Fine Residual Carrier Frequency and Sampling Frequency Estimation in Wireless OFDM Systems

Chen Chen, Yun Chen*, Na Ding, Qiang Zhang, Wenzhuo Bao, Yuanzhou Hu, and Xiaoyang Zeng
ASIC and System State Key Laboratory, Fudan University, Shanghai, China, 201203
Email: chenyun@fudan.edu.cn

Abstract—This paper presents a novel algorithm for residual phase estimation in wireless OFDM systems, including the carrier frequency offset (CFO) and the sampling frequency offset (SFO). The subcarriers are partitioned into several regions which exhibit pairwise correlations. The phase increment between successive OFDM blocks is exploited which can be estimated by two estimators with different computational loads. Numerical results of estimation variance are presented. Simulations indicate performance improvement of the proposed technique over several conventional schemes in a multipath channel.

I. INTRODUCTION

Orthogonal-Frequency-Division-Multiplexing (OFDM) significantly enhances the system performance under dispersive channels. However, it is vulnerable to synchronization non-idealities, including the symbol timing offset (STO), carrier frequency offset (CFO), and sampling frequency offset (SFO).

The previous works including [1]–[3] dealt with the coarse STO and CFO estimation in time domain before Fast Fourier Transform (FFT). Nevertheless, with imperfect compensation, after FFT, the residual part of CFO remains to be corrected in frequency domain. Also, residual SFO should be removed, or otherwise it produces a phase rotation not only proportional to the tone index within one OFDM block (inter-block increment), but also grows linearly for successive OFDM blocks (intra-block increment) [4]. Several schemes were proposed to estimate the residual CFO and SFO in frequency domain using pilots [5]–[7]. [5] utilized the symmetry of pilot locations to obtain CFO and SFO jointly with performance degradation in multipath environment. [6] suggested three variants relying on least square estimation (LSE). An improved weighted LSE scheme was proposed in [7] requiring the second-order statistics of the channel state information (CSI).

In this paper, a novel technique is proposed taking advantage of the intra-block increment spanning a number of consecutive OFDM blocks. By dividing the subcarrier index into several regions, the scheme exploits the pairwise correlation yielding accurate results after least square fitting. Two variants differing in computation complexity are presented. Different from [8] which suits baseband schemes, the method here turns to passband transmission and thus requires channel estimation.

The rest of the paper is structured as follows. Section II presents the signal model in presence of CFO and SFO. Section III elaborates the proposed technique and presents some analytical results. Simulation results are given in Section IV.

II. OFDM SIGNAL MODEL WITH CFO AND SFO

We consider an OFDM system where the transmitted data is modulated by an N-point Inverse FFT (IFFT). Assuming a total of M OFDM blocks with each block consisting of K data samples (K ≤ N), the complex baseband signal is described by

\[ s(t) = \frac{1}{\sqrt{N}} \sum_{l=0}^{M-1} \sum_{k \in \mathbb{X}} X_{l,k} e^{j 2 \pi k (t - (N_g + N_g) T_s)} \cap (t - l N_B T_s) \] (1)

where \( \mathbb{X} \) are the locations of the K data subcarriers; for \( k \notin \mathbb{X} \), \( X_{l,k} \) is either pilot or null subcarrier. \( N_g \) is the length of the guard interval, \( N_B \) the total length of an entire OFDM block given by \( N_B \approx N + N_g \), and \( T_s \) the sampling interval. \( \cap(\cdot) \) is the rectangular windowing function. The multipath channel is

\[ h(t, \tau) = \sum_{\ell=0}^{L-1} h_{\ell}(t) \delta(\tau - \tau_{\ell}) \] (2)

where \( L \) is the total number of taps, \( \{ h_{\ell}(t) \}_{\ell=0,1,\ldots,L-1} \) the independent and Rayleigh distributed complex channel gains, \( \{ \tau_{\ell} \} \) the timing delay of path \( \ell \), and \( \delta(\cdot) \) the delta function.

Without loss of generality, we assume \( \tau_{\ell} \approx \ell T_s \). Up-converting \( s(t) \) to the carrier frequency \( f_r \), the post-channel equivalent signal takes the form

\[ y(t) = [s(t) e^{j 2 \pi f_r t} * h(t, \tau)] + w(t) \] (3)

where the notation \( * \) stands for linear convolution, and \( w(t) \) the complex, identically independently distributed (i.i.d.), additive white Gaussian noise (AWGN) with zero mean and variance \( \sigma_w^2 \). It is also assumed to be wide sense stationary (WSS), with independent real and imaginary part with equal variance in both parts. Now, assuming a CFO \( \Delta f \) and a SFO \( \eta \) given as \( \Delta f \approx f_R - f_R, \eta \approx (T_s^r - T_s) / T_s \) where \( f_R \) is the carrier frequency at the receiver, and \( T_s^r \) the sampling interval at the receiver. The received \( n \)-th sample in the \( l \)-th OFDM block is

\[ r_{l,n} = y(t) e^{-j 2 \pi f_a t} \big|_{t = l N_B T_s + N_g T_s + n T_s^r}, \quad n = 0, 1, \ldots, N - 1 \] (4)

Discarding the \( N_g \) samples in the guard interval, the complex data for the \( l \)-th block and on the \( k \)-th subcarrier is

\[ R_{l,k} = X_{l,k} H_{l,k} \Omega(k) \left( e^{i \pi \theta_k (N - 1) / N} \cdot e^{j 2 \pi ((l N_B + N_g) / N) \theta_k} \right) + \text{ICI}_{l,k} + W_{l,k} \] (5)
where $H_{l,k}$ is the channel transfer function (CTF) in frequency domain; $\Theta_q \approx \epsilon + \eta k$ and $\epsilon = \Delta f N_B T_s$ the normalized CFO to the subcarrier spacing. $\alpha(\epsilon)$ is the amplitude attenuation approaching unity and can be neglected.ICI$_{l,k}$ is the inter-carrier interference (ICI) caused by distorted orthogonality of subcarriers due to the presence of CFO and SFO. $W_{l,k}$ is the WSS i.i.d. Gaussian noise in frequency domain. $\epsilon$ and $\eta$ can be regarded as the residual part of CFO and SFO after coarse synchronization or imperfect channel estimation and equalization.

III. PROPOSED TECHNIQUE

Define the full set of subcarrier index as $K = \{k|0 \leq k \leq N - 1\}$, which can be further divided into $Q$ equally-spaced regions, denoted as $K_q = \cup_{k=1}^{Q} \cup K_q \cdots \cup K_Q$ where

$$K_q = \left\{ k \in \mathbb{Z} \right\} \left( (q-1)N < k < qN \right)$$

where $\mathbb{Z}$ denotes integers. Ignoring ICI, using equation (5), for the $q$-th segment in the $l$-th OFDM block, the pair-wise correlation takes the form below:

$$V_{l,k_1,k_2}^q = R_{l,k_1}R_{l,k_2} e^{j\theta_{l,k_1,k_2}}$$

$$= \left\{ X_{l,k_1}H_{l,k_1}X_{l,k_2}H_{l,k_2} + X_{l,k_2}H_{l,k_2}W_{l,k_1} + W_{l,k_1}, W_{l,k_2} \right\}$$

where

$$N_q = \frac{N + 2N(q-1)}{Q}, \quad q = 1, 2, \ldots, Q$$

$$\frac{N}{k_1} = X_{l,k_1}X_{l,k_2}H_{l,k_1}H_{l,k_2}$$

$$\theta_{l,q,k_1,k_2} = (2\epsilon + \eta N_q) \left\{ 2\pi l(1 + g) + 2\pi g + \pi \frac{N - 1}{N} \right\}$$

$$\theta_{l,q,k_1,k_2} = \pi \Theta_{l,k_1,k_2} + 2\pi \left( \frac{N}{N_B} + \frac{N}{N_2} \right)$$

and $g = N_q/N$. Clearly, the extra phase rotation of the useful part in (10) is irrelevant to subcarrier index $k_1$ and $k_2$; it is only pertinent to the OFDM block index $l$ and segment index $q$. The cross terms are the main disturbance in estimation. In practice, the contribution of signal and channel ($X_{l,k_1}$) should be replaced by

$$\lambda_{l,k_1,k_2}^q = X_{l,k_1}X_{l,k_2}H_{l,k_1}H_{l,k_2}$$

where

$$X_{l,k} = \left\{ \begin{array}{ll} X_{l,k}, & k \in \mathbb{P} \\ \tilde{X}_{l,k}, & k \in \mathbb{U} \\ 0, & k \in \mathbb{U} \end{array} \right.$$ (14)

$\mathbb{P}$ denotes the full set of pilots and $\mathbb{U}$ the full set of null subcarriers. $\tilde{X}_{l,k}$ is the estimation of $X_{l,k}$, obtained from the

1Here, we assume even $N$ and $Q$, and $N$ is divisible by $Q$
choose $\left[ b_{q}\right]_{1,1}^2$ and arrange all $\tilde{c}_q$ into the $Q \times 1$ vector $c = [\tilde{c}_1 \; \tilde{c}_2 \; \cdots \; \tilde{c}_q \; \cdots \; \tilde{c}_Q]^T$ which leads to

$$c = B\mu + \chi_c$$

where $\mu = [\eta \; \epsilon]^T$, $\chi_c$ is the $Q \times 1$ error vector, and

$$B = \begin{bmatrix}
\frac{N}{Q} & \frac{3N}{Q} & \cdots & \frac{N_q}{Q} & \cdots & \frac{N+2N(Q-1)}{Q} \\
2 & 2 & \cdots & 2 & \cdots & 2
\end{bmatrix}^T$$

is the $Q \times 2$ observation matrix. Another least square fitting yields

$$\hat{\mu} = (B^TB)^{-1}B^Tc$$

(28)

The estimated $\tilde{\eta}$ and $\tilde{\epsilon}$ are $[\tilde{\mu}]_{1,1}$ and $[\tilde{\mu}]_{1,2}$ respectively. A simple sketch with $Q = 2$ is drawn in Fig. 1. Assuming correctness in tackling the phase ambiguity in the linearization process, estimation using either the weighted or simplified method is unbiased. On the other hand, the numerical variances of $\tilde{\eta}$ and $\tilde{\epsilon}$ are

$$\text{Var} \{\tilde{\eta}\} = \frac{81 \sum_{q=1}^{Q} U_q \left[ \sum_{l=0}^{M-1} \frac{2l+M+1}{F_{l,q}} \right]^2}{32N^4(Q^2-1)^2\pi^2(1+g)^2M^2(M^2-1)^2}$$

(29)

$$\text{Var} \{\tilde{\epsilon}\} = \frac{81 \sum_{q=1}^{Q} Y_q \left[ \sum_{l=0}^{M-1} \frac{2l+M+1}{F_{l,q}} \right]^2}{32N^4(Q^2-1)^2\pi^2(1+g)^2M^2(M^2-1)^2}$$

(30)

where

$$F_{l,q} = \begin{cases}
\frac{\phi_{l,k_1,k_2}^+}{\phi_{l,k_1,k_2}^-} & \text{Weighted} \\
\frac{\phi_{l,k_1,k_2}^-}{(\sum_{(k_1,k_2)\in C_q} \phi_{l,k_1,k_2}^+)^2} & \text{Simplified}
\end{cases}$$

and

$$\phi_{l,k_1,k_2}^+ = \frac{|X_{l,k_1}|^2|H_{l,k_1}|^2 + |X_{l,k_2}|^2|H_{l,k_2}|^2}{\sigma_W^2}$$

(31)

(32)

\(^2\)The performance would suffer if $[b_{q}]_{2,1}$ is used for estimation, which is omitted here.

IV. SIMULATION

In this section, we consider a wireless OFDM system with FFT size $N = 512$. Other parameters are set as: $N_g = 64$, $N_B = 576$, $M = 10$, $Q = 4$, $f_T = 5$ GHz, $T_s = 100$ ns. For the best performance of the proposed estimators and other conventional pilot-assisted schemes, we set all subcarriers as pilots. The cardinality of each set $C_q$ is $N_{C_q} = N/2Q = 64$. The signal is drawn from 16-PSK. The channel consists of $L = 32$ i.i.d. Rayleigh taps with a power delay profile decaying exponentially: $P(\ell) \propto \exp(-\ell/L)$, $\ell = 0, 1, 2, \cdots, L - 1$

(36)

For the proposed estimators and the scheme in [7], CTF is assumed to be known perfectly as well as the noise variance, unless otherwise mentioned. Mean squared errors (MSEs) serve as benchmarks for performance evaluations, defined as $\text{MSE}\{\tilde{\eta}\} = E \left[ |\tilde{\eta} - \eta|^2 \right]$ and $\text{MSE}\{\tilde{\epsilon}\} = E \left[ |\tilde{\epsilon} - \epsilon|^2 \right]$ respectively.

Fig. 2 highlights the comparison of MSE$\{\tilde{\eta}\}$ among the proposed estimators with other schemes in [5]–[7]. Perfect CSI is assumed. Numerical result of $\text{Var} \{\tilde{\eta}\}$ is drawn by assuming A1–A2. Under multipath, the proposed estimators achieve

\(^3\)For OFDM systems containing null subcarriers, $\sigma_W^2$ could be estimated, which is omitted in this paper.
their best performances. Under flat fading, the weighted estimator reduces to the simplified one. Var \( \{ \hat{\eta} \} \) provides a tight bound in moderate SNR.

Fig. 3 shows the performance comparison of MSE\( \{ \hat{\epsilon} \} \) in multipath channel with perfect CSI. [5] could achieve the best performance when SNR \( \leq 7 \) dB, which cannot be sustained into higher SNR. Again, Var \( \{ \hat{\epsilon} \} \) provides a tight bound in moderate SNR.

Also shown in Fig. 2 and Fig. 3 are the performances given imperfect CSI. The inaccurate channel estimation is assumed as \( \hat{H}_{l,k} = \sqrt{1 - \kappa^2} H_{l,k} + \kappa J_{l,k} \) where \( \kappa \) represents the estimation accuracy which is 0.1 in simulations, and \( J_{l,k} \) the additional complex noise with zero mean and unit variance. The performance degradation is more evident in high SNR. However, \( \hat{H}_{l,k} \) deviates from the practical situation, that in general the channel estimation is more accurate in high SNR. Thus, the results under \( \kappa = 0.1 \) can be considered as worst case.

Fig. 4 shows the deviation of MSE\( \{ \hat{\eta} \} \) when SFO \( \eta \) changes under SNR = 20 dB in multipath channel. For the proposed estimators, MSE\( \{ \hat{\eta} \} \) is asymmetric for negative and positive \( \eta \) due to the presence of a positive \( \epsilon \); for \( \eta > 0 \), the performance degrades gradually with a higher \( \eta \) since the ICI is increasing simultaneously. For a major part, [7] and the simplified estimator entangle with each other.

Fig. 5 displays the deviation of MSE\( \{ \hat{\epsilon} \} \) with a varying CFO \( \epsilon \) under SNR = 20 dB in multipath channel. Different from Fig. 4, shape of MSE\( \{ \hat{\epsilon} \} \) is akin to symmetric, since comparing with \( \epsilon \), the contribution of \( \eta \) on ICI is minor. In the full range, both of the proposed schemes outperform others.

V. ACKNOWLEDGEMENT

This work is supported by Project of State Key Laboratory under grant NO.11ZD0005 and Dawn Program of Shanghai Municipal Education Commission 11SG07.

REFERENCES