The Price of Ignorance:  
Distributed Topology Control in Cognitive Networks  

Abstract—In a cognitive network, autonomous and adaptive radios select their operating parameters to achieve individual and network-wide goals. The effectiveness of these adaptations depends on the amount of knowledge about the state of the network that is available to the radios. We examine the price of ignorance in topology control in a cognitive network with power- and spectral-efficiency objectives. We propose distributed algorithms that, if radios possess global knowledge, minimize both the maximum transmit power and the spectral footprint of the network. We show that while local (as opposed to global) knowledge has little effect on the maximum transmission power used by the network, it has a significant effect on the spectral performance. Furthermore, we show that due to the high cost of maintaining network knowledge for highly dynamic networks, the cost/performance tradeoff makes it advantageous for radios to operate under some degree of local knowledge, rather than global knowledge. We also propose distributed algorithms for power and frequency adaptations as radios join or leave the network, and assess how partial knowledge impacts the performance of these adaptations.

Index Terms—Cognitive networks, topology control, channel assignment, local information.

I. INTRODUCTION

The English statesman Sir Francis Bacon once opined that "knowledge is power." While there are many situations in which his proclamation is apt, there are others where having more knowledge, but not the right knowledge, provides considerably less power – even leading to poor decisions. Furthermore, acquiring knowledge is often expensive, and that cost can potentially offset its benefits. This paper investigates the effects of partial knowledge on the performance of distributed topology control in a cognitive network.

Wireless networks, in the most general sense, are unstructured. With increasing programmability, radios are able to autonomously adapt to their environment, setting transmission power and selecting their frequency of operation. Topology control attempts to harness this programmability to build structure into the wireless network. Our work asks, for a distributed topology control architecture: How much knowledge about network state must be available to each radio to enable it to make topology control decisions that are efficient in a network-wide sense? We measure the maximum transmission power and spectral footprint (total number of channels used in the network) of a fully-connected, non-interfering topology, defining efficiency by how well these metrics are minimized.

Managing spectrum utilization and radio connectivity has natural relevance to the field of cognitive radios (CRs) and dynamic spectrum access. A cognitive network (CN) is a system of CRs that collectively attempts to achieve an end-to-end goal [1]. For our approach to topology control, the CN consists of multiple CRs, operating to achieve network objectives of spectrum and power minimization. Each CR makes local decisions that together achieve the network goals.

We investigate the performance of this CN in topology creation and also in its recovery from dynamic link creation and disconnection under a continuum of network state knowledge at each CR. Specifically, we develop two core strategies: LOCAL-RS, to select interference-free transmission channels, and LOCAL-DIA, to minimize the maximum transmission power. These algorithms are the localized variants of algorithms Random Sequential (RS) and $\delta$-Improvement Algorithm (DIA) proposed in [2], [3] respectively. To recover from the impact of network dynamics when radios join and leave the network, we develop the LOCAL-DIA-ADD and LOCAL-DIA-REMOVE strategies. For all these strategies, we quantify the price of ignorance – the ratio of the average performance under local knowledge versus global knowledge.

Our contributions are threefold: First, we present a distributed CN framework for performing topology control. Second, we develop distributed algorithms under this framework for creating and maintaining efficient topologies (in terms of power and spectrum) that take into account dynamic network conditions. Third, we examine the overhead cost and power/spectral performance tradeoffs of these algorithms under a continuum of knowledge at the CR level. We show that these algorithms have provable performance guarantees (under global knowledge) and acceptable heuristic performance (under local knowledge).

These contributions provide insights into the effect of ignorance on CN and topology control performance. We show that while local knowledge has little effect on the maximum transmission power used by the network, it has a significant effect on the spectral performance. Furthermore, we show...
that due to the high cost of maintaining network knowledge for highly dynamic networks, the cost/performance tradeoff makes it advantageous for radios to operate under some degree of local knowledge, rather than global knowledge. We also show that as radios are added to the network, the network performance drifts away from the optimal, with the rate of drift depending on the amount of knowledge available to the radios.

II. BACKGROUND AND RELATED WORK

Reducing the requirements of a network of radios has typically been examined in conjunction with interference avoidance. Previous topological work has assumed full network knowledge and can be broken into two assumptions: the network exercises power control [4]–[7] or the network exercises channel control [8], [9].

To achieve power efficiency, transmit power control based topology control schemes have been developed that generally utilize a single shared channel and assume that a Medium Access Control (MAC) protocol provides temporal separation of interfering transmissions. Utility-based models of such adaptive power control techniques can be found in [10], [11]. Several cooperation-based topology control algorithms have been proposed to create power-efficient topologies; for a recent survey, see [12]. Most of these works cope with network dynamics indirectly, either by developing local, fault-tolerant algorithms such as [13], [14] or by developing probabilistic models that are often argued as being robust to mobility, e.g. [15].

Our idea of k-hop neighborhood for localizing knowledge is closely related to the concept of zone radius in the Zone Routing Protocol (ZRP) [16]. The authors of [17] examine the optimal zone radius for routing to improve network performance. Our work, in contrast, determines the optimal k-hop neighborhood in the context of topology control and not for routing, by examining the performance of various k-hop knowledge power-controlled topology configurations, to improve energy efficiency. Given the costs of signaling overhead associated with achieving cooperation in wireless networks, balancing the costs and benefits of information exchange has been identified as an important design decision task for CNs [18], [19].

Despite these efforts, a thorough evaluation of the impact of partial information on topology control is still missing from the literature. Our work investigates the effect of partial information by extending [20] by considering radios that can regulate both power and channel selections not just in static but also in dynamic conditions i.e., when radios join and depart the network.

III. COGNITIVE NETWORK MODEL

The distinguishing aspect of a cognitive network is the end-to-end scope of the cognitive process [21]. End-to-end, in this context, denotes network objectives that transcend those of the individual network elements. In particular, our CN approach to topology control attempts to optimize two network-wide goals when creating a topology: minimize the maximum transmission power in the network and minimize the total number of channels utilized by the network.

To achieve the dual network objectives of power and spectrum optimization, the CRs perform two sequential, distributed feedback loop algorithms. The first algorithm is for power control, where CRs attempt to minimize their transmit power level while maintaining network connectivity. The output of this algorithm is a power-efficient topology, which is fed into the channel control algorithm. Here, CRs select orthogonal channels to avoid interference at neighboring receivers. Both algorithms can be envisioned as being part of a cognitive cycle: the CRs select (and possibly revise) their optimum settings based on the perceived topology state; these revised decisions induce a change in topology configuration, either in the connectivity or in the channel occupancy profile; the modified topology affects the performance of individual CRs, which in turn update their power or channel settings, and the cycle starts all over again.

We examine the performance of the CN under a continuum of knowledge at the CR level. We define local knowledge1 to be an understanding by a CR of the state of its k-hop neighborhood (the set of radios reachable within k hops; for a more formal discussion, see Section V). When the k-hop neighborhood incorporates the entire network for every radio, we say that the network operates under global knowledge.

As we are interested in the effect of partial knowledge on the performance of the cognitive network, we utilize the metric expected price of ignorance first introduced in [18]. The price of ignorance2 (ρ) is the relative change in the performance achieved by the network under partial knowledge as compared to that of the network under global knowledge. Under fixed initial network conditions, ρ is defined as:

$$\rho = \frac{C_{\text{knowledge}} - C_{\text{ignorance}}}{C_{\text{knowledge}}}$$  \hspace{1cm} (1)

The expected price of ignorance E(ρ) can be determined by averaging over all possible initial network topology configurations. Positive values indicate that the cognitive network has worse performance under ignorance than with global knowledge; negative values indicate better performance under partial knowledge. In Section VII, we estimate E(ρ) for static and dynamic network cases by simulating a diverse set of initial conditions.

IV. SYSTEM MODEL

We model the network topology as a directed graph $G = (N, E)$, where $N$ is the set of radios and $E$ is a set of unidirectional connections. Equation (2) defines the set of connections: a connection $e_{ij}$ exists if the transmission power ($p_i$) is greater than the thermal noise ($\sigma$) and Signal to Noise Ratio (SNR) requirement ($\gamma$) at the receiving radio, given the gain (loss) factor between the two (contained in the matrix $[g_{i,j}]$). We only consider thermal noise and not interference because of the orthogonal nature of the to-be

1The terms local knowledge and partial knowledge are used interchangeably in this paper.

2The word ignorance used throughout this paper refers to the relative ignorance of local/partial knowledge with respect to global knowledge.
selected channels.

\[ E = \left\{ e_{ij} \mid p_i \geq \frac{\sigma g_{ij}}{g_{ij}} \right\} \quad (2) \]

The value of \( \frac{\sigma g_{ij}}{g_{ij}} \) is referred to as \( \omega(t_{ij}) \), which is the transmission power required to form a connection from radio \( i \) to radio \( j \). We assume the matrix \( [g_{ij}] \) to be symmetric, so that \( \omega(t_{ij}) = \omega(t_{ji}) \).

We assume that topological connectivity comes from bi-directed connections. The set of bi-directed connections consists of elements of \( E \) that have their reverse also in \( E \). \( G \) is said to be connected if and only if there exists a bi-directed path—a collection of contiguous bi-directional links—between every radio pair \( i, j \in N \).

In addition to the communication graph \( G \), we also define an interference graph \( U = (N, E') \) that specifies the set of transmissions that can potentially interfere with one another if those transmissions occur simultaneously on the same channel. Transmissions from one radio may interfere with transmissions from every other radio in the network (as in a physical network model), or with only a subset of those transmissions (as in a protocol model). To model possible conflicts, \( G \) is transformed to an undirected conflict graph \( U \). This conflict graph is created by placing an undirected edge between each pair of possibly conflicting transmissions in the communication graph. For our purpose, we leave the exact transform from \( G \) to \( U \) unspecified, except that we require \( U \) to be undirected. In some sense, \( U \) may be considered exogenous, and our results hold for any undirected \( U \). Note also that while it is common to consider interference terms only from the strongest interferers as in the protocol model [22], our model is general enough to accommodate any undirected conflict graph.

We assume that radios can access multiple channels and can receive on all channels simultaneously if necessary; each radio can only transmit on a single channel at a time, although it can use any of the channels. The sensed spectrum is divided into orthogonal channels. For any meaningful communication to take place over a link, the transmitter interface of the link makes the channel selection and then informs the receiver interface. It is possible for the forward and reverse links (\( e_{ij} \) and \( e_{ji} \)) to be on different channels, thus allowing for full-duplex operation.

We model dynamic changes in the network as the addition or removal of a radio from the network. Under discrete updates, mobility will appear to the network as radios appearing and disappearing. This also represents dynamic changes that do not involve radio mobility. For example, in Wireless Local Area Networks (WLANs) radios often drop in or out as users turn their machines on or off. Wireless sensor networks also consist of radios that wake up or go to sleep periodically, adding or removing themselves from the network.

V. ANALYSIS OF POWER CONTROL

A. Topology Creation

In a CN, radios are expected to have only a partial and incomplete picture of the network. Because radios must contend with limited information during decision-making, we develop a distributed, localized topology control algorithm, LOCAL-DIA.

To quantify knowledge, we define a \( k \)-hop neighborhood. The \( k \)-hop neighborhood of radio \( i \) is defined as the set of radios that are reachable within \( k \) hops via a bi-directional path. The members of each successive \( k \)-hop neighborhood of \( i \) can be described recursively:

\[ \mathcal{N}_i^k = \left\{ \{i\} \cup \left\{ j \mid e_{ji}, e_{ij} \in E, l \in \mathcal{N}_i^{k-1} \right\} \right\}_{k=1,2,\ldots} \quad (3) \]

We assess the effectiveness of the cognitive processes when radios have \( k \)-hop knowledge, which means radios have knowledge of the network state in their \( k \)-hop neighborhood and no more. More sophisticated models of knowledge can be developed, such as models that incorporate transitive (i.e. sharing information beyond a \( k \)-hop neighborhood) or passive (i.e. utilizing information overheard in the environment) learning components. Analyzing these models is beyond the scope of this paper. Our model of \( k \)-hop knowledge presents a first order approximation. In some sense, our model provides a worst case bound on system performance, as more advanced learning models will further reduce the cost of acquiring \( k \)-hop knowledge.

These assumptions allow \( k \)-hop knowledge to be an experimental parameter that can be tuned to study the role of partial knowledge in network design. Note that if \( k \geq \text{dia}(G) \) (where \( \text{dia}(G) \) is the diameter of the network) then the network is said to be operating under global knowledge since every radio’s \( k \)-hop neighborhood includes the full network. For all \( k < \text{dia}(G) \), there is some degree of partial knowledge, in the sense that some or all radios may be unaware of a portion of the network state. Also note that the fraction of the total network that the CRs are aware of is a function of their \( k \)-hop knowledge and the topological connectivity; we explicitly examine this relationship in Section VII.

For power control adaptations, the objective function for each radio can be expressed as:

\[ u_i^{rc} (p) = M_i f_i (p) - p_i \quad (4) \]

where \( f_i \) is the number of the radios in the \( k \)-hop neighborhood of radio \( i \), and \( M_i \geq p_i^{\text{max}} \) is a scalar multiplier. The objective function captures the fact that radios prefer to maintain their \( k \)-hop neighborhood over a power decrease, but get greater reward for maintaining their neighborhood at lower power levels. Because the traffic requirements and selection of destinations are typically unavailable at the time of topology formation, topology control seeks to maintain a connected topology, without making any assumptions about traffic conditions in the network.

Given the objective function (4), radios adapt their power levels according to a LOCAL-DIA algorithm—a generalized version of the DIA algorithm found in [3]. LOCAL-DIA operates on a finite set of transmission powers, elements of which are denoted by \( p_i^{(m)} \), for radio \( i \) (\( m \) is the set element index). In LOCAL-DIA, described in Algorithm 1, each CR attempts to set its transmit power level one step lower than its previous level. The search space for each radio consists of set \( A_i^{rc} \), with elements \( p_i^{\text{max}} = p_i^{(0)} > p_i^{(1)} > \ldots > p_i^{(M)} \). The set is constructed such that at most one connection would
be dropped if all radios \( j \) in the \( k \)-hop neighborhood of \( i \) adapt their powers from \( p_j^{(m)} \) to \( p_j^{(m+1)} \) (we call the difference between these powers the step size). A small enough constant step size can be used to ensure that at most one connection will be dropped per round in each \( k \)-hop neighborhood; given the compactness of the search space, such a step size is guaranteed to exist. Note that for \( k < \text{dia}(G) \), not every radio in \( i \)'s \( k \)-hop neighborhood will have the same search space, due to differences in their \( k \)-hop neighborhood sets. It is also worth noting that larger \( k \) will, on average, require smaller step sizes and, hence, result in the algorithm taking more iterations to converge. This will further exacerbate the problem of maintaining high knowledge levels, as large \( k \) values already will require more time to disseminate information across \( k \)-hop neighborhoods.

Each radio is chosen to update its action according to a random permutation, which may be realized by allowing each radio to randomly choose a backoff period within a fixed window. When the radio’s backoff period ends, it selects an action from the set \( \tilde{A}_i^\text{RC} \). The backoff periods induce an ordering that is represented by \( \pi \) in Algorithm 1.

**Lemma 1:** In LOCAL-DIA the optimal strategy for every radio is to preserve its \( k \)-hop neighborhood.

**Proof:** We prove this by contradiction. Suppose radio \( i \) reduces its power level from \( p_i \) to \( q_i \) to reduce its \( k \)-hop neighborhood (from, say, \( N_i^k \) to \( N_i^k \)) and increases its utility. This implies that \( u_i(q_i, p_{j\pi(i)}) - u_i(q_i, p_{j\pi(i)}) = M_i(N_i^k) - p_i > M_i(N_i^k) - p_i \), where \( |N_i^k| < |N_i^k| \). This implies, \( M_i'(N_i^k - N_i^k) < p_i - q_i \), an impossible inequality, because the term on the left hand side is larger than \( p_i^\text{max} \) and the term on the right hand side is smaller than \( p_i^\text{max} \).

Note that while each radio maintains its \( k \)-hop neighborhood, according to Lemma 1, its decision may reduce another radio’s \( k \)-hop neighbor set; for an illustration, see Figure 1. Using the above lemma, we can show that LOCAL-DIA converges.

**Theorem 1:** If every radio makes power selections according to LOCAL-DIA, then within a finite number of adaptations, the CRs will reach a stable state in which they cease to make further adaptations.

**Proof:** There are only a finite number of power levels that can be stepped through (corresponding to the power thresholds \( \omega(\tau_j) \)). From Lemma 1 we see that radios will reduce their power only if it preserves their \( k \)-hop neighborhood. Note, however, that radios may reduce the \( k \)-hop neighborhood of other radios (see Figure 1). In this case, radios with a reduced neighborhood are no longer aware of radios that are now outside their new \( k \)-hop neighborhood (due to our model of local knowledge) and thus will not increase their power. Furthermore, increasing power under any other conditions will not add any \( k \)-hop neighbors, and will not increase a radio’s objective function. Since transmission powers will only be reduced and there are a finite number of power levels, the algorithm must eventually stabilize.

**Theorem 2:** If initial topology \( G \) is connected, then LOCAL-DIA converges to a network that is also connected for all \( k > 0 \).

**Proof:** A connected network can be disconnected in two ways:
1) if a radio (say \( i \)) disconnects itself from another radio, while executing LOCAL-DIA or,
2) if radio \( i \) disconnects two previously connected radios, say \( j \) and \( m \), in the process of reducing its power during the course of LOCAL-DIA.

We know that case 1 violates Lemma 1. The latter case is not possible unless \( j \) and \( m \) are connected to each other through \( i \). If \( j \) and \( m \) are \( k \)-hop neighbors of \( i \), \( i \) will not lose connection with either \( j \) or \( m \) by virtue of Lemma 1. If \( j \) and \( m \) are beyond the \( k \)-hop neighborhood of \( i \), the only way for \( i \) to lose connectivity with either of them is to disconnect from an existing member of its \( k \)-hop neighborhood, which is disallowed by Lemma 1. It follows that the topologies remain connected in equilibrium.

Under global knowledge, the search space \( (A_i^\text{RC}) \) of every radio is the same. Furthermore, this same global knowledge can be used to synchronize the search through \( A_i^\text{RC} \), so that every radio reduces their power by one step at a time (if it increases their objective function). When this done, LOCAL-DIA has the property of arriving at an optimal power efficient topology, minimizing the maximum transmission power in the topology. This special topology will be referred to as \( G_{\text{dia}} \). On the other hand, under partial knowledge, radios do not have knowledge of \( \omega(\tau_j) \) for the entire network and therefore cannot select the same \( A_i^\text{RC} \) or synchronize their searches through it. For this reason, LOCAL-DIA, under partial knowledge, does not necessarily converge to a topology that minimizes the maximum transmission power in the network.

**Theorem 3:** Under global knowledge LOCAL-DIA converges to topology \( G_{\text{dia}} \) that preserves network connectivity and maximizes the maximum power of any given radio.

From Algorithm 1, observe that in each iteration, the longest edge that ensures connectivity. In some sense, this topology is a superset of the Minimum Spanning Tree (MST) connections, containing the bi-directional links that make up the MST and the additional extraneous links induced by the wireless broadcast property of the medium. The MST minimizes the maximum edge weight of a graph, and therefore, the maximum radio power is also minimized. For a formal proof, see [3].

Under local knowledge, the sub-optimality of the resultant LOCAL-DIA topology is exacerbated as the amount of knowledge is decreased. The resultant topologies are over-connected under local knowledge, given that radios will not remove any connection that decreases the size of their \( k \)-hop
neighborhood. Furthermore, there is a “first mover advantage” inherent in \textsc{Local-Dia}. The first radio to act has the most actions available to it; subsequent radios have their action spaces reduced by previous radios’ action choices.

B. Topology Recovery

1) Adding Radios: When adding radios to the topology, the new radio needs to connect into the existing topology and the existing topology needs to add and remove bi-directional connections such that, under global knowledge, the MST properties of $G_{dia}$ are recovered.

This strategy for recovering the MST properties of $G_{dia}$ after the addition of a radio can be summarized as follows: The new radio forms a least-power connection with some existing radio in the topology. Under global knowledge, each radio resets its transmit power to the current maximum power in the network. Under local knowledge, each radio with $k$-hop knowledge of the new connection resets its power to the current maximum power in its $k$-neighborhood. At this point, the radios that reset their powers reconstruct the topology using \textsc{Local-Dia}.

$\textsc{Local-Dia-Add}$, described in Algorithm 2 (from a radio’s perspective), contains two operations that can be triggered. First, “Hello” is triggered when a new radio, $x$, joins and attempts to connect into the topology. The radio closest to $x$ (this is denoted by radio $i$ in Line 2) responds by forming a bi-directional connection with it, then notifies its $k$-hop neighbors to perform a “Local Restart.” This second operation instructs each $k$-hop neighbor to increase power to the maximum transmission power in its $k$-neighborhood, and then run \textsc{Local-Dia}.

We now show that, starting from a $G_{dia}$ topology, under global knowledge, \textsc{Local-Dia-Add} restores the $G_{dia}$ topology. We begin with a basic lemma that identifies the bi-directional connections that make up a MST:

Lemma 2: Partition radio set $N$ into two subsets $M$ and $O$. Let $F$ be the set of all possible bi-directional connections between $M$ and $O$. If $e = \arg \min_{f \in F} \omega(f)$ then $e$ is part of the MST.

We now examine the impact of adding a radio on the maximum transmission power of the recovered topology, using Lemma 2. Adding a new radio gives rise to one of two scenarios: The added radio falls inside the region covered by the transmission ranges of the existing radios, or the added radio falls outside this region.

Lemma 3: If radio $i$ requires the least transmission power to reach radio $x$ and $\omega(x) \leq \max_{e \in G_{dia}} \omega(e)$, then the maximum transmission power in the new topology $G'_{dia}$ is less than or equal to that in the initial $G_{dia}$ topology:

$$\max_{e \in G'_{dia}} \omega(e) \leq \max_{e \in G_{dia}} \omega(e)$$

Proof: From Lemma 2, in a MST, radio $x$ will be connected via bi-directional connection $\overline{xt}$, which requires less power than all other bi-directional connections from $x$. From our assumptions, $\omega(\overline{xt}) \leq \max_{e \in G_{mt}} \omega(e)$, so this has not increased the transmission power in $G_{dia}$.

We now examine the remaining bi-directional connections in $G'_{dia}$. Let $E$ be the set of all possible connections in $G$ and let $E'$ be the set of all possible connections in $G'$ after adding radio $x$. Note that $E' \supseteq E$, meaning we have not removed any possible connections, only added possible connections. Returning to the notation of Lemma 2, for all cutting subsets of $N \cup \{x\}$, the set of possible connections $F' \supset F$. Thus

$$\min_{f \in F'} \omega(f) \leq \min_{f \in F} \omega(f)$$

This
and we have proven that addition of $x$ will not increase the maximum power in the network.

Next we examine the case when radio $x$ falls outside the region covered by the maximum transmission range of the radios in the existing topology.

**Lemma 4:** If radio $i$ requires the least transmission power radio to reach radio $x$ and $\omega(\text{in}_i) > \max_{e \in G_{dia}} \omega(e)$, then the minimum power connections in the new topology $G'_{dia}$ is the connection between $x$ and $i$:

$$\omega(\text{in}_i) = \max_{e \in G_{dia}} \omega(e)$$

**Proof:** From the assumption that $\omega(\text{in}_i) > \max_{e \in G_{dia}} \omega(e)$ and Lemma 2 we see that all cutting subsets of $N \cup \{x\}$ that include $x$ (except that with only $x$) will not select a connection originating from $x$, since it is not the minimum power connection between the cut. Thus adding radio $x$ will not change the connections in the topology, with the exception of the min-power connection between $x$ and $i$. \hfill \blacksquare

Following lemmas 3 and 4, LOCAL-DIA-ADD ensures that, in the event a new radio joins the network, optimal topology $G_{dia}$ can still be re-constructed with enough network awareness (i.e. no edge in $G_{dia}$ is ever removed by LOCAL-DIA-ADD). The following theorem is an immediate consequence of these two lemmas, and it establishes the correctness of LOCAL-DIA-ADD under global knowledge.

**Theorem 4:** Under global knowledge, LOCAL-DIA-ADD converges to the $G'_{dia}$ topology.

**Proof:** Lemmas 3 and 4 prove that regardless of the power required to connect radio $x$, LOCAL-DIA-ADD will set all radios to the maximum required connection power, $\max_{e \in G_{dia}} \omega(e')$. Theorem 3 proves that this connected topology will converge to the $G'_{dia}$ topology. \hfill \blacksquare

2) Removing Radios: The other network dynamic we consider recovering a topology from is the case where radios leave the network. The removal of a radio from the network can potentially split the existing connected topology into multiple partitions.

The remove case does not have the same foundational property that the add case does: the guarantee that the network is fully connected with the exception of the added radio. The removal of a radio may create, in the worst case, as many partitions as the degree of the removed radio. Without global knowledge of the required power for all possible connections in the network and the members of all partitions, it is not possible to guarantee that all radios know when the topology is fully connected and what optimal connections to use. For an illustration of the remove scenario, see Figure 2.

With these limitations in mind, we develop a localized strategy, called LOCAL-DIA-REMOVE that guarantees a re-connected network by first restoring all prior $k$-hop neighbors from the partitioned topology (by resetting power levels appropriately). It then proceeds with LOCAL-DIA to improve this reconstructed topology recovered from the removal of radios. This is described in Algorithm 3 from a radio’s perspective, using $N_i$ to denote the original $k$-neighborhood (before the radio’s removal) and $N'_i$ to denote the current $k$-neighborhood (after the radio’s removal).

### Algorithm 3 LOCAL-DIA-REMOVE($x$, $m$) \rightarrow $\hat{p}_i$

1: $K = N_i \setminus (N_{ki} \cup \{x\})$
2: while $K \not\subseteq N_{ki}$ do
3: \hspace{1em} $m = m - 1$
4: \hspace{1em} $p_i = p'_m \in A_{pc}$
5: end while
6: $\hat{p}_i = \text{LOCAL-DIA}(p)$

In LOCAL-DIA-REMOVE, each radio sequentially increases its transmission power by one level (as specified by line 4) in $A_{pc}$ until its $k$-hop neighborhood is recovered (as specified by the while loop in line 2). These power increases will create uni-directional connections, eventually to be complemented with their reverse, creating bi-directional connections that add at least one more radio into the $k$-hop neighborhood. By following a random ordering, eventually all radios will have recovered their original $k$-hop neighborhood. This sequential stepwise strategy is expected to reconnect the topology with fewer connections.

Note that even when LOCAL-DIA-REMOVE has global knowledge, it is not guaranteed to recover the $G_{dia}$ topology. This is because, unlike LOCAL-DIA-ADD, which resets the entire network to a common power level under global knowledge, LOCAL-DIA-REMOVE resets the network to differing power levels and because of the order of updates, it may not re-connect the least-power connections between partitions. This means under global knowledge LOCAL-DIA will not operate in a synchronized manner, a requirement to construct $G_{dia}$. Having said this, we show in Section VII, that LOCAL-DIA-REMOVE probabilistically approaches the optimal performance under global knowledge.

### VI. ANALYSIS OF CHANNEL CONTROL

The second phase of our distributed algorithm is the channel control phase. Here the output of the previous power control algorithm is optimized with respect to its spectral footprint, i.e., the number of channels utilized by such a power-optimized topology. Unlike in the power control algorithm, the CRs employ a simple greedy strategy during the course of the channel assignment that can be easily implemented in real systems. This strategy is used for channel assignment for topology creation and recovery.

To assign non-interfering channels, a distributed channel assignment algorithm can be constructed as follows: the radio set $N$ is the same as before; each radio picks a channel $c_i \in A_{cc} = \{0, 1, \ldots, n\}$ from a palette of $|N|$ channels\(^3\) (all radios share a common palette); the radio’s objective function is given by:

$$u_{ci}^e(e) = \begin{cases} 1 & \text{if } c_i \notin \{c_j \mid e_{ij} \in U\}; \\ 0 & \text{otherwise}. \end{cases}$$

The objective in channel assignment is the same as that of graph coloring, thus there are many different heuristic strategies in the literature. In order to minimize the total

\(^3\)It is trivial to prove that any channel assignment scheme can determine a non-conflicting assignment with a palette size equal to the number of vertices in the graph.
number of colors, a possible greedy strategy is for each randomly ordered radio to choose the lowest non-conflicting channel index; we call this strategy LOCAL-RS, as it is a localized version of the Random Sequential coloring algorithm described in [2]. A formal description of the operation of LOCAL-RS is provided in Algorithm 4 (from the network’s perspective).

Algorithm 4 LOCAL-RS(U, π) → ě
1: ěi = 0 ∀i
2: for i in π do
3:     ěi = min {A^CC \ {e_{ij} | e_{ij} ∈ U}}
4:     ci = ěi
5: end for

As in LOCAL-DIA, radios are chosen from a random permutation π. When the radio’s backoff period ends, it selects the lowest channel that does not conflict with its neighbors to transmit on. This repeats until a non-conflicting channel assignment is reached and the algorithm stabilizes.

Theorem 5: When radios follow LOCAL-RS, the adaptation process is guaranteed to stabilize to a conflict-free channel assignment after a finite number of adaptations.

Proof: The proof of convergence follows from a simple observation: Whenever a radio improves its objective function by selecting a non-interfering channel, it never decreases the objective function of any other radio in the network. This is due to the symmetry in interference relationships (given that U is undirected). In other words, say i selects a new channel distinct from those of all its potential interferers, and switches from channel ci to ěi to obtain a utility $\hat{u}^CC_i = 1$. Then, the utility of every interferer j (of i) on ci would increase (if j’s color matched only that of i to begin with) or would remain the same (if j’s color matched that of i and/or that of some other neighbor of j). Thus, the utility of every radio is non-decreasing in every iteration of the algorithm. Because the channel search space $A^CC$ is finite, the LOCAL-RS adaptation process must converge.

To show that the convergent state is conflict-free, notice that if a radio’s channel selection conflicts with that of another radio’s, it can improve its objective function by choosing a channel different from its neighbors (because, by assumption, there are enough channels in the palette).

One advantage of LOCAL-RS is that there exist no hard-to-color topologies\(^4\). A disadvantage of LOCAL-RS is that it does not assign any priority to the radios, and so misses out on simple optimizations such as allowing highly connected radios to select channels earlier.

LOCAL-RS, being a strictly local algorithm, does not have the same knowledge requirements as LOCAL-DIA, with the conflict neighborhood consisting of channel selection information from itself and 1-hop neighbors. Thus, there is no concept of partial knowledge. Having less than 1-hop knowledge will prevent any strategy from avoiding interference, while having more than 1-hop knowledge provides no advantage to LOCAL-RS.

To cope with dynamic changes in the network due to the addition and removal of radios, LOCAL-RS will continue running after the dynamic event. The topology of the network will have changed, changing the conflict neighborhood of some radios. Those radios that share channel assignments with their new conflict neighbors will update their channel selections according to the same rules and order of selection as used under LOCAL-RS prior to the dynamic event.

VII. SIMULATION RESULTS

We developed a simulation consisting of radios placed according to a uniform random distribution within a square 2-D map (with a density of |N| radios/unit\(^2\)). When examining the performance of LOCAL-DIA and LOCAL-RS, we set the initial power levels such that the $G_{max}$ was 1-connected with 90% probability (see [23] for how to calculate initial power assignments). In our simulation setup, conflicting pairs are chosen according to a variant of the distance-2 interference model proposed in [24]: conflicting radios include those that share a bi-directed connection or those that share an intermediary radio that is within the transmission range of both radios, and has a bi-directed connection with at least one of radios. Specifically, a topology G is transformed into a conflict graph U by placing undirected edges between all conflicting one-hop and two-hop neighbors, where two-hop neighbors share a common intermediary radio.

\(^4\)A hard-to-color topology means that no implementation of the algorithm can exactly color the topology.
A. Topology Creation

We first examine the performance of LOCAL-DIA by varying the $k$-hop knowledge available to radios and evaluating the efficiency of topologies created for different values of $k$, in terms of the total power consumed (and averaged over 1000 randomly generated topologies). Figure 3 shows the price of ignorance in a 50-radio network. Clearly, at low $k$ values, the resulting topologies created by LOCAL-DIA are highly sub-optimal as compared to those created under global knowledge. With low knowledge, radios do not have sufficient information of what the network-optimal connections are. With increasing knowledge, however, radios are able to preserve more optimal connections, thus improving the network performance. While the performance increases monotonically, we observe that beyond $k = 7$, this increase is insignificant. (Similar results were observed for the maximum power performance metric as well.) As an engineering tradeoff, choosing $k$ close to this value is perhaps sufficient, when one considers the cost of increasing knowledge. The cost/performance tradeoff of LOCAL-DIA is examined in subsection VII-C.

To assess the spectral efficiency, we evaluated LOCAL-DIA under global knowledge and six other well-known interference-reducing algorithms on the same sets of scenarios ranging from 5 to 100 radios. The results of this comparison are found in Figure 4, which shows the average minimum number of channels required by each algorithm. We observe that, compared to other interference-reducing algorithms, LOCAL-DIA produces topologies with low conflicts under global knowledge. To determine the performance of LOCAL-RS, we compare the average additional channels required over the minimum number of channels in Figure 5. The exact minimum number of channels required was determined by running the exact coloring algorithm [25] on the conflict graph used by LOCAL-RS. We observe that LOCAL-RS, on average, requires less than 12% additional spectrum over the optimum. This indicates that although more complex coloring strategies are possible for channel assignment, the cost of their complexity may make them poor candidates in comparison to a localized strategy such as LOCAL-RS.

B. Topology Recovery

In order to accurately evaluate the performance of the recovery algorithms LOCAL-DIA-REMOVE and LOCAL-DIA-ADD, and have an unbiased baseline from which to analyze the effect of local knowledge on these algorithms, we standardize on the initial recovery conditions. This is done by first creating a $G_{dia}$ topology, then dynamically changing the network by adding or removing a radio followed by recovering the topology under various levels of ignorance ($k$-hop knowledge). By starting all recoveries from an initial $G_{dia}$ topology, we ensure that a common performance baseline underlies all comparisons. If partial information were used for topology creation as well as maintenance, the price of ignorance would be magnified, as both the topology creation and recovery would be subject to its effects. By separating these two effects, we evaluate the price of ignorance in the presence of network dynamics alone.

We specifically investigate the expected price of ignorance when recovering a topology from a dynamic event, knowing
that under global knowledge the network can use strategies that successfully minimize both objectives. In Figure 6, the price of ignorance is measured for both channel and power objectives for a 50-radio network. In this figure, a price of ignorance of (for instance) 1.5 indicates that the network objective measured is 150% worse under that degree of ignorance than under global knowledge.

Figure 6 shows that the price of ignorance is uniformly low for the maximum power objective. This is not unexpected, as most of the time little in the topology needs to change to incorporate a new radio. Recall from Lemma 3 that if the least power connection for a new radio is less than the current maximum power in the network, the maximum power will not increase in the new topology. Furthermore, from Lemma 4, if this least power is greater than the current maximum, this connection will be the new maximum. LOCAL-DIA does not increase any radio’s transmission power beyond these limits and only in very special cases will the addition of a radio reduce the maximum transmission power. Particularly as the number of radios in the network increases, the probability that adding a single radio will reduce the maximum transmission power decreases.

While the objective of minimizing the maximum power is relatively unaffected by ignorance, the spectral efficiency objective does not fare so well. The average channel usage in Figure 6 was calculated using the exact coloring algorithm using the continuation strategy. The large price of ignorance for small $k$ is correlated to the fact that the total power under partial knowledge is much greater than that under global knowledge, creating more connections. In the expected sense, increasing the connectivity increases the maximum degree and clique size of the conflict graph. This in turn increases the number of channels required for conflict-free operation.

Figure 7 shows the price of ignorance under radio removal for both transmission power and spectral efficiency. Unlike the results in Figure 6, the removal case shows that increased knowledge has a negative effect on the network objectives, performing worse under knowledge than ignorance. This surprising and counter-intuitive result can be explained by the oft heard expression “what you don’t know can’t hurt you.” Under ignorance (such as 3-hop knowledge), most radios are not aware that a partitioning has occurred, and hence do not react in any way to the radio removal, causing little change to the original topology. Under larger $k$-hop knowledge, more radios have more $k$-hop neighbors that they are attempting to reconnect with, thus skewing the $G_{dia}$ topology. As discussed earlier, LOCAL-DIA combined with LOCAL-DIA-REMOVE may result in many additional connections over $G_{dia}$. Due to the sub-optimal effects of LOCAL-DIA-REMOVE, we observe an increasing trend in the price of ignorance until $k = 11$-hop knowledge. Beyond this point, the effect of LOCAL-DIA having more knowledge dominates. This allows LOCAL-DIA to offset the effects of the high power connections retained by LOCAL-DIA-REMOVE. Nevertheless, the sub-optimal effects are still not fully negated. Thus, in most topologies, having more knowledge degrades the overall performance and a little knowledge is in fact sufficient to select low-power connections for use in re-connecting the network.

### C. Knowledge and Performance

The performance benefits of increasing the amount of knowledge available to the radios in the network are clear. However, this knowledge comes at a cost, requiring more transactions as the amount of knowledge increases. To understand the impact of acquiring knowledge, we need a metric that allows a comparison between the network performance (with respect to the power and spectrum objectives) and the cost of acquiring knowledge.

As a proxy measurement for these objectives, we measure the total packet energy. The total packet energy is calculated as the amount of energy required to transmit, via unicast, a data packet from every radio to every other radio, using the least-power route between every pair of radios in the network. The power used by the radio at each hop along the route in the topology is summed and this value is totaled for every radio.
radio pair. To convert from power to energy, we multiply this power total by a constant equal to the packet transmission time, which assumes that all packets are of equal transmission length.

To measure the cost of maintaining the network topology, we measure the total packet energy required for knowledge by calculating the amount of energy needed to transmit an update message from every radio to each of its $k$-hop neighbors. As with the data measurement, this is calculated by determining the least power route from each radio to each of its $k$-hop neighbors. The power used by each radio to reach, via unicast, every $k$-hop neighbor is summed. We use the same time constant as with the data packets to convert from power to energy.

Figure 8 shows the total packet energy required for just data and also for the sum of the data and control packets (those used to disseminate knowledge), if data packets are sent with the same frequency as control updates. This shows that increasing knowledge decreases the total packet energy required for data. It also briefly decreases the total packet energy required for control packets used to disseminate $k$-hop information, but then this begins to climb. There is a "sweet spot" for energy around 5-hop knowledge, in which the sum total of packet energy is lower than at global knowledge. This is the point at which the total energy cost of knowledge is minimized. To make better sense of the amount of partial knowledge radios have, we calculated the average fraction of network a radio is aware of. For 5-hop knowledge, radios have, on average, awareness of 70% of all network radios.

This example is for a low ratio of data to updates. Assuming the amount of data stays constant, as a network becomes more mobile, the number of updates required to maintain $k$-hop knowledge increases proportionally to the data. Figure 9 illustrates the percent additional energy required to have complete knowledge over partial (5-hop) knowledge for different ratios of data to updates. As expected, when the network is relatively stable, and the ratio of data to updates is high, having global knowledge gives the best performance. When the network is dynamic, and the ratio of data to updates is low, having partial knowledge results in a lower total packet energy.

We next study the impact of adding radios, one after another, on the network performance when using LOCAL-DIA-ADD. Intuitively, we expect that after each addition, the network will "drift" away from its optimal operating point. At low knowledge, the network drifts away much faster from its optimal state, than at high knowledge. From Figure 10, we observe that the rate of drift at 2-hop knowledge is almost twice that at 10-hop knowledge. Recall from Figure 6 that at low knowledge, the resultant steady-state networks are highly sub-optimal. This, coupled with the cascading effects, pushes the low knowledge network scenarios further away from the optimal; the less knowledge, the more rapidly this occurs.

As the drift continues to grow, it is probably beneficial to restart the topology control algorithm at some point, rather than to "fix" the network after each topology change event. Naturally, the restart period is longer with more network-awareness. However, the exact restart period is a design decision that is based on the tradeoff between the performance benefit of restarting (which depends on level of sub-optimality that can be tolerated) and the associated cost of restart. This decision may be a good problem for a cognitive controller to analyze and act on.

VIII. CONCLUSIONS

CNs present an approach to achieving end-to-end objectives through learning and reasoning. By breaking down network objectives into multiple local cognitive element goals, CNs operate in a distributed and self-organized manner. The network performance these cognitive elements achieve is dependent on the amount of knowledge they have about the network. The lack of knowledge may lead to some degree of sub-optimality. Depending on the problem to be solved and the strategy employed, sometimes having more knowledge illuminates better solutions, while other times it may just add overhead to the system. Regardless of the network benefit these partial-knowledge solutions provide, there is always a network cost to acquiring, communicating and maintaining knowledge. These
Fig. 10. Illustrating the drift performance at different levels of knowledge: price of ignorance for channel assignment in a 50-radio network as radios are added, one at a time.

factors must be taken into account to determine how much knowledge the cognitive elements need.

For the particular topology control problem examined here, we show that for dynamic networks, as radios join the network, more knowledge provides better spectral performance; on the contrary, when radios leave the network, some ignorance in the network results in better performance. We also determine that the cost of maintaining global knowledge is justified only when the network is fairly static. We also observe a tradeoff between restarting the topology control algorithm and allowing the topologies to drift further from their optimal states. This design problem is left open as a possible cognitive task.

REFERENCES


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