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Inference on Stress-Strength Reliability for Weighted Weibull Distribution

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Abstract This paper deals with the estimation of the reliability $R = p(Y < X)$ when X and Y are independent variables distributed as Weighted Weibull Distribution. Different methods for estimating the reliability are obtained such as Maximum Likelihood Estimators, Least Square Estimators and Bayesian Estimators which are based on non-informative and informative prior distributions. A comparison of the estimates obtained is performed as well. Finally a numerical investigation is carried out to study the properties of the new estimators.

Keywords Reliability, Stress-Strength, Maximum Likelihood Estimator; Bayesian Estimator, Weighted Weibull Distributions

1. Introduction

It has become a fact that improving the quality of products depends mainly on the on going demand for these products. In addition, staying in the market has become also associated with the reliability of such products. For these reasons, companies need to meet specific measure to compete in world wide markets such as, its ability to high quality, competitive prices, and reliable goods on time. In this regar ds, manufactures use formative assessment for there products to achieve reliability which is closely related to durability, accessibility and survival. Failure can be conside red a random variable since it is so difficult to tell precisely when a specific product will fail under use condition.

Under normal condition, measure of reliability for device becomes too difficult and it requires very long time. So, accelerated life testing induces failures and the failure data at the accelerated conditions are used to estimate the reliability at normal operating conditions when the reliability of a component is "high" and failure data of the component may not be attainable during its expected life. The problem of estimating the reliability $R = p(Y < X)$ arises in the situation of mechanical reliability of a system with strength X and stress Y, and R is a determine of system reliability. The system fails if and only if, at any time the stress exceeds the strength.

Many authors have measured different choices for stress and strength distributions. Johnson (1988)[10] summarized some of these choices. Awad & Gharraf (1986)[5] considered the case when X and Y are independent and have Burr Type XII random variables. They obtained a maximum likelihood estimator, minimum variance unbiased estimator, and a Bayesian estimator for R. They used a numerical procedure for evaluating their Bayesian estimator. Alam & Roohi (2002)[3] studied stress and strength having exponential distribution but in (2003)[4] have studied the problem of $R = p(Y < X)$ in a different way. They have set up the required parametric values of the assumed distributions as a replacement for finding P (Y<X) for a given set of distribution, For them idea, they assumed exponential strength and exponential stress. Kotz *et al.* (2003)[12] presented a review of all methods and results on the stress-strength model in the last four decades. Abd-Elfattah & Mandouh (2004)[1] obtained the three estimators of R for Lomax distribution with known scale parameter, these estimators are maximum likelihood estimator, uniformly Minimum variance unbiased estimation (UMVUE) and Bayes estimator. Mokhlis (2005)[14] studied the case when X and Y are independent random variables Burr type III, he obtained the maximum likelihood estimator (MLE), Minimum variance unbiased estimation (UMVUE) and Bayesian estimates of R, he compared between the estimators by using Mont Carlo simulation. Later Khan & Islam (2007)[11] dealt with problem for power function distribution.

The Weibull distribution is one of the most widely used distributions in the reliability and survival studies. Baklizi (2012)[7] considered the stress-strength reliability based on record values from the Weibull distribution. He obtained the Bayes estimator based on squared error loss and the maximum likelihood estimator of the reliability R. Akbar *et al* (2012))[2] focus on the inference for the stress- strength

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reliability R when X and Y are two independent Weibull distributions with the same shape parameter, but having the different scale parameters. They obtained The maximum likelihood and the approximate maximum likelihood estimators of R.

Azzalini (1985)[6] suggested a method of obtaining weighted distributions from independently identically distributed random variables. He used the density function of one random variable and the distribution function of the other random variable as follows:

$$F_X(x)=\frac{1}{P(\alpha X_1 > X_2)} f_Y(x) F_Y(\alpha x) \quad (1.1)$$

Recently, Gupta & Kundu, (2009)[9] presented a new class of weighted exponential distributions. Makhdoom (2012)[13] studied the estimation of Stress-Strength reliability when X and Y are two weighted exponential distributions with different parameters. He obtained the MLE of R based on one simple iteration procedure, and he carried out Bayesian estimators of parameters with real data. Saman *et al.* (2010)[15] proposed The weighted Weibull model based on an idea of Azzalini (1985)[6]. They studied basic properties of the distribution including moments, generating function, hazard rate function and estimation of parameters. The proposing model which in Saman *et all* (2010)[15] to evaluate the parameters of Weighted Weibull distribution is considered here. Then, the cumulative distribution function is:

$$F_X(x)=\frac{\alpha+1}{\alpha}\left[\left\{1-\exp\left(-x^{\beta}\right)\right\}-\frac{1}{\alpha+1}\left\{1-\exp\left(-(\alpha+1)x^{\beta}\right)\right\}\right] \quad (2.1)$$
$$;\ \alpha,\ \beta,\ \lambda,\ x>0$$

and the probability density function is:

$$f_X(x)=\frac{\alpha+1}{\alpha}\lambda\beta x^{\beta-1}\exp\left(-\lambda x^{\beta}\right)\left[1-\exp\left(-\alpha\lambda x^{\beta}\right)\right] \quad (2.1)$$
$$;\ \alpha,\ \beta,\ \lambda,\ x>0$$

hence $\alpha,\ \lambda$ are shape parameters and β is scale parameter. Now, let $\lambda=1$ as the same used manner at Saman *et all* (2010)[15].

This paper consists of three sections corresponding to sections 2, 3, and 4, respectively. section 2 provides an approach for scheming Stress-Stress reliability system. section 3, discusses the maximum likelihood Estimators, Least Square Estimators and Bayesian estimators of the reliability R. Finally a numerical investigation will be carried out to study the properties of the new estimators.

2. System Reliability

Let X be the strength of a system and Y be the stress acting on it. X and Y are the random variables from Weighted Weibull distributions with parameters (α_1,β) and (α_2,β) respectively. Therefore, the probability density functions of X and Y are, respectively:

$$f_X(x)=\frac{\alpha_1+1}{\alpha_1}\beta x^{\beta-1}\exp\left(-x^{\beta}\right)\left[1-\exp\left(-\alpha_1 x^{\beta}\right)\right] \quad (1.2)$$
$$;\ \alpha_1,\ \beta,\ x>0$$

$$f_Y(y)=\frac{\alpha_2+1}{\alpha_2}\beta x^{\beta-1}\exp\left(-x^{\beta}\right)\left[1-\exp\left(-\alpha_1 x^{\beta}\right)\right] \quad (2.2)$$
$$;\ \alpha_2,\ \beta,\ y>0$$

where α_1 and α_2 are shape parameters and β is scale parameter. Then, the reliability function R is:

$$\begin{aligned}
R=P(y<x)&=\int_0^{\infty} F_Y(x)\ f(x)dx \\
&=\int_0^{\infty}\frac{\alpha_2+1}{\alpha_2}\left[\left\{1-\exp\left(-x^{\beta}\right)\right\}-\frac{1}{\alpha_2+1}\left\{1-\exp\left(-(\alpha_2+1)\,x^{\beta}\right)\right\}\right] \\
&\quad\times\frac{\alpha_1+1}{\alpha_1}\beta x^{\beta-1}\exp\left(-x^{\beta}\right)\left[1-\exp\left(-\alpha_1 x^{\beta}\right)\right]dx \\
&=c\left[\left(\frac{1}{\beta}-\frac{1}{\beta(\alpha_1+1)}-\frac{1}{2\beta}+\frac{1}{\beta(\alpha_1+2)}\right)\right. \\
&\quad\left.-\bar{c}\left(\frac{1}{\beta}-\frac{1}{\beta(\alpha_1+1)}-\frac{1}{\beta(\alpha_2+2)}+\frac{1}{\beta(\alpha_1+\alpha_2+2)}\right)\right.
\end{aligned} \quad (3.2)$$

where $c=\beta\cdot\frac{\alpha_1+1}{\alpha_1}\cdot\frac{\alpha_2+1}{\alpha_2}$ and $\bar{c}=\frac{1}{\alpha_2+1}$.

3. Point Estimator of the Reliability R

3.1. Maximum Likelihood Estimator

Let $X_1, X_2, \cdots, X_n$ and $Y_1, Y_2, \cdots, Y_m$ be the two independent random samples taken from the Weighted Weibull distributions with parameters (α_1, β) and (α_2, β) respectively, then, the likelihood and log-likelihood function based on the above samples are respectively given as:

$$L = \left(\frac{\alpha_1 + 1}{\alpha_1}\right)^n \left(\frac{\alpha_2 + 1}{\alpha_2}\right)^m \beta^{n+m} \prod_{i=1}^{n} x_i^{\beta-1} \prod_{i=1}^{m} y_i^{\beta-1} \; e^{-\sum_{i=1}^{n} x_i^\beta} \; e^{-\sum_{i=1}^{n} y_i^\beta}$$
$$\times \prod_{i=1}^{n}\left(1 - e^{-\alpha_1 x_i^\beta}\right) \prod_{i=1}^{m}\left(1 - e^{-\alpha_2 y_i^\beta}\right), \tag{1.3}$$

$$\ell = n \ln(\alpha_1 + 1) - n \ln \alpha_1 + m \ln(\alpha_2 + 1) - m \ln \alpha_2 + (n+m) \ln \beta$$
$$+ (\beta - 1)\left[\sum_{i=1}^{n} \ln x_i + \sum_{i=1}^{m} \ln y_i\right] + \sum_{i=1}^{n} \ln\left(1 - e^{\alpha_1 x_i^\beta}\right) + \sum_{i=1}^{m} \ln\left(1 - e^{\alpha_2 y_i^\beta}\right). \tag{2.3}$$

The derivatives of $\ln(\alpha_1, \alpha_2; x, y)$ with respect to α_1 and α_2 are, respectively:

$$\frac{\partial \ell}{\partial \alpha_1} = \frac{n}{\hat{\alpha}_1 + 1} - \frac{n}{\alpha_1} + \sum_{i=1}^{n} \frac{x_i^\beta \, e^{\alpha_1 x_i^\beta}}{1 - e^{\alpha_1 x_i^\beta}} = 0, \tag{3.3}$$

$$\frac{\partial \ell}{\partial \alpha_2} = \frac{m}{\hat{\alpha}_2 + 1} - \frac{m}{\alpha_2} + \sum_{i=1}^{m} \frac{y_i^\beta \, e^{\alpha_2 y_i^\beta}}{1 - e^{\alpha_2 y_i^\beta}} = 0, \tag{4.3}$$

$$\frac{\partial \ell}{\partial \beta} = \frac{n+m}{\beta} + \sum_{i=1}^{n} \ln x_i + \sum_{i=1}^{m} \ln y_i + \alpha_1 \sum_{i=1}^{n} \frac{x_i^\beta \ln x_i \, e^{-\alpha_1 x_i^\beta}}{1 - e^{-\alpha_1 x_i^\beta}} + \alpha_2 \sum_{i=1}^{m} \frac{y_i^\beta \ln y_i \, e^{-\alpha_2 y_i^\beta}}{1 - e^{-\alpha_2 y_i^\beta}} = 0. \tag{5.3}$$

Unfortunately, there is no demand value for $(\alpha_1, \alpha_1, \beta)$ in (3.3), (4.3) and (5.3), so with that, Newton–Raphson method in an iterative approach to solve this equations in numerical analysis is considered. It is an iterative method for solving equations $f(t) = 0$ where $f(t)$ is assumed to have a continuous derivative $f'(t)$. Given a function $f(t)$ and its derivative $f'(t)$, a first guess t_0 is initialed. Then, an approximation of t_1 is $t_0 - \frac{f(t_0)}{f'(t_0)}$ and an approximation of t_2 is $t_1 - \frac{f(t_1)}{f'(t_1)}$ and so on for number of iterations r or if $|t_{r+1} - t_r| \le \tau$ where t_r is the r^{th} estimate.

Now, replace $\hat{t}$ which gives from iteration with the parameter t after iteration process for estimating the parameter t in equations (3.3), (4.3) and (5.3).

Since, Maximum Likelihood Estimators are do not change, So it becomes:

$$\hat{R}_{MLE} = c\left[\left(\frac{1}{\hat{\beta}} - \frac{1}{\hat{\beta}(\hat{\alpha}_1 + 1)} - \frac{1}{2\beta} + \frac{1}{\beta(\alpha_1 + 2)}\right) - \bar{c}\left(\frac{1}{\beta} - \frac{1}{\beta(\alpha_1 + 1)} - \frac{1}{\beta(\alpha_2 + 2)} + \frac{1}{\beta(\alpha_1 + \alpha_2 + 2)}\right)\right] \tag{6.3}$$

where $c = \hat{\beta} \cdot \frac{\hat{\alpha}_1 + 1}{\hat{\alpha}_1} \cdot \frac{\hat{\alpha}_2 + 1}{\hat{\alpha}_2}$ and $\bar{c} = \frac{1}{\hat{\alpha}_2 + 1}$.

3.2. Least Square Estimators

Suppose that (2.1) is a linear relation between the two variables and take the logarithm of the two sides as follows:

$$\ln\left[F\left(x_i\right)\right]=\ln\left(\alpha_1+1\right)-\ln\left(\alpha_1\right)+\ln\left[1-e^{-x_i^{\beta}}-\frac{1}{\alpha_1+1}\left(1-e^{-\left(\alpha_1+1\right)x_i^{\beta}}\right)\right] \tag{7.3}$$

So, if this function is minimized with respect to (α_1,β), we end up with the Least Square Estimators. This method is known as Least Square method (more details, see Flaih *et al* (2012)[8].

Now, using the mean rank method, we can estimate $F(x_i)$ from this relation $F(x_i)=\frac{i}{n+1}$, where $x_1,x_2,\cdots,x_n$ are the rank failure times.

Let $y_i=\ln(\frac{i}{n+1})$, so, (7.3) becomes straight line equation. Therefore, the least square estimators of α_1 and β come from minimizing the following equation:

$$Q(\alpha_1,\beta)=\sum_{i=1}^{n}\left[y_i-\ln\left(\alpha_1+1\right)+\ln\left(\alpha_1\right)-\ln\left[1-e^{-x_i^{\beta}}-\frac{1}{\alpha_1+1}\left(1-e^{-\left(\alpha_1+1\right)x_i^{\beta}}\right)\right]\right]^2 \tag{8.3}$$

The first partial derivatives of $Q(\alpha_1,\beta)$ with respect to α_1 and β, respectively, are:

$$\frac{\partial Q(\alpha_1,\beta)}{\partial\alpha_1}=2\sum_{i=1}^{n}\left[y_i-\ln\left(\alpha_1+1\right)+\ln\left(\alpha_1\right)-\ln\left[1-e^{-x_i^{-\beta}}-\frac{1}{\alpha_1+1}\left(1-e^{-(\alpha_1+1)x_i^{\beta}}\right)\right]\right]$$

$$\times\left[-\frac{1}{\alpha_1+1}+\frac{1}{\alpha_1}+\frac{\frac{1}{(\alpha_1+1)^2}\sum_{i=1}^{n}\left(1-e^{-(\alpha_1+1)x_i^{-\beta}}\right)-\frac{1}{\alpha_1+1}\cdot\sum_{i=1}^{n}x_i^{-\beta}\left(1-e^{-(\alpha_1+1)x_i^{-\beta}}\right)}{\left[1-e^{-x_i^{-\beta}}-\frac{1}{\alpha_1+1}\sum_{i=1}^{n}\left(1-e^{-(\alpha_1+1)x_i^{-\beta}}\right)\right]}\right], \tag{9.3}$$

$$\frac{\partial Q(\alpha_1,\beta)}{\partial\beta}=2\sum_{i=1}^{n}\left[y_i-\ln\left(\alpha_1+1\right)+\ln\left(\alpha_1\right)-\ln\left[1-e^{-x_i^{-\beta}}-\frac{1}{\alpha_1+1}\left(1-e^{-\left(\alpha_1+1\right)x_i^{\beta}}\right)\right]\right]$$

$$\times\left[-\frac{\sum_{i=1}^{n}x_i^{-\beta}\cdot\ln\left(x_i\right)\cdot e^{-x_i^{-\beta}}-\frac{1}{\alpha_1+1}\cdot(-\alpha_1-1)\sum_{i=1}^{n}x_i^{-\beta}\cdot\ln\left(x_i\right)\left(1-e^{-\left(\alpha_1+1\right)x_i^{-\beta}}\right)}{\left[1-e^{-x_i^{-\beta}}-\frac{1}{\alpha_1+1}\sum_{i=1}^{n}\left(1-e^{-\left(\alpha_1+1\right)x_i^{-\beta}}\right)\right]}\right]. \tag{10.3}$$

Similarly, the first partial derivatives of $Q(\alpha_2,\beta)$ with respect to α_2 is:

$$\frac{\partial Q(\alpha_2,\beta)}{\partial\alpha_2}=2\sum_{i=1}^{m}\left[y_i-\ln\left(\alpha_2+1\right)+\ln\left(\alpha_2\right)-\ln\left[1-e^{-y_i^{-\beta}}-\frac{1}{\alpha_2+1}\left(1-e^{-\left(\alpha_2+1\right)y_i^{\beta}}\right)\right]\right]$$

$$\times\left[-\frac{1}{\alpha_2+1}+\frac{1}{\alpha_2}+\frac{\frac{1}{(\alpha_2+1)^2}\sum_{i=1}^{m}\left(1-e^{-\left(\alpha_2+1\right)y_i^{-\beta}}\right)-\frac{1}{\alpha_2+1}\cdot\sum_{i=1}^{m}y_i^{-\beta}\left(1-e^{-\left(\alpha_2+1\right)y_i^{-\beta}}\right)}{\left[1-e^{-y_i^{-\beta}}-\frac{1}{\alpha_2+1}\sum_{i=1}^{m}\left(1-e^{-\left(\alpha_2+1\right)y_i^{-\beta}}\right)\right]}\right] \tag{11.3}$$

If both $\frac{\partial Q(\alpha_1,\beta)}{\partial \alpha_1}$, $\frac{\partial Q(\alpha_1,\beta)}{\partial \alpha_2}$ and $\frac{\partial Q(\alpha_1,\beta)}{\partial \beta}$ equal zero, then, the Least Square Estimators of α_1, α_2 and β will result immediately. However, this process is too difficult to be done without numerical solution. So, we adapt the previous technique of , Newton–Raphson method.

3.3. Bayes Estimator of Reliability R

Let $X_1, X_2, \cdots, X_n$ and $Y_1, Y_2, \cdots, Y_m$ be two independent random samples, drawn from the Weighted Weibull distributions with parameters (α_1, β) and (α_2, β) respectively, an associate gamma prior distributions for α_1 and β with parameters $(\lambda, \theta)$and (ε, τ) are employed respectively, so that the prior distribution for α_1 and β are:

$$\pi(\alpha_1) \propto \alpha_1^{\lambda-1} e^{-\theta\alpha_1} \quad , \alpha_1 > 0 \tag{12.3}$$

$$\pi(\beta) \propto \beta^{\varepsilon-1} e^{-\tau\beta_1} \quad , \quad \beta > 0 \tag{13.3}$$

where $(\lambda, \theta, \varepsilon, \tau)$ are independent. And the joint density function for data and the parameters α_1 and β is:

$$\Pi(data;\alpha_1,\beta) \propto (\alpha_1+1)^n \alpha_1^{\lambda-n-1} \beta^{n+\varepsilon-1} \prod_{i=1}^{n} x_i^{\beta-1} \times e^{-\sum_{i=1}^{n} x_i^{\beta} - \theta\alpha_1 - \tau\beta} \prod_{i=1}^{n} \left(1 - e^{-\alpha_1 x_i^{\beta}}\right) \tag{14.3}$$

Then, the posterior density function for α_1 and β based on data is:

$$\Pi(\alpha_1,\beta|data) = \frac{\Pi(data;\alpha_1,\beta)}{\int_0^{\infty}\int_0^{\infty} \Pi(data;\alpha_1,\beta)\, d\alpha_1\, d\beta} \tag{15.3}$$

Now, by using Posterior Mode Method to obtain the Bayesian estimators of parameters as follows:

The log posterior distribution for the sample $X_1, X_2, \cdots, X_n$ taken from Weighted Weibull distribution with Gamma priors $(\lambda, \theta)$and (ε, τ) respectively is:

$$\begin{aligned} \ln\Pi(\alpha_1,\beta|data) &\propto n\ln(\alpha_1+1) + (\lambda-n-1)\ln(\alpha_1) + (n+\varepsilon-1)\ln(\beta) \\ &+ (\beta-1)\sum_{i=1}^{n}\ln(x_i) - \sum_{i=1}^{n} x_i^{\beta} - \theta\alpha_1 - \tau\beta + \sum_{i=1}^{n}\ln\left(1-e^{-\alpha_1 x_i^{\beta}}\right) + \ln(C) \end{aligned} \tag{16.3}$$

where C does not dependent on the parameters α_1, β and the parameters λ, θ, ε and τ are fixed.

The first derivatives of $\ln(\alpha_1, \alpha_2; x, y)$ with respect to α_1 and β are, respectively:

$$\frac{\partial \ln\Pi(\alpha_1,\beta|data)}{\partial\alpha_1} = \frac{n}{\alpha_1+1} + \frac{\lambda-n-1}{\alpha_1} - \theta + \frac{1}{\alpha_1}\sum_{i=1}^{n} \frac{d}{g\,.\ln(x_i)} \tag{17.3}$$

$$\frac{\partial \ln\Pi(\alpha_1,\beta|data)}{\partial\beta} = \frac{n+\varepsilon-1}{\beta} + \sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n} x_i^{\beta}\ln(x_i) - \tau + \alpha_1\sum_{i=1}^{n}\frac{d}{g} \tag{18.3}$$

Let $h_1 = \frac{n}{\alpha_1+1} + \frac{\lambda-n-1}{\alpha_1} - \theta + \frac{1}{\alpha_1}\sum_{i=1}^{n}\frac{d}{g\,.\ln(x_i)}$, and $h_2 = \frac{n+\varepsilon-1}{\beta} + \sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n} x_i^{\beta}\ln(x_i) - \tau + \alpha_1\sum_{i=1}^{n}\frac{d}{g}$.

Then,

$$\frac{\partial^2 h_1}{\partial\alpha_1} = -\frac{n}{(\alpha_1+1)^2} - \frac{\lambda-n-1}{\alpha_1^2} - \sum_{i=1}^{n}\frac{d}{\alpha_1 . g^2} \tag{19.3}$$

$$\frac{\partial^2 h_2}{\partial\beta} = -\frac{n+\varepsilon-1}{\beta} - \sum_{i=1}^{n} x_i^{\beta}(\ln(x_i))^2 + \sum_{i=1}^{n}\left[\frac{d}{g}\left(\frac{2}{x_i} - \frac{d}{g}\right)\right] \tag{20.3}$$

$$\frac{\partial^2 h_1}{\partial \beta} = \frac{\partial^2 h_2}{\partial \alpha_1} = -\sum_{i=1}^{n}\left[\frac{d}{g}\left(x_i^{\beta} - \frac{d}{\alpha_1 g}\right)\right] \tag{21.3}$$

where $d = \alpha_1 x_i^{b} . \ln(x_i) e^{-\alpha_1 x_i^{\beta}}$ $g = 1 - e^{-\alpha_1 x_i^{\beta}}$. So, the Hessian matrix of the derivatives I is :

$$I = \begin{bmatrix} \frac{\partial^2 \ln \prod(\alpha_1, \beta | data)}{\partial \alpha_1^2} & \frac{\partial^2 \ln \prod(\alpha_1, \beta | data)}{\partial \alpha_1 \partial \beta} \\ \frac{\partial^2 \ln \prod(\alpha_1, \beta | data)}{\partial \beta \, \partial \alpha_1} & \frac{\partial^2 \ln \prod(\alpha_1, \beta | data)}{\partial \beta^2} \end{bmatrix} \tag{22.3}$$

The object of this work is estimate $\hat{\delta} = (\alpha_1, \beta)$ numerically to find $\hat{\delta}_{i+1} = \hat{\delta}_{i+1} I^{-1} \varphi$ where $\varphi = [h_1 \quad h_2]$. Solution can be converged using Newton–Raphson algorithm to estimate α_1, β which are replaced by $\hat{\alpha}_1, \hat{\beta}$ gleaned from iteration.

4. Simulation Study

Table (1). When $\alpha_1 = 3 \ , \alpha_2 = 2 \ , \lambda = 4 \ , \theta = 9, \ \varepsilon = 3, \ \tau = 1$

n	m	R_{MLE}	R_{BN-IF}	R_{B-IF}	MSE_{MLE}	MSE_{BN-IF}	MSE_{B-IF}
5	5	0.4063	0.3975	0.4094	0.0222	0.0182	0.017
	10	0.4179	0.3996	0.4256	0.0161	0.0142	0.0138
	20	0.4222	0.3993	0.4324	0.0155	0.0146	0.012
	30	0.4233	0.3988	0.4344	0.0119	0.014	0.0115
10	5	0.3939	0.3988	0.3913	0.0156	0.0131	0.149
	10	0.4023	0.3977	0.4042	0.0095	0.0085	0.0081
	20	0.4059	0.3965	0.4102	0.00617	0.00571	0.0056
	30	0.4085	0.3976	0.4136	0.00511	0.00484	0.0048
15	5	0.3852	0.3963	0.3798	0.0122	0.0104	0.0099
	10	0.3974	0.3999	0.3961	0.00601	0.00535	0.0051
	20	0.401	0.3986	0.402	0.00277	0.00253	0.0024
	30	0.4018	0.3979	0.4036	0.00194	0.00179	0.0017
20	5	0.386	0.3996	0.3792	0.0112	0.0092	0.009
	10	0.3929	0.3976	0.3905	0.00471	0.00421	0.004
	20	0.3955	0.3955	0.3954	0.00182	0.00164	0.0016
	30	0.3992	0.3976	0.3999	0.00075	0.00065	0.0006

Table (2). When $\alpha_1 = 2 \ , \alpha_2 = 3 \ , \lambda = 20 \ , \theta = 40, \ \varepsilon = 10, \ \tau = 5$

n	m	R_{MLE}	R_{BN-IF}	R_{B-IF}	MSE_{MLE}	MSE_{BN-IF}	MSE_{B-IF}
25	25	0.5807	0.5895	0.5777	0.0194	0.0234	0.0182
	50	0.5916	0.5869	0.5943	0.0139	0.0164	0.123
	75	0.5995	0.5881	0.6051	0.0111	0.0129	0.0105
	100	0.6027	0.5892	0.6094	0.0105	0.0122	0.0132
50	25	0.5744	0.5926	0.5666	0.0152	0.0173	0.0148
	50	0.588	0.5927	0.5861	0.0087	0.0096	0.0083
	75	0.5926	0.5903	0.5938	0.0061	0.0068	0.0059
	100	0.5964	0.5918	0.5987	0.0051	0.0056	0.0048
75	25	0.5677	0.5907	0.5575	0.0131	0.0141	0.0083
	50	0.5814	0.5907	0.5781	0.0067	0.0072	0.0132
	75	0.5898	0.5922	0.5887	0.0035	0.0037	0.0034
	100	0.5902	0.5902	0.5903	0.0028	0.0029	0.0027
100	25	0.5636	0.5882	0.5526	0.0124	0.0491	0.0126
	50	0.5794	0.5903	0.5742	0.0057	0.0061	0.0054
	75	0.5842	0.5881	0.5825	0.0027	0.0028	0.0026
	100	0.5874	0.5889	0.5846	0.0018	0.0019	0.0017

The computer programs MathCAD (2001) is used to obtain numerical illustration for the last theoretical results. A comparison between the three estimators, MLE, Bayes based on non-informative estimator and Bayes based on informative estimator is performed . 200 samples generated from Weighted Weibull distributions with parameters (α_1, β) and (α_2, β) are used, respectively, and different values of α_1, α_2 and β with various sizes $(5, 10, 20, 30)$ for both n and m in table 1. Also, various sizes $(25, 50, 75, 100)$ for them in table 2. Then the means of these replicates are calculated to obtain maximum likelihood estimator of R (R_{MLE}), Bayes estimator based on non-informative (R_{BN-IF}) and Bayes estimator based on informative R_{B-IF} . The mean square error of R_{MLE}, R_{BN-IF} and R_{B-IF} are computed as well.

From Table (1) and (2), note that the mean square error of R_{B-IF} is smaller than each of the mean square error of R_{MLE} and R_{BN-IF} if the sample sizes n, m are small or large. But, in table (2), when the sample sizes are large the mean square error of all estimators are less than the other. For example, when the sample sizes $n = 5$ and $m = 20$, we find that the mean square errors of R_{MLE} and R_{BN-IF} and R_{B-IF} are (0.0155, 0.0148 and 0.0120) respectively, while, they are (0.0111, 0.0129 and 0.0105) when the sample sizes are $n = 25$ and $m = 75$. This behavior applies on the rest of parameters $\alpha_1, \alpha_2, \beta, \lambda, \theta, \varepsilon$ and τ as well.

Also, the three estimators R_{B-IF}, R_{MLE} and R_{BN-IF} are increase with decreasing the sample size *n* and take the sample size *m* is fixed, and they are increase with decreasing the sample size *m* and take the sample size *n* is fixed. On the other hand, they are increase with decreasing both the sample sizes n and m.

5. Conclusions

In this paper, the estimation problem of the reliability of a system R in stress-strength model when X and Y are independent variables distributed as Weighted Weibull Distribution is considered. Maximum Likelihood Estimators, Least Square Estimators and Bayesian Estimators based on non-informative and informative prior distributions were discussed via numerical results.

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