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A Guide to Capital Allocation
Attribution and Optimization

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The phrase "capital allocation" has come to encompass activities ranging from measuring the capital used by transactions or business units to calculating performance measures to determining the optimal division of the institution's capital among competing transactions or business units. We find it helpful to make a distinction between the "attribution" of capital to transactions (or business units) and the process of "optimizing" the capital allocations to those transactions (or business units).

Attributing Capital to Transactions

The institution can attribute either "diversified" or "marginal" capital to a specific transaction.[1]

> ***Diversified capital*** (also referred to as *allocated capital*) assigns capital based on a transaction's volatility net of the diversification benefits achieved with the larger portfolio. Diversified measures are sometimes referred to as "portfolio beta" measures because the attribution of risk is based on the covariance of each transaction with the entire portfolio in the same way that a stock's beta is calculated from its covariance with the market portfolio.
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> ***Marginal capital*** measures the amount of capital that the transaction adds to the entire firm's capital (or, conversely the amount of capital that would be released if the transaction were transferred). Marginal capital for individual transactions is typically calculated by estimating total portfolio capital at a high confidence level, e.g. the 99.9^{th} percentile, both with and without the transaction in question, and then taking the difference.

Capital attribution for transactions is generally accomplished "bottom-up," meaning that individual transactions are modeled and then aggregated to arrive at portfolio capital. Alternatively, a "top down" measure employs the observed volatility of earnings (or cash flow) to estimate the volatility of a business line's net assets. Top down measures are most appropriate for high volume businesses, where transaction level detail is unavailable and the allocation of capital to specific transactions is not required. The diversified and allocated capital concepts apply whether the unit of analysis is a transaction or a business. Often separate measures of credit risk capital, market risk capital, and other risk capital (possibly operational risk capital) are obtained and then summed to get total capital.[2]

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[1] For attributing capital to business units, instead of individual transactions, a financial institution would also consider "stand-alone" capital, the amount of capital that the business unit would require, if it were viewed in isolation. Stand-alone capital would be determined by the volatility of each unit's earnings.

[2] Summing those measures assumes that they are perfectly correlated, clearly an overly conservative assumption.

The most common approach to attributing capital on a diversified basis is a transaction's (or business unit's) contribution to the standard deviation of the portfolio (σ_p). For example, if the exposure to asset A is x_A , then its contribution to standard deviation is given by the change in standard deviation for a small increment in the transaction, multiplied by the amount of the transaction.

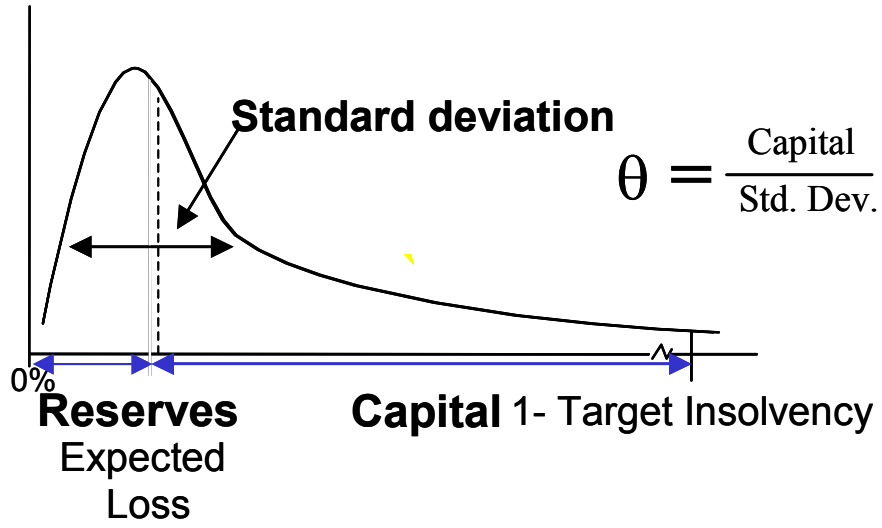
$$Capital\ Attribution_A = x_A \left[\frac{\partial \sigma_p}{\partial x_A} \right]$$

The contribution to standard deviation (unexpected loss in credit portfolios) can be converted into a capital allocation using a capital multiplier. Use of a multiplier maintains the summing up property, i.e. total allocated capital equals total portfolio capital. To implement this, most models create a "capital multiplier". The capital multiplier, θ, is defined by

$$CL = \varepsilon + \theta \sigma_p$$

where *CL* is total capital measured at the desired percentile and ε is the expected loss.

Exhibit 1 -- The Capital Multiplier



The multiplier, θ, scales the standard deviation based attributions upward so that they sum to total capital.

$$\widetilde{CapitalAttribution}_A = \varepsilon_A + \theta(CapitalAttribution_A)$$

A multiple of standard deviation is used, because there is no analytic solution to the problem of attributing a percentile of the loss distribution.[3]

Optimizing the Allocation of Capital

Since capital markets are competitive, financial institutions must offer equity investors a return sufficient to justify their investment. The ***optimal*** capital allocation is one that deploys capital to maximize shareholder value.

Metrics for Comparison

Before the institution can optimize the allocation, it must first measure the return being earned on capital already employed.

RAROC -- A "generic" method for calculating RAROC is illustrated in Exhibit 2.[4]

Exhibit 2 A Generic Model for Return on Risk Capital

$$\text{RAROC} = \frac{\text{Net Operating Revenue} = \begin{cases} \text{Revenues} \\ - \text{Credit provisions } \textit{(for expected credit losses)} \\ - \text{Direct and Allocated costs} \\ - \text{Taxes and adjustments} \\ - \text{Capital adjustments} \end{cases}}{\text{Risk-Adjusted Economic Capital} = \begin{cases} \text{Credit risk capital} \\ + \\ \text{Market risk capital} \\ + \\ \text{Other risk capital} \end{cases}}$$

The denominator is diversified economic capital. The numerator represents the economic earnings of the unit. Note that credit risk enters the numerator as a provision for expected losses, since expected losses are considered a *cost of being in the credit business* rather than a *risk*. While institutions differ in the way they treat capital, a common approach is to give each unit

[3] The appropriateness of this method is called into question by the fact that the relative riskiness of transactions change as one moves from standard deviation out to the tail of the loss distribution. This issue will be covered in an article by Greg Hayt and Jack Praschnik in the July issue of *Risk*.

[4] RAROC literally stands for "Risk-Adjusted Return on Capital" but, as is illustrated in Exhibit 2, the most common calculation is actually "Return on Risk-Adjusted Capital" – i.e., we calculate RORAC, but call it RAROC.

credit for the risk capital deployed, i.e. to reflect earnings on the risk capital as if it were invested in a risk-free asset. Others attribute this return on capital to a central "treasury" function.

Economic profit -- The cost of capital deployed is deducted from net operating revenue:

$$\text{Economic Profit} = \text{Net Operating Revenue} - \text{Economic Capital} \times \left[\begin{array}{c}\text{Rate of Return} \\ \text{Required by Shareholders}\end{array}\right]$$

With respect to the rate of return required by shareholders, financial institutions generally use a single number, estimated at the institution level, to all transactions and business units. This bank-wide required return might be estimated using the Capital Asset Pricing Model, the firm's stock beta, and the equity risk premium; but, more likely, it is a blended rate, reflecting the bank's actual costs of permanent financing, and input from outside sources on equity financing costs.

We perceive economic profit to be a more direct measure of whether a firm (or a business unit within that firm) is creating value for its shareholders, because of the direct relationship between economic profit and value creation. While it is true that a RAROC above the cost of capital indicates positive economic profit, economic profit provides a clearer signal: RAROC eliminates the effect of scale, emphasizing percentage results over total results. Maximizing economic profit provides incentive to look at the portfolio of businesses.

Optimizing the Allocation of Capital

In an optimal portfolio, ***marginal return per unit of risk will be equal across business units***. (Taking a unit of capital from one business and giving it to another will not add additional value to shareholders, because the return gained equals the return lost.) As more and better data becomes available, we expect to see formal optimization models and processes implemented. Until then, most financial institutions employ an informal process – based on the optimization principle -- to determine how to deploy the institution's capital.

Within the optimization process, an often-voiced question is how to treat regulatory capital. If economic capital exceeded regulatory capital for all transactions, regulatory capital could be ignored – i.e., the regulatory capital "constraint" would be non-binding. When this is not the case, we have observed firms using a variety of *ad hoc* approaches.

- Some firms impose an explicit charge for regulatory capital in the calculation of economic profit. While such an approach identifies businesses that require high amounts of regulatory capital in the creation of economic profit, it can mask real economic value.

  A business unit that requires a lot of regulatory capital could be highly profitable on an economic basis. The regulatory capital charge could result in the unit's net economic profit being lower than other business units. This may be the wrong signal; the optimal path might be for the bank to figure out how to reduce the unit's regulatory capital requirement or to obtain additional capital to grow the otherwise highly profitable business.

- Other firms work with whichever capital number is larger -- i.e., calculate economic profit relative to regulatory capital if that exceeds economic capital. This method has some appeal since it charges each business for the larger of the two capital numbers, but it suffers from a "summing up" problem. By taking the larger of the two capitals for each business the bank will appear to have deployed more capital than it actually has, and the profit calculations will not make sense when aggregated bank-wide.

Financial institutions can combine the two approaches and look at the costs and benefits, at the margin, of another unit of regulatory capital. In a formal optimization model, this would be known as evaluating *shadow prices*; in less-formal processes it simply involves calculating the benefit achieved if the regulatory capital constraint could be loosened slightly relative to the cost of the additional capital. This idea of course applies equally well to other constraints, such as computing capacity or human resources. Shadow prices send the correct signal about where the most value can be created for an additional unit of capital or any other resource.

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