Stability of additive mappings in non-Archimedean fuzzy normed spaces

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Received 20 April 2008; received in revised form 27 October 2008; accepted 31 October 2008
Available online 8 November 2008

This paper is dedicated to our teacher, Professor Mohammad Ali Pourabdollah

Abstract

In this paper we introduce a notion of a non-Archimedean fuzzy norm and study the stability of the Cauchy equation in the context of non-Archimedean fuzzy spaces in the spirit of Hyers–Ulam–Rassias–Găvruta. As a corollary, the stability of the Jensen equation is established. We indeed present an interdisciplinary relation between the theory of fuzzy spaces, the theory of non-Archimedean spaces and the theory of functional equations.

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MSC: primary 54C30; secondary 39B22; 39B82; 46S10; 54C05; 54D30

Keywords: Fuzzy stability; Cauchy equation; Jensen equation; Fuzzy norm; Non-Archimedean fuzzy normed space

1. Introduction

A basic question in the theory of functional equations is as follows: “When is it true that a function, which approximately satisfies a functional equation must be close to an exact solution of the equation?”

If the problem accepts a solution, we say the equation is stable. The first stability problem concerning group homomorphisms was raised by Ulam [1] in 1940 and affirmatively solved by Hyers [2]. The result of Hyers was generalized by Aoki [3] for approximate additive mappings and by Rassias [4] for approximate linear mappings by allowing the difference Cauchy equation \(|f(x+y) - f(x) - f(y)|\) to be controlled by \(ax^p + by^p\). Taking into consideration a lot of influence of Ulam, Hyers and Rassias on the development of stability problems of functional equations, the stability phenomenon that was proved by Rassias is called the Hyers–Ulam–Rassias stability. In 1994, a generalization of Rassias’ theorem was obtained by Găvruta [5], who replaced \(ax^p + by^p\) by a general control function \(\varphi(x, y)\). Since then the stability problems of various functional equations and mappings, such as the Cauchy equation \(f(x+y) = f(x) + f(y)\), the Jensen equation \(2f((x+y)/2) = f(x) + f(y)\), the quadratic equation, the cubic

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