A heuristic procedure for the Capacitated \(m\)-Ring-Star Problem

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Abstract: In this paper we propose a heuristic method to solve the Capacitated \(m\)-Ring-Star Problem which has many practical applications in communication networks. The problem consists of finding \(m\) rings (simple cycles) visiting a central depot, a subset of customers and a subset of potential (Steiner) nodes, while customers not belonging to any ring must be “allocated” to a visited (customer or Steiner) node. Moreover, the rings must be node-disjoint and the number of customers allocated or visited in a ring cannot be greater than the capacity \(Q\) given as an input parameter. The objective is to minimize the total visiting and allocation costs. The problem is a generalization of the Traveling Salesman Problem, hence it is NP-hard. In the proposed heuristic, after the construction phase, a series of different local search procedures are applied iteratively. This method incorporates some random aspects by perturbing the current solution through a “shaking” procedure which is applied whenever the algorithm remains in a local optimum for a given number of iterations. Computational experiments on the benchmark instances of the literature show that the proposed heuristic is able to obtain, most of the optimal solutions and can improve some of the best known results.

Keywords: Capacitated \(m\)-Ring-Star problem, Heuristic Algorithms, Networks.

1. INTRODUCTION

The Capacitated \(m\)-Ring-Star Problem (C\(m\)RSP) has been introduced by Baldacci et al. [1]. In the C\(m\)RSP, we are given a mixed graph \(G = (V, E \cup A)\), in which \(V\) is the set of nodes, \(E = \{(i, j) : i, j \in V, i \neq j\}\) is the set of edges and \(A\) is the set of arcs. The node set \(V\) is defined as \(V = \{0\} \cup U \cup W\) in which node 0 represents the depot, \(U\) is the set of customers and \(W\) is the set of Steiner nodes. Each customer \(i \in U\) can be connected to a subset of nodes denoted by \(C_i \subseteq U \cup W\), so the arc set \(A\) can be written as \(A = \{(i, j) : i \in U, j \in C_i\}\). We consider a non negative routing cost \(c_e\) for each edge \(e \in E\) and a non negative allocation cost \(d_{ij}\) for each arc \((i, j) \in A\). A ring \(R\) is a simple cycle visiting a subset of nodes including the depot. A customer \(i\) is assigned to a ring \(R\) if it is visited by the ring or allocated to a node on the ring. The number of rings, \(m\), and the capacity of each ring, \(Q\), are given as input parameters, and it is assumed that \(mQ \geq |U|\). In each feasible solution of the C\(m\)RSP, each customer has to be assigned to exactly one ring, each Steiner node can be visited at most once, and the number of customers assigned to a ring cannot be greater than the capacity \(Q\).

The goal of the C\(m\)RSP is to find \(m\) rings so that the global cost, given by the sum of the routing costs and of the allocation costs, is minimized. The C\(m\)RSP is NP-hard, since it generalizes the Symmetric Traveling Salesman Problem (TSP).

Baldacci et al. [1] proposed two heuristic procedures, H1 and H2, for the C\(m\)RSP. Moreover, they defined two Integer Linear Programming (ILP) formulations and developed a Branch and Cut (BC) approach for the C\(m\)RSP. Mauttone et al. [2] proposed a hybrid metaheuristic approach for the C\(m\)RSP in 2007. In their approach a combination of GRASP and Tabu Search algorithms has been proposed for solving the problem. Finally in 2008 Hoshino and de Souza [3] proposed an ILP formulation based on a Set Covering model and developed a Branch-and-Price (BP) algorithm for the C\(m\)RSP.

In this paper we propose a heuristic method for the C\(m\)RSP, which is able to obtain, within a short computing time, most of the optimal solutions and can improve some of the best known results proposed in the literature.

2. PROPOSED HEURISTIC

In the proposed algorithm, to construct the initial solution, we apply the clustering algorithm proposed by