

# Population Implosion in Genetic Programming

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**Abstract.** With the exception of a small body of adaptive-parameter literature, evolutionary computation has traditionally favored keeping the population size constant through the course of the run. Unfortunately, genetic programming has an aging problem: for various reasons, late in the run the technique become less effective at optimization. Given a fixed number of evaluations, allocating many of them late in the run may thus not be a good strategy. In this paper we experiment with gradually decreasing the population size throughout a genetic programming run, in order to reallocate more evaluations to early generations. Our results show that over four problem domains and three different numbers of evaluations, decreasing the population size is always as good as, and frequently better than, various fixed-sized population strategies.

## 1 Introduction

In generational EC, the choice of how to allocate evaluations may be cast as a tradeoff between exploration and exploitation. A maximally large population, run for a single generation, is an extreme in exploration, approximating random search. On the other hand, depending on the locality of the modification operator, a minimal (2 person) population exploits local features to the point of hill-climbing.

Choosing a good population size vs. runlength tradeoff is a long-explored topic in evolutionary computation. This choice has traditionally hinged on two factors: theoretical justifications for a given population size; and practical necessity for capping population size, primarily because of memory capacity. Recent advances in computer technology have obviated much of the second factor: genetic programming has occasionally seen population sizes around a million individuals divided among multiple machines in Beowulf clusters [1,2]. Contrast this to early 1 + 1 evolution strategies techniques which used a population size of 2!

The most common runlength for a genetic programming problem is a very short 50 generations, and the standard population sizes have ranged from 500 to 2000. The reason for these layouts is mostly tradition: they were the layouts popularized by Koza in [3]. Koza established these layouts through trial and error, and his decisions seem to be justified for GP problems. As was discussed

in [4], it appears that many of the canonical test problem domains for GP have a surprisingly short maximum useful runlength. In the Symbolic Regression domain, for example, beyond 32 generations it becomes advantageous to split the run into two or more shorter runs.

This paper examines the effects of decreasing the population size towards zero during the course of a genetic programming run. Early in the run the system would evolve with a large population of individuals, but late in the run the system would be effectively reduced to hill-climbing. This idea was inspired by simulated annealing, which also explores early and exploits (hill-climbs) late in the run, albeit in a very different fashion. We imagined that this “population implosion” would be effective because it would reallocate evaluations away from late generations where the population would have mostly stagnated anyway.

Decreasing the population size has another unforeseen advantage in genetic programming: it counters the effects of bloating on memory consumption. The total memory consumed by the system is a function largely of the size of the average individual times the number of individuals in the population. As the first factor goes up, a drop in the other factor may maintain current memory consumption levels. However, this issue is not considered in the presented experiments.

We define a *layout* as a choice of population size and runlength in order to allocate  $N$  evaluations for a run. In generational evolutionary computation, layouts are almost always rectangular, with the minor exception of the  $\mu + \lambda$  evolution strategy, which may have a small initial generation. In this paper we have chosen to decrease the population using a *diagonal* layout. The diagonal layout starts with a large population, then linearly decreases the population size at each generation until it reaches zero.

The paper discusses previous approaches to modifying the population size during the search process, followed by three experimental investigations. The first experiment compares a diagonal layout and three traditional layouts over several GP problem domains and evaluation lengths. The second experiment broadens the comparisons with a wider range of standard layouts. The third experiment applies the new layout to two GA domains. The paper concludes with a brief discussion and directions for future work.

## 2 Previous Work

Setting the population size has been a challenging issue in the EC domain for a very long time. Initial work concentrated on deciding the optimal number of individuals in fixed-sized populations. Interesting research has recently suggested methods for adapting the size of the population during the search process, depending on various online parameters like fitness improvement or size of individuals.

In [5], De Jong investigated the influence of population size, mutation, and crossover rates on the efficiency of the search process. He also suggested parameter values that showed good performance in his experiments. Later, Grefen-

stette used a meta-GA to find good parameters for the search process and recommended a smaller population size than the one suggested by De Jong [6]. Grefenstette’s results were later supported by theoretical investigations [7]. [8] provides a theoretical and empirical investigation on the relation between population size and crossover probability, while [9] presents a theoretical analysis meant to determine how to set the population size in order to promote the selection of correct building blocks. More recently, [10] describes another theoretical investigation of the impact of the population size on the performance of the GA algorithms in the OneMax problem domain, and [11] finds statistical significant differences when using different population sizes and classifier lengths in an SCS/LCS system.

Some studies have also raised the possibility of modifying the population size during the search process. [12] introduces the GAVaPS algorithm, which assigns fitness-dependent lifetimes to individuals in a steady-state EC system. Individuals are removed from the population only when their lifetime is exceeded. Another interesting approach is presented in [13], where an adaptive mechanism adjusts the population size as a way to control selective pressure.

Population size has been studied in the GP domain as well. Koza ([3]) advocated using large population sizes, but small populations have also been espoused [14]. An interesting approach is reported in [15], where the number of *tree nodes*, not individuals, in the population is kept constant in order to prevent bloating. This approach is reported to give similar results to the standard method, however it reduces the use of computational resources.

### 3 First Experiment

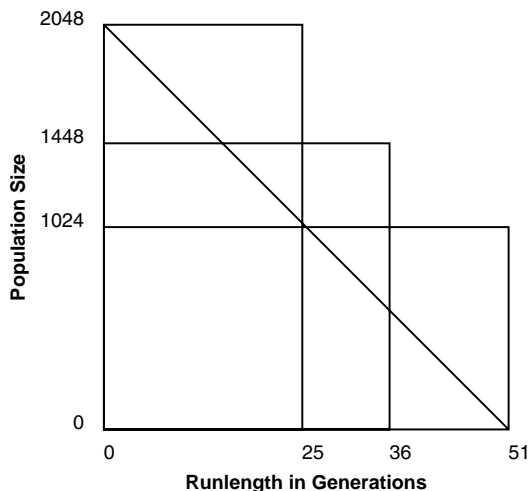
In our first experiment, we compared four layouts against three different numbers of evaluations and four different genetic programming problem domains. The problem domains were Symbolic Regression, Artificial Ant, 5-bit Parity, and 11-bit Boolean Multiplexer. We chose to include three choices of number of evaluations: 26600, 52224, and 102400 evaluations (hereafter referred to as, inaccurately, 25K , 50K, and 100K).<sup>1</sup> For each number of evaluations, we picked rectangular layouts with population sizes of 1024, 1448, and 2048. We compared these against a diagonal layout starting at a population size of 2048, and decreasing linearly towards zero.

1448 was included because we were concerned that the diagonal might outperform other choices simply because it struck a middle-ground between large population sizes and long runlengths. Therefore we included our own rectangular middle-ground. Note however that the 1448 rectangular layout cannot have *exactly* the same number of evaluations as the others: we chose to err on the side of very slightly fewer evaluations. For the 50K evaluations runs, for example, the 1448 layout had 520 fewer evaluations than the others did.

Figure 1 illustrates the four layouts used.

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<sup>1</sup> These slightly odd evaluation sizes are due to our decision to run for 51 generations.



**Fig. 1.** Four layouts for allocating  $\sim 50\text{K}$  (52224) evaluations: 2048x26; 1448x36; 1024x51; and Diagonal 2048x51

### 3.1 Other Parameters

Our evolutionary computation system was ECJ [16]. Unless stated otherwise, parameters have values as described in [3]. Symbolic Regression used no ephemeral constants and a function of  $x^4 + x^3 + x^2 + x$ . Artificial Ant used the Santa Fe trail. We used 7-tournament selection. All other initialization, modification, and representation parameters are the same as those used in [3].

We compared mean best fitness of run among experiments, and performed 200 runs for each experiment setup. Our difference of means test was an ANOVA at 95% confidence, plus a Tukey post-hoc comparison.

### 3.2 Results

We show the mean fitness results in Tables 1 through 4. Layouts are ordered left-to-right in worsening fitness. Horizontal bars above and connecting different layouts indicate no statistically significant difference between them.

The four problems vary significantly in problem difficulty; some prefer much larger populations, others prefer longer runlengths. The diagonal layout often outperformed all the other layouts, though not always by a statistically significant margin. Nonetheless, what was interesting was that the diagonal layout consistently appeared in the best class of layouts for every single problem domain and every single number of evaluations attempted; and it was the only layout to do so. The diagonal layout did particularly well with smaller numbers of evaluations; for two of the domains (Multiplexer and Ant) it had no peer.

There is an interesting trend among the rectangular layouts. In all four problems, for smaller numbers of evaluations, smaller-population layouts are pre-

**Table 1.** Statistical significance groupings for mean best fitness of run for the Symbolic Regression domain (First Experiment)

Problem and Evals	Layouts			
	Diagonal	1024	1448	2048
Regression 25K Mean Best Fitness	0.09131543	0.103177731	0.145182262	0.169305242
Regression 50K Mean Best Fitness	Diagonal 0.05270353	2048 0.057168302	1448 0.075613984	1024 0.079054647
Regression 100K Mean Best Fitness	Diagonal 0.037023631	2048 0.038560673	1448 0.039290056	1024 0.060902268

**Table 2.** Statistical significance groupings for mean best fitness of run for the Artificial Ant domain (First Experiment)

Problem and Evals	Layouts			
	Diagonal	1024	1448	2048
Ant 25K Mean Best Fitness	Diagonal 22.325	1024 24.63	1448 24.79	2048 25.99
Ant 50K Mean Best Fitness	Diagonal 17.71	2048 18.505	1024 20.015	1448 21.87
Ant 100K Mean Best Fitness	2048 15.865	Diagonal 16.18	1024 18.16	1448 18.775

**Table 3.** Statistical significance groupings for mean best fitness of run for the 5-bit Parity domain (First Experiment)

Problem and Evals	Layouts			
	1024	Diagonal	1448	2048
5-bit Parity 25K Mean Best Fitness	1024 6.68	Diagonal 6.885	1448 7.785	2048 8.85
5-bit Parity 50K Mean Best Fitness	1024 4.77	Diagonal 4.835	1448 4.835	2048 5.825
5-bit Parity 100K Mean Best Fitness	Diagonal 3.35	2048 3.44	1448 3.46	1024 3.55

ferred; and for larger numbers of evaluations, larger-population layouts are preferred. This suggests that there is an optimal useful runlength for each of the problems. As the number of evaluations increases, the smaller-population layouts are extended beyond this runlength and are essentially wasting time. Similarly, for small numbers of evaluations, the large population layouts do not run for enough generations to produce competitive results.

**Table 4.** Statistical significance groupings for mean best fitness of run for the 11-bit Multiplexer domain (First Experiment)

Problem and Evals		Layouts			
11-bit Multiplexer 25K Mean Best Fitness	Diagonal	1024	1448	2048	
		197.16	230.475	274.695	352.72
11-bit Multiplexer 50K Mean Best Fitness	Diagonal	1448	2048	1024	
		92.38	119.33	122.43	128.32
11-bit Multiplexer 100K Mean Best Fitness	2048	Diagonal	1448	1024	
		46.59	53.32	70.43	90.41

Why did the diagonal perform well? We include Fig. 2, showing results in the Artificial Ant domain, to illustrate the general trend during the course of evolution. We think that the diagonal layout is effectively taking advantage of the maximal-runlength phenomenon by providing *both* the runlength of the longer layouts *and* the emphasis on population size of the larger layouts. We note that in both cases, up until 10,000 evaluations, the diagonal layout closely resembles the 2048-population layout in performance, and then in the 20,000-30,000 range the diagonal layout “speeds up” and surpasses the smaller 1024-population layout, which has mostly converged.

## 4 Second Experiment

Our second experiment added additional layouts and repeated the 50K evaluation runs for each problem domain, to see if the diagonal was outperforming other layouts simply because it started at 2048K or was able to eke out long numbers of generations in its tail. The additional layouts included ones with both larger and smaller population sizes (and correspondingly shorter and longer runlengths).

Our additional layouts brought the total rectangular layouts to 4096, 2048, 1448, 1024, 501, 251, and 125 population sizes. The diagonal layout again started at 2048 and decreased to 0.

We again compared mean best fitness of run among experiments, and performed 100 runs for each experiment setup. Our difference of means test was an ANOVA at 95%, plus a Tukey post-hoc comparison.

### 4.1 Results

In the second experiment we changed the random seeds from the first experiment; hence the means for the original layouts are slightly different. We show the mean fitness results in Table 5. Layouts are again ordered left-to-right in worsening fitness, and horizontal bars above and connecting different layouts indicate no statistically significant difference between them.

Keep in mind that due to outlier biases from the worst layout results, the conservative Tukey test now cannot discern statistically significant differences

**Table 5.** Statistical significance groupings for mean best fitness of run for the Symbolic Regression, Artificial Ant, 5-bit Parity, and 11-bit Multiplexer domains (Second Experiment)

Problem and Evals	Layouts								
	Diagonal	2048	1024	1448	4096	251	502	125	
Regression 50K Mean Best Fitness	0.051	0.063	0.078	0.084	0.102	0.198	0.232	0.373	
Ant 50K Mean Best Fitness	17.7	20.23	21.25	21.31	22.49	26.63	28.45	29.68	
Parity 50K Mean Best Fitness	4.67	4.93	4.95	5.32	5.78	6.01	6.16	8.48	
Multiplexer 50K Mean Best Fitness	54.68	54.72	69.64	81.38	247.26	296.98	301.84	363.03	

among the original four layouts. This is as expected: the goal of the second experiment was only to see if the original four layouts were good choices, and it appears that they were. The new rectangular layouts were usually poor performers, and the diagonal layout usually outperformed all of them by a statistically significant margin.

## 5 Third Experiment

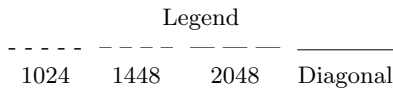
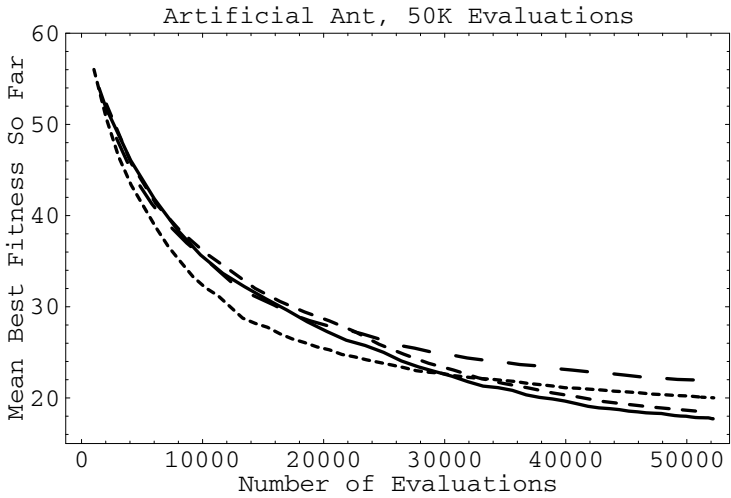
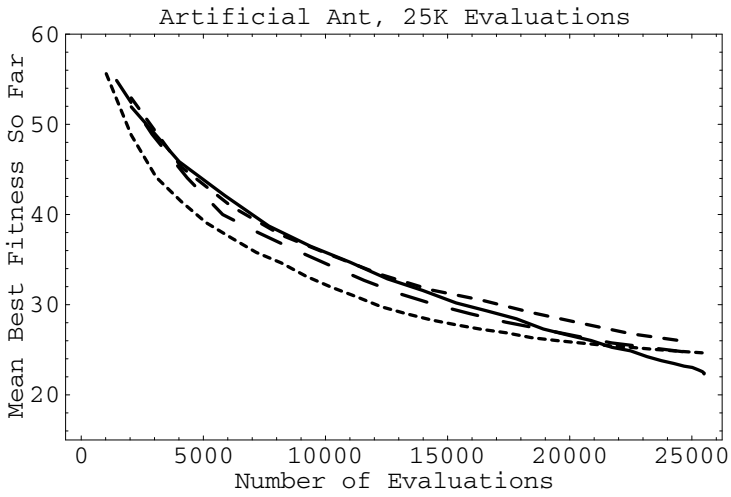
We also performed a similar experiment using two well-known GA problem domains, the Rastrigin and Rosenbrock problems. Each GA individual's representation was a vector of 100 floating-point genes each ranging from -5.12 to 5.12. We used one-point GA crossover and a gene-independent mutation probability of 0.01, where mutation consisted of gene randomization. We applied tournament selection of size 2, plus one-individual elitism.

Both functions are minimization functions. The Rosenbrock problem [5] computes fitness over a genome of size  $n$  using the function

$$\text{Rosenbrock}(x_1, \dots, x_n) = \sum_{i=1}^n 100(x_i^2 - x_{i+1})^2 + (1 - x_i^2)$$

Similarly, the difficult Rastrigin problem [17] computes fitness using the function

$$\text{Rastrigin}(x_1, \dots, x_n) = \sum_{i=1}^n x_i^2 + a(1 - \cos(2\pi x_i))$$



**Fig. 2.** Best-So-Far Curves of Four Layouts, Artificial Ant Problem, at 25K and 50K Evaluations



**Table 6.** Statistical significance groupings for mean best fitness of run for the Rosenbrock and Rastrigin domains (Third Experiment)

Problem and Evals		Layouts			
Mean Best Fitness	Rosenbrock	32	Diagonal	45	64
		2256.585	2415.601	3281.273	5131.429
Mean Best Fitness	Rastrigin	32	Diagonal	45	64
		136.0726	142.2995	155.8251	184.689

We adjusted the size of layouts to make them more appropriate to the GA realm: 32x1024, 45x724, 64x512, and a diagonal layout with initial population size of 64, running for 1024 generations. We performed 100 independent trials for each problem. As shown in Table 6, for both problem domains the ordering was the same, namely: 32x1024 outperformed diagonal, which in turn outperformed 45x724, which in turn outperformed 64x512. However, on the Rastrigin domain the difference between the 32x1024 and the diagonal layout was not statistically significant.

This is a mixed result. The result is similar to that of the 5-bit Parity GP domain, namely that longer runs are consistently preferred over shorter ones. But for the first time in this paper, diagonal has come in second in one of the experiments (Rastrigin). We note that the number of evaluations (32K, a typically size for GAs) is nonetheless similar to the smaller GP experiments. Further runs may suggest that GAs too have “diminishing returns” late in the run: but this is not yet borne out from the evidence here.

## 6 Conclusions and Future Work

In this paper we examined the possibility of imploding the population — gradually decreasing it towards zero as the run progressed — in the context of genetic programming. A linear decrease in population size proved effective regardless of the number of evaluations used. A diagonal layout was consistently in the top tier in every GP experiment, and usually gave the best results in the experiments. In initial GA experiments however, the results were mixed.

From this we can draw two conclusions: the primary conclusion is of course that non-rectangular layouts may yield better results than rectangular ones in environments like GP. But the second more troubling conclusion is that our experiments add to existing evidence that GP may have an aging problem. Whether due to premature convergence, bloat, or other factors, GP does not appear to use large-population resources effectively late in the run. While methods like diagonal layouts may serve to work around the issue, there is need for a closer examination as to *why* GP has diminishing returns, and how the representation or breeding methods may be changed to alleviate the problem.

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