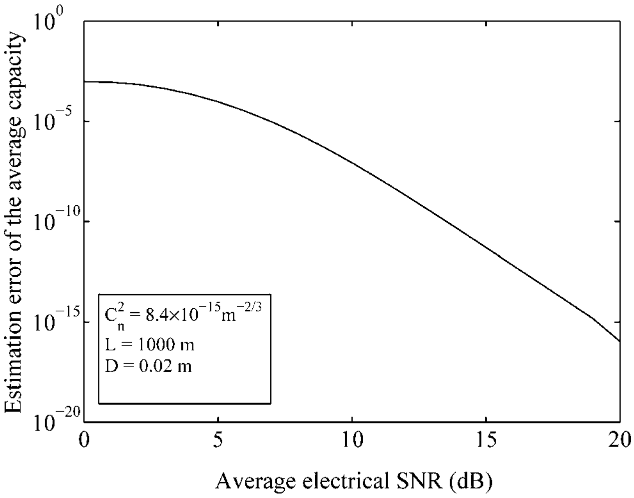

Figure 9 *Estimation error of the average capacity, $|\langle\tilde{C}\rangle - \langle C\rangle|/\langle C\rangle$, against the average electrical SNR, $\bar{\mu}$, for log normal turbulence channels, for the case with $C_n^2 = 8.4 \times 10^{-15}\ m^{-2/3}$, L = 1000 m and D = 0.02 m*

but similar results can be obtained for all the other cases. From this figure, it is obvious that as the average electrical SNR increases, the estimation error decreases rapidly. Thus, for $\bar{\mu} = 20$ dB the estimation capacity error is less than 10^{-15}.

The above results clearly show that the influence of the atmospheric turbulence conditions depends strongly on the length of the link. For a relatively short link, the fluctuations of C_n^2 can cause a small variation of the performance of the optical system. In this case, increasing the aperture diameter of the receiver may anticipate these variations. For longer links, the atmospheric conditions play a very significant role and for large values of C_n^2 (strong turbulence), the system cannot operate properly. In these cases, the increasing of D may improve the efficiency. However, if this is not enough, relay techniques [10] may be incorporated but this is beyond the scope of this work.

6 Conclusions

In this work, we have derived closed-form expressions for the evaluation of the average capacity and the outage probability of a typical FSO communication system under weak-to-moderate and moderate-to-strong turbulence atmospheric conditions modelled by the log-normal and gamma–gamma distributions, respectively. We studied the dependence of the reliability and the performance of the system as a function of the principal parameters of such a link, being the length of the link, the aperture diameter of the receiver and the atmospheric turbulence conditions between the transmitter and the receiver. Furthermore, we investigated their particular influence on a typical FSO system and proposed ways to increase the practical link efficiency.

The analysis presented in this paper can be further extended to the study of the capacity in multiple-input multiple-output (MIMO) FSO systems or to other modulation schemes such as pulse position modulation (PPM).

7 References

[1] KEDAR D., ARNON S.: 'Urban optical wireless communication network: The main challenges and possible solutions', *IEEE Optical Communications Supplement to IEEE Communications Magazine*, 2004, pp. S1–S7

[2] ARNON S.: 'Optical wireless communications', chapter in the Encyclopedia of Optical Engineering (EOE), R.G. DRIGGERS (ED.) (Marcel Dekker, 2003), pp. 1866–1886

[3] MAJUMDAR A.K.: 'Free-space laser communication performance in the atmospheric channel', *Journal of Optical and Fiber Communications Research*, 2005, **2**, pp. 345–396

[4] VETELINO F.S., YOUNG S., ANDREWS L.: 'Fade statistics and aperture averaging for gaussian beam waves in moderate to strong turbulence', *Appl. Opt.*, 2007, **46**, (18), pp. 3780–3789

[5] TSIFTSIS T.A.: 'Performance of heterodyne wireless optical communications systems over gamma–gamma atmospheric turbulence channels', *Electron. Lett.*, 2008, **44**, (5), pp. 373–375

[6] AL-HABASH M.A., ANDREWS L.C., PHILIPS R.L.: 'Mathematical model for the irradiance PDF of a laser beam propagating through turbulent media', *Opt. Eng.*, 2001, **40**, (8), pp. 1554–1562

[7] SANDALIDIS H.G., TSIFTSIS T.A.: 'Outage probability and ergodic capacity of free-space optical links over strong turbulence', *Electron. Lett.*, 2008, **44**, (1), pp. 46–47

[8] UYSAL M., LI J., YU M.: 'Error rate performance analysis of coded free-space optical links over gamma–gamma atmospheric turbulence channels', *IEEE Trans. Wirel. Commun.*, 2006, **5**, (6), pp. 1229–1233

[9] ANDREWS L.C., AL-HABASH M.A., HOPEN C.Y., PHILLIPS R.L.: 'Theory of optical scintillation: Gaussian beam wave model', *Waves Random Med.*, 2001, **11**, pp. 271–291

[10] TSIFTSIS T.A., SANDALIDIS H.G., KARAGIANNIDIS G.K., SAGIAS N.C.: 'Multihop free-space optical communications over strong turbulence channels'. IEEE ICC Proc, 2006

[11] LAZARAKIS F., TOMBRAS G.S., DANGAKIS K.: 'Average channel capacity in a mobile radio environment with Rician statistics', *IEICE Trans. Commun.*, 1994, **E77-B**, (7), pp. 971–977

[12] LI J., YU M., RICKLIN J., UYSAL M.: 'On the capacity and BER performance of SLSD and MLMD optical wireless communications systems'. CISS'05 (Conf. on Information Sciences and Systems), Baltimore, MD, USA, 2005

doi: 10.1049/iet-com.2008.0212

[13] HAAS S.M., SHAPIRO J.H.: 'Capacity of wireless optical communications', *IEEE J. Sel. Areas in Commun.*, 2003, **21**, (8), pp. 1346–1357

[14] LI J., UYSAL M.: 'Optical wireless communications: system model, capacity and coding'. Proc. Vehicular Technology Conf., 2003, pp. 168–172

[15] LAOURINE A., STEPHENNE A., AFFES S.: 'Capacity of log normal fading channels'. Proc. of the International Conf. on Wireless Communications and Mobile Computing 2007 (IWCMC2007), Honolulu, Hawaii, USA, 12–16 August 2007, pp. 13–17

[16] KATSIS A., NISTAZAKIS H.E., TOMBRAS G.S.: 'Bayesian and frequentist estimation of the average capacity of log normal wireless optical channels'. IeCCS 2007 (Int. e-Conf. on Computer Science), December 2007, AIP Proc., 2008, **1060**, pp. 26–29

[17] LAOURINE A., STEPHENNE A., AFFES S.: 'Estimating the ergodic capacity of log normal channels', *IEEE Commun. Lett.*, 2007, **11**, (7), pp. 568–570

[18] NISTAZAKIS H.E., TOMBRAS G.S., TSIGOPOULOS A.D., KARAGIANNI E.A., FAFALIOS M.E.: 'Average capacity of wireless optical communication systems over gamma–gamma atmospheric turbulence channels'. 2008 IEEE MTT-S, Atlanta, USA, 2008, pp. 1561–1564

[19] ZHU X., KAHN J.M.: 'Free-space optical communications through atmospheric turbulence channels', *IEEE Trans. Commun.*, 2002, **50**, (8), pp. 1293–1300

[20] SANDALIDIS H.G., TSIFTSIS T.A., KARAGIANNIDIS G.K., UYSAL M.: 'BER performance of FSO links over strong atmospheric turbulence channels with pointing errors', *IEEE Commun. Lett.*, 2008, **12**, (1), pp. 44–46

[21] GRADSHTEYN I.S., RYZHIK I.M.: 'Table of integrals, series, and products' (Academic, New York, 2007, 7th edn.)

[22] ADAMCHIK V.S., MARICHEV O.I.: 'The algorithm for calculating integrals of hypergeometric type function and its realization in reduce system'. Proc. Int. Conf. on Symbolic and Algebraic Computation, Tokyo, Japan, 1990, pp. 212–224

doi: 10.1049/iet-com.2008.0212