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Using what matters to students in bilingual mathematics problems

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Abstract In this study, the author represented what matters to bilingual students in their everyday lives—namely bilingualism and everyday experiences—in school-based mathematical problems. Solving problems in pairs, students demonstrated different patterns of organizing and coordinating talk across problem contexts and across languages. Because these patterns bear consequences for how bilinguals experience mathematics learning, the author takes these patterns as the basis to argue that mathematics education for bilingual students should capitalize on bilingualism and experiences as cognitive resources.

Keywords Elementary mathematics · Bilingualism · Everyday experiences · Languaging

1 Situating this research: Recognizing what matters in students' lives

School is an experience that matters in bilingual students' everyday lives. However, students' out-of-school experiences, including the languages in these experiences, are often not recognized as having mathematical value in schools (Civil, 2002a; Masingila, 2002; Resnick, 1987). Instead, traditional school mathematics instruction purports the relationship between school-based learning and out-of-school learning to be unidirectional, with out-of-school contexts as opportunities for students to apply what they learned in school. In a recent dialog between mothers of bilingual students and their teacher in an elementary school in the Midwestern region of the USA, the teacher told the mothers: "Y yo creo que, todo lo que están aprendiendo en el salón, cuando salen de la escuela, uh, lo importante es que lo apliquen en una situación real, porque de eso se trata, poder utilizar la información que estamos aprendiendo en el salón y ponerlo en una situación real" [*I think that, everything that they're learning in the classroom, when they get out of school, uh, what's important is that they apply this in a real situation, because that's what it's all about, to be*

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able to use the information that we're learning in the classroom and to apply it in a real situation]. Situated cognition theorists have questioned this assumption of a straightforward transfer of school-based mathematical knowledge into people's everyday activities (Lave & Wenger, 1991; Brown, Collins, & Duguid, 1989; Young, 1993). In this study, I view students' everyday activities not as a field of application (traditional school view) but as a source of conceptual development (Streefland, 1991). Following this view, I represented in school-based mathematical problems the experiences and languages that matter in bilingual students' everyday lives. To investigate what students did with these problems, I formulated the following research question.

2 Research question

Given the institutionalized practices in schools (short-lived bilingual programs, inattention to inadequate supply of bilingual teachers, mathematics instruction that does not capitalize on bilingual students' resources) that effectively subtract resources from the education of bilingual students (Valenzuela, 1999), and given the repertoires of everyday experiences in which students participate (Gutierrez & Rogoff, 2003), this study contributes to answering the question: How are bilingualism and everyday experiences cognitive resources for learning mathematics?

To consider this question in light of what is known from research, I review relevant studies by looking at the various approaches that have been used to investigate the roles of everyday experiences and bilingualism in learning mathematics.

3 Literature review

With few exceptions, the roles of everyday experiences and bilingualism in mathematics learning have not converged in research. Therefore I review these studies separately, not without acknowledging that findings from each tradition are foundational to the dual focus of this paper. The review ends with a few studies that focus on both everyday experiences and bilingualism. In these studies, bringing students' experiences and bilingualism into classrooms makes irrelevant the delayed application of school knowledge by emphasizing the process of making sense *in* the classroom *with* resources from students' everyday lives.

4 The role of everyday experiences in mathematics learning

The idea that students—and people in general—develop out-of-school knowledge from meaningful participation in everyday experiences has been investigated in various settings, including street vendors (Carraher, Carraher, & Schliemann, 1985), candy sellers (Saxe, 1988), carpet layers (Masingila, 1994), grocery shopping (Lave, Murtaugh, & de la Rocha, 1984; Lave, 1988; Taylor, 2004), or tailoring and pattern making (González, Andrade, Civil, & Moll, 2001). Because these studies compared and claimed that people perform problem solving better outside the school than in school, they suggest drawing on the informal problem-solving practices observed in these contexts (particularly the invention of meaningful and intuitive methods) as support for reinvigorating the classroom problem-solving practices with meaning. Carraher and Schliemann (2002) note that the power of everyday mathematics "is not the concreteness of the objects or the everyday realism of the situations, but the meaning attached to the problems" (p. 137).

Besides investigating mathematics practices away from school settings, researchers have also developed projects focused on redesigning the mathematics curriculum by taking into account students' everyday experiences. Research in this category rejects the deficit-thinking hypothesis and instead locates resources in students' cultures and everyday lives. These projects include realistic mathematics education (Freudenthal, 1991; Gravemeijer, 1994; Streefland, 1991; Treffers, 1987), with its emphasis on "recognisable situations and use [of] terms and expressions from everyday life" (Elbers & de Haan, 2005, p. 46), and the BRIDGE Project (Civil, 1995, Civil & Khan, 2001, Civil, 2002a, 2002b) with its emphasis on pedagogical transformations of students' everyday knowledge into modules that connect with school mathematics. These studies attribute consistent mathematics achievement gains to innovative curricula that use students' everyday practices in meaningful mathematics instruction.

Findings from everyday mathematics research are foundational for the present study. Firstly, the out-of-school settings provide important scaffolds for solving problems. Secondly, problem solvers in these settings take ownership of problems via the uninterrupted grasp of meaning. Thirdly, replicating these problems in schools is not a realistic goal, rather researchers suggest looking into how to promote meaning making by encouraging students to develop problem-solving strategies that make sense to them. This research is based on careful analysis of contrasting ways of using strategies between out-of-school and in-school settings (Carraher & Schliemann, 2002). Finally, researchers who have incorporated local knowledge into mathematics curricula have recognized the danger of trivializing or stereotyping everyday practices (Lipka, 2002), thus contributing improved understanding of how context should figure in mathematics education. This contribution is developed in the conceptual framework.

5 The role of bilingualism in mathematics learning

The recent focus on communication in mathematics learning (National Council of Teachers of Mathematics, 2000; National Research Council, 2002) has attracted considerable research attention to how bilingualism (and multilingualism) figures in mathematics learning. Several research approaches are reviewed next.

One approach focuses on how bilinguals allocate their two languages as they solve classroom-based problems. These studies include Spanish–English bilingual students who are becoming dominant in English (Cuevas, 1984; Valverde, 1984; Secada, 1991), Yoruba–English bilingual children from grades 1 through 4 (Adetula, 1989), and Vietnamese–English bilingual students in grade 4 (Clarkson, 2006). The consistent finding across these studies is that these diverse bilingual students perform better when solving problems in their native language than in their second language, especially when the problems are difficult (Clarkson, 2006; Secada, 1991), for which they also tend to use more advanced strategies in their native language (Adetula, 1989), or choose the native language to address open ended questions that require mathematical reflection (Clarkson, 2006). As language choice is aligned with problem difficulty and improved performance, these studies contribute evidence of the role of bilingualism as a cognitive resource for mathematics learning.

A different approach to investigate how students use bilingualism in mathematics considers code-switching as a strategy that supports their mathematical activity. For example, researchers have documented consistent use of code-switching when learners are exploring new ideas (Adler, 2001), or when they are in classrooms with teachers who publicly code-switch during conversations focused on exploring students' reasons for their mathematical ideas (Setati, 2005). In contrast, when students are in classrooms with teachers who do not code-switch publicly or do it only as a tool for discipline, this conceptual discourse is replaced by students' calculational discourse, or "discussions in which the primary topic of conversation is any type of calculational process" (Sfard, Nesher, Streefland, Cobb, & Mason, 1998, p. 46). In another study, Moschkovich (2005) offers compelling evidence of two grade 9 students using code-switching to clarify mathematical meanings, state assumptions, support mathematical claims, and make connections to mathematical representations, all of which are "valued mathematical discourse practices" (p. 138). Moschkovich raises a question regarding students' bilingualism in mathematics that the present study contributes to answer, "What are the student's experiences with each language in and out of school, in particular, past experiences with mathematics instruction in each language?" (p. 132). Therefore, whether students code-switch (bilingual mode) as in these studies, or code alternate (monolingual mode) (Grosjean, 1999), as in studies reviewed earlier, they demonstrate that their bilingualism is to be used as a cognitive resource whose importance is on a par with the cognitive demands of their mathematical work.

Some researchers use an interactional approach to investigate how bilingualism figures in students' participation in mathematical discussions. For example, Turner, Dominguez, Maldonado, and Empson (2010) investigated a set of classroom practices that facilitated English learners' participation in mathematical discussions, concluding that these students can and do display mathematical competence when instructional features such as positioning students in English and Spanish in particular roles relative to tasks and one another support their participation in classroom discussions. In another interactional study, Barwell (2003) discussed the case of one bilingual and one monolingual student constructing and solving mathematics problems. Students' talk displayed various patterns of attention, including joint attention to their personally meaningful experiences and to the genre of school-based mathematical problems. Interactional studies support the view of bilingualism as a shareable resource from which all students, including non-bilinguals, benefit.

6 Bilingualism and everyday mathematics: Learning with applications

Very few studies in mathematics education have broadened their focus to include both everyday knowledge and bilingualism. These include the following: the Yup'ik Project (Lipka, 1994, 1998, 2002, 2005), with its focus on the use of native language and knowledge from everyday practices, and the Kamehameha Early Education Project (Brenner, 1998a), with its concern about how Hawaiian children developed mathematical knowledge in everyday situations, shopping and family interactions, and using this knowledge in an experimental math curriculum, along with the use of pidgin. Other studies with similar orientation but smaller scale and less explicit use of bilingualism include Simic-Muller, Turner and Varley (2009); Barta, Sánchez, and Barta. (2009); and Cicero, Fuson, and Allexsaht-Snider (1999). Besides documenting improved mathematics performance, these studies concur that conceptual development in mathematics does not need to take the form of delayed application, as implied by the traditional school view, but it can proceed by grounding such development in the practices that are relevant in students' communities.

The literature reviewed offers important contributions to this study. Firstly, everyday mathematics research has identified supports that are typically absent in classroom instruction, and it has refined the concept of context, which I discuss in the conceptual framework. Secondly, the bilingualism research has demonstrated that bilinguals make strategic use of their two languages (either by code-switching or code-alternating) in order to maximize their mathematics performance, especially when the tasks they solve are challenging. An implication therefore is that if bilinguals are strategic language users, they may also be strategic problem solvers, especially if learning to problem solve is viewed as a communication-based experience.

However, the view of bilingualism and everyday experiences as inseparable phenomena that I propose in this study calls for an integrated treatment of these processes. This integration has been hinted at by at least two researchers. For example, Secada (1991), intrigued by how students in his study and those in Adetula's (1989) study used languages, advised: "there may be some subtle effects of schooling and of the larger social context that are worth examining" (p. 227). Similarly, Moschkovich (2005) points out that besides code-switching, "other aspects of bilingual learners' language practices and features of talk may be relevant to learning mathematics, yet are less salient to the untrained ear, and should be explored" (pp. 138–139).

In light of these findings, the contribution of this study is that it *specifies* how bilingualism and everyday experiences are cognitive resources that can support students' reasoning within a context where learning mathematics matters: the classroom.

7 Conceptual framework

I conceptualize bilingualism and everyday experiences as cognitive resources for solving school-based mathematics problems. I draw from the literature reviewed, specifically from the general finding that bilingualism and everyday experiences facilitate the learning of mathematics. I also draw from Bruner's (1990) theory of language as praxis for making meaning, particularly from his view of language and practice as inseparable phenomena. From this theory, I conceptualize bilingualism as social action: doing things with and to others, or languaging, and everyday experiences as culturally inseparable from language and facilitating meaning making.

8 Bilingualism as a resource for doing things with and to others

In previous work, I argued that bilingualism is a cognitive resource because it connects students with social practices (Dominguez, 2008). More specifically, bilingualism serves to connect the ways students talk in school-based tasks with culturally specific ways of talking in everyday practices. This is not a trivial matter, for this connection enhances meaning making. For example, in an after-school project with my colleagues we reported how one bilingual student framed a powerful proportional reasoning idea by using the structure of an everyday saying in Spanish, his primary language. He said, "Cada 8 valía por 2" *(Each [bag of] 8 is worth 2 [bags of 4]*) (Turner, Dominguez, Maldonado, & Empson, 2007). This cultural-mathematical connection resulted in an effective way of framing a personally meaningful mathematical idea that other students appropriated later. The student, in other words, is languaging (Swain, 2006): transforming his thoughts about a proportional reasoning situation into a shareable resource—shareable with himself and others.

The cultural practices in which students participate every day and from which they draw to frame mathematical arguments are, as the above example shows, languaging practices.

As Bruner (1990) states, "*Logos* and *praxis* are culturally inseparable" (p. 81). In these languaging practices, students learn how to do things with words (Austin, 1962). But what does doing things with words look like when students solve problems in mathematics classrooms? According to Bruner, "the very act of speaking is an act of marking the unusual from the usual" (p. 79). This may explain why in most mathematics classrooms, students barely talk—they rarely do things with words—as they may be dealing primarily with the usual, that is, repeated exercises of the same kind. Doing things with words in mathematics classrooms means learning from one's ideas and the ideas of others, which is consistent with the view of mathematics learning as enhanced by communication (National Council of Teachers of Mathematics, 2000).

The contexts in which children participate are always contexts of practice; that is, children are always *doing* or *trying* to do something (e.g., solving problems at school). Moreover, children are continuously languaging in these contexts of practice, and are consistently driven by "the child's push to give meaning or 'structure' to experience" (Bruner, 1990, p. 90). Children's engagement with meaning making is social, because "meaning itself is a culturally mediated phenomenon that depends upon the prior existence of a shared symbol system" (Bruner, 1990, p. 69). The social dimension of meaning making helps us to understand that what bilingual students do with words, whether in classrooms or at home, is always mediated by what the child perceives as acceptable languaging practices. In these contexts of practice, "The child is not learning simply what to say but how, where, to whom and under what circumstances" (Bruner, 1990, p. 71). The view of bilingualism as situated in social experiences makes it imperative to consider the conditions that these experiences come with that either encourage or discourage bilinguals to do things with words. So, once children "grasp the significance of situations (or contexts)" (p. 71), they are prepared to use this grasp to influence others' grasp, that is, the child is ready to do things with words, or to be agentive with language. Contained in this grasp, though, is a set of conditions that children learn to recognize and that mediates their ability to do things with words. For bilingualism to be a cognitive resource, conditions must be favorable for students to do things with words. For Halliday (1978), the view of language as a resource implies "the choices that are available, the interconnection of these choices, and the conditions affecting their access" (p. 192).

Therefore, this framework moves our understanding beyond what students can achieve with bilingualism—which has been highlighted by previous research—to how what students can achieve is mediated by culturally established contexts of practices. These contexts of practice are discussed next.

9 Everyday experiences as (unfinished) contexts of practice where students do things with and to others

Students' contexts of practice are so varied and complex that often it is more productive to consider what they are not. According to Carraher and Schliemann (2002), "contexts are not fully constituted by their physical properties but always involve issues of meaning and interpretation" (p. 147). Integrating bilingual students' everyday experiences—their contexts of practice—into mathematics problem solving implies integrating the students' two languages. As stated in the previous section, contexts of practice are, at the same time, languaging practices. These languaging contexts of practice constitute cognitive resources because, as Bruner (1990) explains, they enable children to grasp the significance of contexts and facilitate the meaning they are continuously trying to give to their experiences.

In this study, contexts are not reduced to physical or social settings. Instead, I follow Carraher and Schliemann's (2002) view that "contexts can be imagined, alluded to, insinuated, explicitly created on the fly, or carefully constructed over long periods of time by teachers and students" (p. 146). Representing bilingual students' everyday experiences as contexts in mathematics problems does not reproduce those experiences in school. On the contrary, I acknowledge that once imported into classrooms, out-of-school problems are no longer the same (Civil, 2002b; Brenner, 2002) because the experiences are redefined (Carraher & Schliemann, 2002). Consistent with these ideas, I represented everyday experiences as contexts in mathematics problems in order to address two related issues. The first is a redefinition of the problems that bilingual students typically receive in school. In a recent course that I taught, I asked experienced teachers, including bilinguals, to incorporate students' prior knowledge into mathematics lessons. Most teachers equated prior knowledge with previously taught skills. This predisposition to ignore the informal knowledge that students learn in everyday experiences maintains these contexts of practice disconnected from school. Arguing for including experiences that matter to students in school mathematics problems, Lesh, Hoover, Hole, Kelly, and Post (2000) advise on this matter, "To develop problems that encourage students to base their solutions on extensions of their personal knowledge, the topics that work best tend to be those that fit the current local interests and experiences of specifically targeted groups of students" (p. 619).

The second issue addressed by using everyday experiences as contexts in mathematics problems is related to the role of contexts as sources for students to grasp the significance and meaning of situations (Bruner, 1990). Contexts in this framework are viewed as foundational to the view of learning mathematics as systematizing what in students' lives is already a continuous search for meaning. This view is aligned with Carraher and Schliemann's (2002) ideas that "These situations do not provide finished knowledge. However, they provide rich repertoires of experience, data, and schematized understandings of how things work without which advanced mathematical understanding would be inconceivable" (p. 148).

10 Bilingualism in contexts of practice

Considering bilingualism and everyday experiences as inseparable adds the final dimension to this framework. This dimension refers to the fact that when students enter these languaging contexts of practice, others have already begun shaping and establishing these contexts, an idea that is consistent with the view of school and home as established cultural practices (Cobb, 2007). By being established, these cultural practices influence what one can do with language, or as Bruner (1990) says, this agentivity with language "will... vary from person to person and, as we also know, vary with one's felt position within the culture" (p. 119). Therefore, key in the framework is considering the influence of these established contexts of practice—including their contrasting ways of using language (González, Moll, & Amanti, 2005)—on how students organize and coordinate talk when solving mathematics problems.

11 Context and participants

This study was done in a pre-kindergarten to grade 5 elementary school in Austin, Texas, located in a working-class, predominantly Mexican/Mexican-American/recent immigrant neighborhood. The school had an enrollment of approximately 400 students and a transitional bilingual program that exits students to English-only classrooms by grade 4, using multiple criteria, including a state mandated test of English, academic content exams, and teacher nominations. Participant students were in grades 4 and 5 at the time of the data collection. They were Spanish–English bilingual at home and community but interacted only in English during mathematics instruction, which was in English.

12 Methods and data collection

To address the question, How are bilingualism and everyday experiences cognitive resources for learning mathematics? I designed the study with a focus on the problem-solving actions that bilingual students will generate when solving (a) problems about familiar experiences in Spanish, (b) mathematically similar problems about unfamiliar experiences in Spanish, (c) problems about familiar experiences in English, and (d) mathematically similar problems about unfamiliar experiences in English. Actions here refer to things that students achieve through languaging. To collect data on these actions, I developed methods that included classroom observations, home interviews, creation of bilingual problems with contexts from students' everyday experiences, and small-group problem-solving interviews. Next I describe each method, specifying how the conceptual framework was used.

13 Classroom observations

The purpose of these observations was to describe the languaging contexts of practices in which students participated regularly and that were established in these classrooms (Cobb, 2007). These descriptions can help understand how students organized and coordinated problem-solving actions later on in the small-group problem-solving interviews.

I observed participating students in their mathematics classes for about 2–3 weeks, and I took field notes of peer interactions, teacher–student interactions, the kind of teaching and learning, languages used, and the teachers' preferred tasks for instruction. Students' mathematical work was characterized by physical proximity but intellectual distance; that is, they were seated in small groups around a table but they were working individually on practice exercises after having received whole group instruction on similar problems. Teacher-student interactions were predominantly one-to-one interactions, with teachers either coming to each group and checking on individual students or individual students approaching the teacher for assistance. During instruction, in English, teachers would present primarily decontextualized problems, and would use these problems to teach students procedures (mainly standard algorithms), followed by practice exercises that aligned perfectly with the procedures just taught (See Boaler, 1998, for description of a similar kind of instruction).

Teachers valued quiet work. In fact, in one observation, one teacher discouraged a group of students who were discussing the practice exercises. When one student explained that his group talk was about the exercises, the teacher did not believe him and threatened to subtract points if they continued talking. Teachers also valued memorization of procedures. For example, one teacher had student-created posters displayed outside the classroom with memorized phrases, such as *multiplication makes bigger* or *you divide a larger number by a smaller number.*

14 Home interviews with parents and students

The purpose of these interviews was to document the kinds of languaging contexts of practices that are established in these homes and that include students' direct participation. Another purpose of these interviews was to use students' everyday practices as contexts for school-based mathematics problems. I used a semi-structured interview to find out about the activities that mattered in students' lives, that is, activities in which the child was a regular participant with and without the family being involved. For each activity, I asked them whether the child used Spanish or English or both. To facilitate the brainstorming of activities, I asked them to think about objects or artifacts that the child was using regularly in these activities.

All parents welcomed me at their homes for these interviews, a privilege facilitated by the fact that I am Mexican and share the culture, the language, and even the age of these parents. In these conversations I learned the following about bilingual students' languaging contexts of practice: In their everyday activities, these children participate in unrestricted and purposeful bilingual practices. This means that students in these contexts control when to use Spanish, when to use English, and when to use both, with whom, and for what purposes. Another characteristic of these languaging contexts of practice is the high value that parents place on the education of their children. In the interviews, parents consistently viewed schoolwork as the most important responsibility for these bilingual children, thus demonstrating that school is an experience that matters in students' everyday lives.

As for the second purpose of investigating students' everyday practices, these interviews revealed a broad range of direct participation in student-selected activities such as art classes in the community, swimming, playing soccer, playing videogames, riding bicycles, reading, grocery shopping, as well as parent-selected activities such as preparing small meals for younger siblings, helping mothers with small house chores, assisting fathers with their jobs, and attending church. From all these activities in which students participated either voluntarily or as required by parents, I selected the most common ones to represent as mathematical problems, thus providing more meaningful alternatives than the decontextualized exercises given to them at school. It is important to note that these students' interests are not static and I view them as dynamic participation in repertoires of practices (Gutierrez & Rogoff, 2003) that evolve as students continue exploring more interests. Next I describe the process of creating mathematics problems using the contexts that matter in students' everyday lives.

15 Using what matters to students as contexts in mathematics problems

The purpose for using what matters to students in their everyday lives as contexts in mathematics problems was to provide an opportunity for students to grasp the significance of contexts with meaning (Bruner, 1990). Rarely do schools offer students the possibility to identify in school-based problems their everyday practices and develop a sense of themselves within those practices as capable problem solvers (Nasir, Hand, & Taylor, 2008). For Rich (1986), "When someone with the authority of a teacher, say, describes the world and you are not in it, there is a moment of psychic disequilibrium, as if you looked into a mirror and saw nothing" (p. 99).

I used three criteria for representing students' everyday activities in problems:

1. Problems described situations that could in fact happen and for which students would not know the answer immediately. I wanted students to solve true problems as opposed to exercises, and feel the need to mark the unfamiliar from the familiar (Bruner, 1990).

2. When possible, I left out key information so students could supply it from their own repertoires of knowledge and familiarity with these contexts. Contexts, according to the framework, are not fully constituted but waiting for student interpretation (Carraher & Schliemann, 2002).
3. Problems followed the language students reported using in the activities, in order to avoid, or at least minimize, the possibility of turning a problem-solving task into a language task. This reflects the view of contexts of practice as always situated (Bruner, 1990, Suchman, 1987) and being simultaneously languaging contexts (Swain, 2006).

Furthermore, I was inspired by Boaler's (1998) approach of using "applied activities situated within school" (p. 53). She explains:

> The ways in which students react to such tasks can never be used to predict the ways in which they will react to real-life mathematical situations. However, I believe that the combination of school settings and realistic constraints provided by applied tasks can give us important insights into the factors that influence a student's use of mathematical knowledge. (p. 53)

Using the above criteria, I created four pairs of mathematical problems. Problems in each pair dealt with the same mathematical content and the same language, and differed only in the context represented. It is important to view each pair of problems as tasks with similar mathematical demands, not as parallel versions. The reason is that for their creation, I tried to keep the contexts as realistic as possible (they could occur in real life), even for the contexts unfamiliar to students.

The first common experience was the community arts classes that most students regularly attended after school. Many parents proudly showed me the art works produced by their children in these classes. I therefore considered this familiar experience to write the following problem in Spanish, the language of the experience, along with a similar problem but with an unfamiliar experience (Table 1). The problems explore the multiplicative structure of fractions, which means, for instance, that finding one third implies creating three times as many pieces.

To create the next set of problems (Table 2), I considered various practices familiar to students, namely grocery shopping, making scrambled eggs (either for themselves or for younger siblings when the mother was at work), and eating breakfast at the school cafeteria. In these practices, students reported speaking Spanish. The problems explore measurement

Table 1 Multiplicative structure of fractions (problems in Spanish)

Problem with familiar experience	Problem with unfamiliar experience
Tu maestra de arte te dio 3 paquetes de papel construcción para que hagan banderitas de México. Un paquete es de hojas verdes, uno es de hojas blancas, y uno es de hojas rojas. Cada paquete trae 25 hojas. ¿Cómo podrías hacer 60 banderas de México?	En el cuarto de materiales de una imprenta quedan 7 paquetes de papel para invitaciones. Cada paquete trae 20 hojas. ¿Cómo puede hacer la imprenta una orden de 270 invitaciones?
(Translation)	(Translation)
Your art teacher gave you 3 packages of construction paper to make flags of Mexico. One package has green paper, one has white paper, and one has red paper. Each package has 25 sheets. How can you make 60 flags?	*In the materials room of a print shop there are 7 packages of invitation paper left. Each package has 20 sheets. How can the print shop make an order of 270 invitations?*

Table 2 Measurement division (problems in Spanish)

Problem with familiar experience	Problem with unfamiliar experience
Para el desayuno escolar, la señora de la cafetería de tu escuela tiene que hacer huevos revueltos para 400 niños. ¿Cuántos cartones de huevo tiene que abrir?	Un organizador de fiestas está organizando un banquete para 500 personas. Necesita comprar platos. Los platos vienen en paquetes de 30. ¿Cuántos paquetes de platos necesita comprar?
(Translation)	(Translation)
For the school breakfast, the school cafeteria lady has to make scrambled eggs for 400 children. How many egg cartons does she have to open?	*A party planner is organizing a banquet for 500 people. He needs to buy plates. Plates come in packages of 30. How many packages of plates does he need to buy?*

division, where the size of the groups (known) can be used to measure the number of groups (unknown). A problem with similar mathematical demands was framed with a context unfamiliar to students.

For the next set of problems (Table 3), I used the common experience of playing videogames. Students said that they preferred English for this practice because of the greater variety of games available in this language, and because they played the games with older siblings or friends who spoke mainly English. I wrote combinations problems based on the fundamental counting principle: If there are n ways of choosing one thing, and there are m ways of choosing another thing, then there are $n \times m$ ways of choosing both things. I matched the familiar videogame problem with a problem about a context unfamiliar to students.

For the final set of problems (Table 4), I used the homework practice of reading English books at home, usually to a parent or sibling, for about 30 min every day. I used the concept of ratios in this familiar experience and in a matching problem but with an unfamiliar experience.

16 Problem-solving interviews

In pairs, half of the students solved the Spanish problems first and then the English problems, and the other half solved them in the reversed order. I wanted students to solve problems in pairs to reflect the joint meaning making and the languaging in these everyday experiences. According to Bruner (1990), "The realities that people constructed were *social* realities, negotiated with others, distributed between them" (p. 105). Similarly, Halliday (1978) conceived of reality as constructed by people through an exchange of meanings.

I presented all problems on my laptop screen to encourage student–student interaction from the start instead of providing individual copies of the problem and thus replicating the individual problem-solving approach observed in the two classrooms. I also made available paper, pencils, pens, markers, and interlocking cubes. After they read each problem

Table 3 Combinations: fundamental counting principle (problems in English)

Problem with familiar experience	Problem with unfamiliar experience
In the game The Sims© you can create and control people. You can choose from 2 genders, 3 skin tones, and 4 hair colors. How many different people can you create?	A print shop makes business cards. It has 5 colors of paper, 3 colors of ink, and 2 paper textures available. How many different business cards can the print shop make?

Table 4 Partitive rate and measurement rate (problems in English)

Problem with familiar experience	Problem with unfamiliar experience
If you can read 5 pages in 20 min, how long is it going to take you to read a book that has 23 pages?	If a painter uses 20 gallons of paint to paint 4 houses, how much paint will he need to paint 15 houses?

together, I reminded them to solve it however they wanted. All problem-solving sessions were videotaped and all student work was collected to help me make sense of the video data. Prior to starting the problem-solving interviews, I asked teachers to nominate pairs of students so as to not have one dominating the conversation and therefore generating most actions. Their nominations proved to be accurate, as talk between students was quite balanced. Also, minimal code-switching occurred as students adhered to the language reported in the original activities. Instead of explicitly asking students to stay in one language, I modeled the speaking of the language of the problems as I walked with them to our problem-solving interview site, a quiet room inside the school library.

17 Data analysis

The data analysis consisted of viewing all videotaped problem-solving sessions, transcribing all student conversations, and finally coding the continuous languaging moments where students framed their problem-solving actions. Next, I explain how key ideas from the conceptual framework informed each step in the data analysis.

18 Preliminary videotape watching: In search of an analytical approach

An initial viewing of selected videotapes revealed a variety of languaging moments saturating these tapes, raising my curiosity about what students were accomplishing through languaging. For example, in some instances students were showing joint attention to an emerging model whereas in others they were working individually. At times, they were mutually exchanging ideas whereas at other times they were struggling with those ideas alone. Also, they were constructing and sharing interpretations of some contexts, and bypassing other contexts in favor of more direct number calculating. What appeared to be unifying these varied languaging processes was that not every action generated the same kind of recipiency (Sacks, 1992a, 1992b) from partner students. Therefore, all videotapes were transcribed with attention to the various actions that students accomplished in talk, including how partners received and responded to these actions. This process generated 80 transcripts from ten groups of students with each group solving eight problems (four in Spanish and four in English, see Tables 1, 2, 3, and 4).

19 Coding

I coded the natural units of communication, that is, discourse units in which participants organized and coordinated actions oriented toward solving a problem. Because the discourse units are constituted by how students organized and coordinated actions, one discourse unit may include more than one action, and certainly more than one category of action. In other words, the actions populated the discourse units. These discourse units, when analyzed for what students achieved through them, generated two categories of codes. *Reinvention actions* are meaning-making actions that contribute to mathematize an unmathematized situation. These actions, housed in the students' languaging with themselves and with one another, reflect students' productive struggle with important mathematical concepts (Hiebert & Grouws, 2007). The discourse units in which these actions occur typically begin with students grasping the significance and meaning of a context and proceed to develop the context in a personally meaningful manner. Finally, reinvention actions promote mathematical learning (and relearning) by maintaining students actively engaged in mathematizing the situation in a problem, and by keeping students in the continuous process of making meaning. *Reproduction actions*, on the other hand, are students' predispositions to reproduce procedures, often incorrectly or incompletely, that they have seen others use (most likely teachers, peers, and textbooks). The discourse units in which these actions emerge typically begin with students bypassing the grasping of the meaning of a context and proceed to a series of trial and errors. De Corte and Verschaffel (1985) refer to these actions as playing the word problem game. Reproduction actions do not promote mathematical learning because they tend to subordinate sense making to procedure-oriented learning. Tables 5 and 6 list and define reinvention actions and reproduction actions, respectively.

The criterion for generating these codes was to restrict descriptions to only observable actions. For example, the codes *contribute*, *notice*, *question*, *use*, *declare*, *defend*, and *explain* all describe observable actions. Names of codes evolved in the beginning of the coding process (first two to three videos) and remained constant for the rest of the coding process.

Although the four conceptual fields (Vergnaud, 1996) covered by the eight problems were grade appropriate, to rule out the possibility that the concepts themselves may have been contributing to how students generated reinvention actions, I calculated reinvention action ratios (Table 7) for each of the four conceptual fields across the two languages using this formula:

$$\text{Reinvention Action Ratio} = \frac{\text{Total Reinvention Actions, Familiar Context Problem}}{\text{Total Reinvention Actions, Unfamiliar Context Problem}}$$

Given the similar mathematical structure of any two problems (familiar and unfamiliar) within each conceptual field, similar ratios across the four conceptual fields and across the two languages would indicate that student productivity, as measured by reinvention action ratios, is primarily a function of the type of context represented in problems. On the other hand, different ratios would suggest that either the language of problems or the conceptual field of problems or some combination of both is having an effect on students' capacity for reinvention actions. Table 7 shows these reinvention action ratios.

Table 5 Reinvention actions

Reinvention action	Definition
Contribute knowledge for construction	Student contributes knowledge that is not part of the problem but that it is necessary to solve the problem
Notice problematic aspect	Student notices that the problem is truly a problem and not just an exercise
Question emergent solution	Student expresses concern for reasonableness of a solution
Use realistic consideration	Student judges feasibility of solution based on a realistic consideration
Offer partner realistic consideration	Same as above but in relation to partner's work

Table 6 Reproduction actions

Reproduction action	Definition
Declare inability for procedure	Student gives up on a procedure due to lack of procedural fluency
Defends wrong procedure	Student chooses a procedure that violates the relations among quantities in problem
Explain by reviewing procedure	Student is asked to explain *why* a certain procedure helps solve a problem and instead demonstrates *how* to execute the procedure
Notice superficial similarity across problems	Student notices similar numbers, or objects, not similar concepts or mathematical structures

Since ratios turned out to be similar across Spanish and English and across the four conceptual fields, they suggest that the 10 groups of bilingual students are more than twice as likely to reinvent mathematical concepts in familiar experience problems than in unfamiliar experience problems. Put differently, these bilingual students' mathematical productivity, as measured by their reinventing capacity in these conceptual fields, is explained, at least initially, by the type of context (familiar versus unfamiliar) rather than by the conceptual field or by the language. Presenting these ratios early in the analysis serves to move the analysis to explanations of how students' use of bilingualism and everyday experiences positioned them as productive problem solvers.

20 Results

With context—or rather languaging contexts of practice—as an identified contributor for how students made sense of the problems, results are organized with a focus on context. Firstly, I present a graphic summary of students' reinvention actions across familiar and unfamiliar contexts, followed by selected discourse units that illustrate how students typically organized and coordinated these actions, therefore contributing to explain how bilingualism and everyday experiences constitute cognitive resources. Findings for reproduction actions follow a similar organization and emphasis.

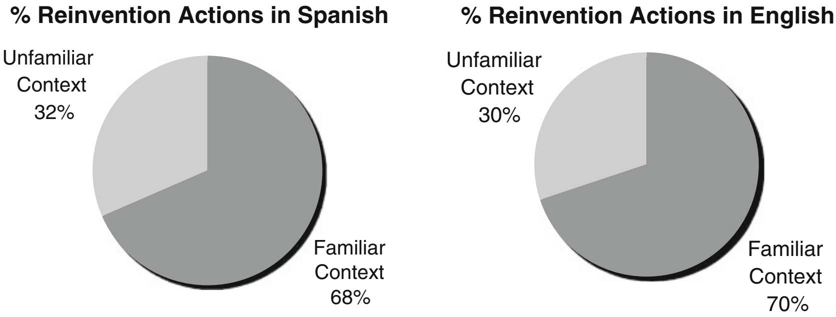
21 Bilingual students' reinvention actions

The following graphs show the percentage distribution of reinvention actions across familiar and unfamiliar contexts in each language. A representation of totals (as percents)

Table 7 Reinvention action ratios by language and conceptual field

Language	Spanish		English	
Conceptual field	Multiplicative fractions	Measurement division	Combinations	Ratios
Reinvention action ratio (familiar/unfamiliar)	2.43	2.35	2.62	2.34

rather than one by one descriptions of these actions is consistent with the focus of the analysis: To understand why contexts attract these many reinventions.



As these graphs indicate, in both Spanish and English, the problems describing familiar contexts attracted more reinvention actions (68% Spanish and 70% English) than the problems with similar mathematical demands but with contexts that were unfamiliar to students (32% Spanish and 30% English). This result is consistent with the conceptual framework's idea that once children "grasp the significance of situations (or contexts)" (Bruner, 1990, p. 71), especially if those contexts reflect what matters in their everyday lives, they are likely to do things with words, such as reinvent concepts rather than reproduce learned procedures. Quite possibly, in classrooms in which mathematical discussions are valued, these students would be considered discussion leaders who are doing an important thing with words: reinventing mathematical concepts.

It is not surprising that students reinvented mathematics concepts when solving familiar experience problems. The mathematics education research community agrees about the importance of including students' experiences in mathematics instruction (Carraher & Schliemann, 2007; Boaler, 1998; Civil, 2002a; Masingila, 2002; Saxe, 1991). What is intriguing about these results is that the actual totals making up the similar percentages of reinvention actions between familiar and unfamiliar situations differ substantially across languages. These totals are as follows: Spanish familiar, 139 (68%) and Spanish unfamiliar, 64 (32%); English familiar, 58 (70%) and English unfamiliar, 25 (30%). In other words, although students demonstrated ability to mathematize the familiar and unfamiliar contexts in similar percentages in each language (roughly 70% and 30%, respectively), the total numbers of these actions reveal a languaging process more saturated with these actions in Spanish than in English. Possible explanations for the relative abundance of reinvention actions in Spanish are found by taking a closer look at how students organized and coordinated these actions in Spanish as compared with English. What I found—I argue—is related to the conditions affecting access to bilingualism as a resource (Halliday, 1978). As Bruner (1990) argues, what students can do with language as a resource—their languaging—depends on students' perception about their position within the well-established cultural practices (Cobb, 2007) represented by school and home. For example, in the next interaction, two students are solving the Spanish problem about how many egg cartons to open for a school breakfast of 400 students. The students are constructing an unfinished but familiar context, by contributing together their everyday knowledge about the size of egg cartons. As students merge this everyday knowledge into the problem situation, it becomes a cognitive resource that serves to support their sense making. This grasp of the meaning of the situation maintains students in an interactive languaging process. It is in fact difficult to attribute the contribution to one single student as both students are co-constructing the idea in a meaning-driven languaging process.

Actions	What is achieved through actions
Sasha: Uh-huh, y yo andaba pensando también que hay doce en un cartón. [*Uh-huh, and I was also thinking that there's 12 in one carton*]	Contributes her own knowledge, not included in problem, therefore grasping the significance of the situation.
Luis: Es una docena. [*It's a dozen*]	Immediately refines contribution.
Sasha: Y si abre, si abre 12 apenas van a ser 144. [*And if she opens 12, it will only be 144*]	Starts mathematizing using knowledge from her own contribution.
Luis: 12 cartones. [*12 cartons*]	Specifies information implied in partner's strategy, therefore contributing to develop idea.

This reinvention process resembling a reasoning system over which cognition and its resources are distributed among several individuals (Collins, 1992; Greeno, 1997) was typical in the familiar experience problems in Spanish. Mercer (1995) points out, "some of the most creative thinking takes place when people are talking together" (p. 4). The high recipiency in Spanish talk generated abundant opportunities for students to coauthor more reinvention actions in this language. In contrast, in the familiar experience problems in English, students tended to generate reinvention actions more individually, relying more on the text of the problem than on partners as they generated these actions. For example, in the next interaction, two students are solving the English problem about how long it takes to read a book with 23 pages, given that it takes 20 min to read five pages. In an effort to encourage more interactive languaging, I asked one student to share with a peer her idea about maintaining the invariance of the given ratio 5:20 by varying each amount proportionally ($5\times3=15$ and $20\times3=60$). The student explained the strategy rather impersonally. Her casual look at her partner produced no response at all.

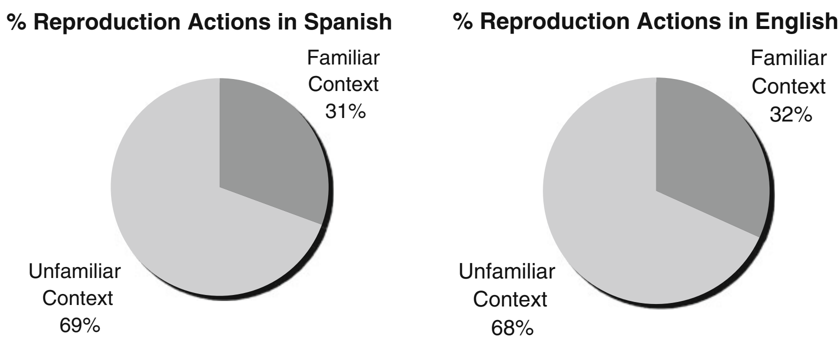
Actions	What is achieved through actions
HD: Can you explain your thinking to Sally?	Encourage student to share strategy with partner.
Cyndi: (turns to look at computer screen and points to problem on screen) Because uh, I add, I use, I know that 5×3 is 15 pages, 5×3 is 15, so I add, I multiplied 20×3, is 60 (turns to look at me first, then looks at Sally, who does not respond).	Student contributes her own knowledge of number facts (I know that 5×3 is 15) but interacts more with computer than with partner. Partner does not respond.

The low recipiency of ideas in the above discourse unit signified fewer opportunities for students to coauthor reinvention actions in English. I therefore argue that, firstly, bilingualism is indexing the everyday knowledge that students contribute to solve problems about everyday experiences—the knowledge of egg cartons in the Spanish problem and the knowledge of multiplication facts in the English problem. But bilingualism is doing more than that. As an inseparable aspect of contexts of experience, bilingualism seems to be bringing in views and predispositions that these students have developed about what they can do with language, including what they can do in mathematics. According to my classroom observations and home visits, these views and predispositions are rooted in students' experiences learning mathematics in classrooms versus learning in activities outside the classroom. As Cobb (2004) observes, "The forms of mathematical reasoning that students develop are bound up with the affordances and constraints of the classroom" (p. 334).

Students' individual approach to solving problems in English resonates with the classroom interaction patterns that I observed. Their disposition to approach Spanish tasks with "penetrating empathy" (Bruner, 1990, p. 87) is consistent with the more collaborative participation in out-of-school situations. The social structure of the familiar activities used in Spanish problems, namely attending art classes in the community center, participating in grocery shopping, preparing meals at home, and eating breakfast at the school cafeteria, are all contexts in which students make negotiations, decisions, and choices with other children and with adults in the community. That is, they constitute languaging contexts of practice. On the other hand, the experiences used in English problems, however familiar to students, seem to be less discursive in nature and may even be less social due to fewer participants involved. Playing videogames and reading in English to an adult (especially if the adult does not ask the reader comprehension questions) may be less discursive activities than helping their mothers with the grocery shopping, interacting with younger siblings at home and with peers in the school cafeteria, or thinking, designing, and making decisions with other participants about art projects at the community art classes in Spanish. The presence of more participants in some activities may stimulate higher levels of languaging and cognitive engagement than in the other more passive activities. As these explanations are speculative, research on the actual engagement of these students in familiar activities in both languages is required. The analysis of reproduction actions, presented next, complements the previous analysis by providing additional explanations to the question of how bilingualism and everyday experiences are cognitive resources.

22 Bilingual students' reproduction actions

The distribution of reproduction actions across the two contexts complements the one found in the reinvention actions: In both Spanish and English, students were more inclined to reproduce ill acquired procedures when they solved problems about unfamiliar contexts (69% Spanish and 68% English) than when they solved problems about familiar contexts (31% Spanish and 32% English). The following graphs show this distribution.



Together with the reinvention actions results, these results suggest even more strongly that representing familiar contexts in mathematical problems supports students' ability to reinvent important mathematical concepts. In other words, it takes good problems to appreciate good problem solvers fully. However, besides responding to good mathematical tasks, a closer look at reproduction actions was in order as a way to find out whether languaging practices in Spanish and English were affecting these actions. The total instances of reproduction actions are as follows: Spanish familiar, 30 (31%) and Spanish unfamiliar, 68 (69%); English familiar, 20 (32%) and English unfamiliar, 43 (68%). Therefore, although students reproduced learned procedures in the familiar and unfamiliar contexts in similar percentages in each language (roughly 30% and 70%, respectively), the total numbers of these reproductions show that students used more of these actions when languaging in Spanish than in English.

A closer inspection of how students responded to reproduction actions revealed this: Reproduction actions in Spanish attracted more responses from a partner than similar reproduction actions in English. Put differently, these actions, just like the reinvention actions, occurred in languaging contexts where problem solvers were more receptive of partners' ideas, even when these ideas were about reproducing procedures. In Spanish, students tended to respond to these unproductive ideas with more penetrating empathy (Bruner, 1990); that is, they were more active at taking risks and sharing them in Spanish than they were in English. Although large amounts of reproduction actions may be undesirable in any language, in this study these amounts occurred because partners received and responded to these actions, and in some cases this recipiency served to turn unproductive ideas into sense making experiences.

Two examples illustrate languaging in reproduction actions. In the next discourse unit, students are solving the Spanish problem about how many packages of 30 plates to buy for a party with 500 people. Dave decides to combine 30, a number in the problem, with 470 in order to get 500, the other number in the problem. Dave's actions illustrate De Corte and Verschaffel's (1985) idea of playing the Word Problem Game. His partner Amanda's persistent questioning illustrates concern for Dave's cognitive activity. Therefore, the students are using bilingualism as a cognitive resource: their interactive languaging helps Dave realize that his idea does not contribute to solve the problem.

Actions	What is achieved through actions
Dave: Yo voy a sumar 30+470 [*I'm going to add 30+470]*.	Dave is bypassing the meaning of the situation in favor of playing Word Problem Game.
Amanda: ¿Por qué? [*Why?* (frowns)]	Amanda responds by asking Dave to justify procedure.
Dave: Porque así me daría 500. [*Because that would give me 500*]	Dave continues playing the Word Problem Game.
Amanda: ¿Y de dónde sacas los 470? [*And where are you getting the 470?*] (looks at Dave)	Amanda continues pressing Dave for a justification.
Dave: ¡Ah sí, son paquetes! [*Oh yes, it's packages!* (smiles)]	Dave regains awareness of referents in the problem.

In contrast, reproduction actions in English tended to be more private and therefore were more immune from peer commentaries. This does not mean that students were not languaging in these interactions. They were indeed, but as Swain (2006) distinguishes, the transformation of one's ideas into shareable resources can be shareable with oneself. Sharing ideas with themselves was often indicated by abundant extralinguistic behavior regulating the expression of ideas. For example, in the following interaction participants are solving the English problem about how many gallons will paint 15 houses if 20 gallons can paint four houses. Students are first languaging with themselves as they generate their individual reproduction actions. In fact, Dave is struggling with his own idea (5+5+5, or 20+20+20), which seems to contain elements of reinvention. However, he is struggling by himself (note his gestural behavior) and so is his partner who seems interested in pursuing her own approach. Interestingly, what brings them together in talk is Dave's miscalculation of $5\times15=525$ ($5\times5=25$, then $5\times1=5$, then writing these products next to each other, a common misconception reflecting weak place value knowledge). This incident connected them in a competition for executing a procedure correctly, without attention to what the procedure does to help them solve the problem.

Actions	What is achieved through actions
Dave: I know the answer. It's, the answer is 16 (shakes head), 60, 60. Yeah, because in 1 house 5...	Dave's strategy: Triples 20 gallons (20×3) by thinking of 15 houses as (5×3), but ignores that 20 gallons is for 4 houses.
Amanda: (doing her own strategy) 100 gallons...	Amanda's strategy: Adding 20 gallons repeatedly (15 times). So far she has added five 20's when she says 100.
Dave: ...5 gallons, and in one, in 15 houses, then 5+ 5+5 (counts each 5 as 1 finger) is 15, and, and then I still, 15... (scratches his eyes) and then it's, no I'm not, I was, I was going to multiply by, I was going to add 20+20+20, no, yeah...	Dave continues struggling, individually, with his strategy. Amanda keeps adding more 20's on her notebook.
HD: I thought you were going to use the 5 instead.	Hearing Dave say "5 gallons" made me remind him of what he had said.
Dave: Yeah but, if I use the 5, ooh! It's multiplying (looks at me with happy face) 5×15... that's uh (looks up, squints) 525?	Dave finds unit ratio: 5 gallons per house; therefore 5×15 gallons for 15 houses. Lack of conceptual understanding of place value in multiplication interferes with his otherwise sound strategy.
Amanda (frowns): No! 15 times what?	Amanda reacts.
Dave: Oh no, it's seven hundred...	Same lack of place value understanding.
Amanda: 200.	Keeps doing her own calculations.
Dave: 700, no.	Questions reasonableness of answer.
Amanda (whispers): 200.	Talks to herself as she keeps working on her own strategy.
Dave: Uh-uh.	Continues struggling.
Amanda: You're saying 15×5. 300.	She may be thinking about her addition of 20's 15 times.
Amanda: (insists) 300. (Dave does calculations in air with finger pointing to screen). Cause 5×5 equals 20, now with the zero.	Tries to convince Dave by explaining a procedure she herself does not understand.
Dave: Seven, seven hundred five, 705.	Lack of place value understanding.
Amanda: No!	Disagrees without justification.
Dave: Yes!	Responds without justification.
Amanda: 300!	Competes for the right answer.
Dave: 705!	Responds in like manner.

This and the previous interaction differ not in the kinds of actions that dominate students' strategies but in the quality of responses from peers. In Spanish, peers responded to reproduction actions by languaging with partners in a process of sharing the transformation of ideas. In these languaging practices, students involved themselves in thinking together about a particular action, sometimes suggesting other reproduction actions in a risk taking way and sometimes, as in the example given earlier, transforming the unproductive action into a productive strategy. In contrast, in English peers responded less to these reproduction actions, or they responded in a manner that was inconsequential for a possible transformation of the unproductive action. For instance, in the above example Amanda is not interested in Dave's struggle with discovering the unit ratio, nor is Dave interested in Amanda's strategy. This low recipiency for each other's thinking has a deleterious effect in that a student's ideas "are not to his own advantage unless he can get them shared by others" (Bruner, 1990, p. 13). The contrasting ways for organizing and coordinating reinvention and reproduction actions in Spanish and English demonstrate that these bilingual students were languaging differently in these contexts of practice. In the final discussion I use these results to further argue for a view of bilingualism and everyday experiences as cognitive resources in mathematics.

23 Discussion

I introduced this study by revisiting the traditional school view of students' everyday experiences as a field of application of school-based knowledge to highlight one serious problem implicated in this view: schools' failure to recognize bilingualism and everyday experiences as cognitive resources that, as this study demonstrates, bring students together in languaging practices that promote a more social, more meaningful, and more persistent participation in problem solving.

The study results suggest various ways in which bilingualism and everyday experiences are cognitive resources that support the important cultural and cognitive process of making sense (Schoenfeld, 1998) in school mathematics. Firstly, representing everyday experiences that matter to students in school-based mathematical problems served as the common ground where students stretched, extended, and pushed their mathematical thinking (Cameron, Hersch, & Fosnot, 2005) in both English and Spanish. Students showed that their capacity to make sense of problems as measured by reinvention actions is enhanced when the problems to be solved include familiar experiences. Giving students these problems served to transform the traditional classroom tasks that they regularly receive and, perhaps more importantly, the relationship between good problems and good problem solvers. Therefore I argue that the view of culturally and linguistically diverse students as not so good at problem solving has more to do with the kinds of problems and the kind of instruction they receive in school than with their mathematical proficiency (see Brenner, 1998b; Khisty & Chval, 2002; Khisty, 1995; and Moschkovich, 1999, for discussions about best instructional practices with these students).

Secondly, representing bilingualism in school-based mathematical problems served to reveal a languaging effect that is consequential for engaging students in discussion-based problem solving. That students were more predisposed to sharing knowledge in Spanish (both their reinvention and reproduction ideas) than in English, is perhaps the most important finding and contribution of this study. Students' joint exploration of ideas in Spanish contrasted with more individual approaches in English. Adler (2001) defines exploratory talk as "informal, and a necessary part of talking to learn because learners need to feel at ease when they are exploring ideas" (p. 70). This penetrating empathy to respond to peers' ideas in Spanish functions as a powerful tool to initiate and sustain mathematical discussions, a feature that is very difficult to find in most mathematics classrooms (Sherin, Louis, & Mendez, 2000). In this sense, bilingualism is a cognitive resource that connects students and their mathematical ideas to other students and their mathematical ideas in discussions.

Third, the fact that students generated more reinventions but also more reproductions in Spanish than in English raised an opportunity to investigate languaging as consistently mediating what students do with and to others in each language. These languaging patterns suggest that in these students' bilingualism, Spanish figures as a language to discuss, argue, take risks, and learn with others, whereas English tends to be reserved for enacting more traditional schoolwork. For example, when students heard a partner generate a reinvention action in English, these students tended to remain unaffected, as if what they heard had little to do with their own work. As Civil (2002b) observes, "traditionally, students in a mathematics classroom do not listen to each other's ideas" (p. 55). In contrast, hearing a partner make a contribution in Spanish made students become involved in their partner's idea. This result has important implications for facilitating the kind of mathematical discussions recommended as essential for learning mathematics (National Council of Teachers of Mathematics, 2000; Lampert, 2001).

Findings point to languaging as an important mediator of students' actions, with languaging in English—the language of instruction—resembling the patterns observed in the students' mathematics classrooms, and languaging in Spanish—the language of the home and community—evoking more socially unrestricted patterns of interaction. In other words, languaging serves to either bring students together or separate them (Zentella, 1997) in their problem-solving efforts. In his analysis of two students constructing and solving mathematics problems at school, Barwell (2003) points out that the observed patterns in students' talk "must to some extent reflect the prevailing discursive practices of their classroom" (p. 53). In the present study, these patterns must also reflect the prevailing languaging practices of the home and community where students live. This languaging effect needs to be considered in connection with the opportunities to learn mathematics that exist in classrooms for these students. Mercer (1995) argues, "one of the opportunities school can offer learners is the chance to involve other people in their thoughts—to use conversations to develop their own thoughts" (p. 4). Unfortunately, when school excludes one of the bilingual students' two languages, it inevitably excludes the repertoire of experiences that coexist with that language. Moreover, it excludes unique ways that students develop in those languaging contexts of practices, including ways to learn mathematics, ways to relate to and discuss ideas with other mathematics learners, and ways to relate to mathematical tasks. English-only instruction in mathematics classrooms with bilingual students misses the opportunity to appreciate the full range of mathematical productivity of these students, as results in this study indicate. In fact, being English dominant, as these students are perceived in school, does not translate as being mathematically productive, especially if productivity is defined as being able to reinvent concepts that students can use to participate in problem-solving discussions.

Finally, Abreu, Bishop, and Presmeg (2002) express concern for "provid(ing) any learner with learning environments conducive to expression, sharing and negotiation of meanings" (p. 10). Based on my results, I argue that by providing bilingual students with opportunities to use their two languages to think mathematically, along with the everyday experiences that matter in their lives, students can express, share, and negotiate meanings and ideas in ways that more fully demonstrate their mathematical productivity. Bilingualism and everyday experiences are cognitive resource as demonstrated by the higher percentages of reinvention actions in both languages. The different languaging patterns in English and Spanish suggest that access to these cognitive resources is mediated by the larger languaging practices that are established in the contexts of home and school where students participate. These patterns also indicate that supporting and sustaining discussion-based problem solving with bilingual students requires recognizing everyday experiences and bilingualism as cognitive resources.

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