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with a budget**

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Abstract

The *Real Options* approach has proved to be a suitable methodology for capturing the flexibility in the investment decision process. This is very useful for the financial evaluation of R&D projects where there are several possible decisions concerning to the investment – delaying, improving or abandoning. Since the risk of an R&D project is usually due to singular characteristics of the project and is uncorrelated with the financial markets, the contingent claims analysis may be not adequate to value R&D projects. Based on a dynamic programming evaluation model developed by Huchzermeier and Loch (Management Science 47(1): 85-101, 2001), we propose two approaches to valuing a portfolio of R&D projects with a budget. Specifically, considering a budget constraint, in the first approach, we make an extension of the model mentioned above for assessing the projects in the portfolio simultaneously. Whereas in the second approach, based on Monte Carlo simulation, we simulate the joint evolution of the projects of the portfolio, taking into account the individual assessment, carried out in accordance with the reference model, and the budget constraint. To test the proposed evaluation procedures, we generated

several R&D portfolios with different dimensions. According to our computational experience, the main conclusions are presented.

Keywords *Real Options; Project Valuation; R&D Project; Stochastic Dynamic Programming; Monte Carlo simulation.*

1. Introduction

From the range of investment projects stand out the research and development projects (R&D projects), which are extremely important in some economy sectors as the pharmaceutical and high tech industries, in order to create new products or services. These projects should be viewed as a collection of sequential decisions, which involve a R&D phase, multi-stage, and a commercialization phase, with different risks and uncertainties, which will decreasing as the project develops (Morris et al. 1991; Luo et al. 2008).

The structure of the flexible decision of real options is valid in R&D projects. Specifically, after an initial investment it is possible collect more information on the evolution of the project and market conditions and, based on this information, modify the action plan (Huchzermeier and Loch 2001; Luo et al. 2008; Dixit and Pindyck 1994; Lint and Pennings 1997; Newton et al. 2004; Chevalier-Roignant et al. 2011).

In most cases, the R&D projects are associated with the development of innovative products for which there are no market benchmarks. Since the underlying asset of the option on the R&D project is not traded, the respective market value cannot be determined (Perlitz et al. 1999). For this reason, the contingent claims analysis, which is based on the principle of replication, should not be applied to evaluate R&D projects (Luo et al. 2008). An alternative technique for evaluating real options is dynamic programming, which is not based on the principle of replication of the risks of the underlying asset (Dixit and Pindyck 1994; Smith and Nau 1995). However, with this technique, the discount rate that reflects the risk attitude of the decision maker has to be specified exogenously (Huchzermeier and Loch 2001). On the other hand, the risk associated with a R&D project is normally attributed to the intrinsic characteristics of the project, that is, the risk is unsystematic or diversifiable. In this way, many researchers argue that a reasonable hypothesis for a large company is a neutral attitude to the risk of the project with the discounted process at the risk-free interest rate ((Huchzermeier and Loch 2001; Luo et al. 2008; Dixit and Pindyck 1994; Smith and Nau 1995; Trigeorgis 2000).

In this perspective, Huchzermeier and Loch (2001) developed a dynamic programming model to evaluate a R&D project. In order to react to the arrival of new information on the evolution of the project, there are two options at each stage of R&D phase: abandon the project or continue the project with an improvement action. Based on a set of levels of performance expected for the project, the backward recursion procedure determines,

in each stage of the R&D phase, the value of the project for each of these performance levels.

In this paper, we present two procedures to evaluate a portfolio of R&D projects considering a limited budget for improvement actions. In the first approach, R&D1, we generalize the evaluation of Huchzermeier and Loch (2001). Specifically, to evaluate simultaneously several R&D projects, we assume that the performance level of a project at every stage of R&D phase is independent of the level reached by other projects. This assumption implies that, at each stage of R&D phase, the evaluation of a project in a given performance level requires the calculation of its conjoint value (that is, the value of the project, when we considered simultaneously all projects of the portfolio) in each of the acceptable performance levels to each one of the other projects. This process can be time consuming when there are many projects and the number of stages of the phase R&D is high. Regarding the second approach, R&D2, based on Monte Carlo simulation, we simulate the joint evolution of the projects values in the portfolio, according to the individual assessment described in Huchzermeier and Loch (2001).

Next section of the paper reports the main features of a R&D project. In the following section, we review the dynamic programming model presented in Huchzermeier and Loch (2001). The Sections 4 and 6 are devoted to the presentation of the evaluation procedures R&D1 and R&D2, respectively, which enable evaluate a portfolio of R&D projects subject to a budget constraint for improvement actions. While in sections 5 and 7, is given an example of application of the approaches R&D1 and R&D2, respectively. Computational experience is reported in Sect. 8 and some final comments are made in the last section of the paper.

2. Characterization of a R&D project

The financial evaluation of R&D projects assumes great importance in industrial sectors that are dedicated to research and development of innovative products or services. Among the various industries, stand out the pharmaceutical and high technology industries.

According to Morris et al. (1991), a R&D project should be interpreted as a collection of sequential decisions, which involve the phase of research and development and a commercialization phase (see Fig. 1). At each phase, it is assumed that there are different risks and uncertainties, which will decreasing as the project evolves. Still, according to these authors, the purpose of an R&D is to maximize future revenues or minimize future costs.

When a company accepts a R&D project, through the payment of an initial premium, the first stage of the R&D phase is initiated and the company acquires the right (that is, the option) to decide if the project proceeds to second stage; if the second stage of R&D

phase is initiated, upon payment of a second premium, the company gains the opportunity to continue to the third step, and so on. Therefore, an investment opportunity in a R&D project can be viewed as a compound option, that is, an option that to be exercised activates new options (Pennings and Sereno 2011; Santos and Pamplona 2005; Herath and Park 2002).

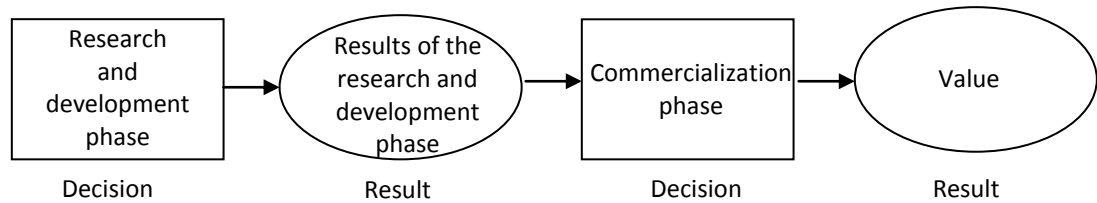


Fig. 1 Interpretation of a R&D project (Morris et al. 1991)

The flexibility of decision associated with each stage of the R&D project is not limited to the option of abandoning the project (Myers and Majd 2001). Also it is possible choose to defer the decision on the project until more information is available on the investment (McDonald and Siegel 1986), or expand or contract the project (Brennan and Schwartz 1977; Dixit 1989), or even change the operating mode of the project depending on the price factor (Kulatilaka and Trigeorgis 2001). In Huchzermeier and Loch (2001) has been introduced a new type of flexibility, which translates into the option of performing, at every stage of R&D phase, an improvement action.

The R&D projects are characterized by a long-term horizon and a high degree of uncertainty, of a technical (due to the characteristics of the projects) and market nature. Therefore the value of decision flexibility in a R&D project (that is, the value of an option on a R&D project) can be substantial (Pennings and Sereno 2011; Luo et al. 2008; Lint and Pennings 1997). The two types of uncertainty influence differently the value of flexibility underlying a R&D project. Indeed, the technique uncertainty reduces the value of flexibility, since it is very likely that the operating costs will increase. While the market uncertainty leads to an increase in the value of flexibility, since there is possibility of high returns, which can be upgraded with the possibility of other uses for the created knowledge (Huchzermeier and Loch 2001).

3. Dynamic programming model of Huchzermeier and Loch (2001)

Before presenting the evaluation model developed in Huchzermeier and Loch (2001), we define the R&D project of the model and we identify the uncertainty factors that affect its value, and therefore its option value.

3.1 R&D project of the evaluation model

The R&D project considered in the evaluation model consists of the phases R&D and of commercialization. The R&D phase is composed of m stages with the same duration (see Fig. 2).

The acceptance of a R&D project involves an initial capital outlay, I , to acquire the necessary infrastructures to start the project. At the beginning of each stage of the R&D phase (review period of the project), according to most recent information, there is an opportunity to abandon, continue or improve the project. In period $t=0, \dots, m-1$, the continuity decision requires the payment of a cost c_t while the improvement decision requires the payment of the cost c_t and a cost of improvement d_t . If the m stages are performed (that is, if the project was not abandoned at some stage) then is necessary to decide the passage or not, to commercialization phase.

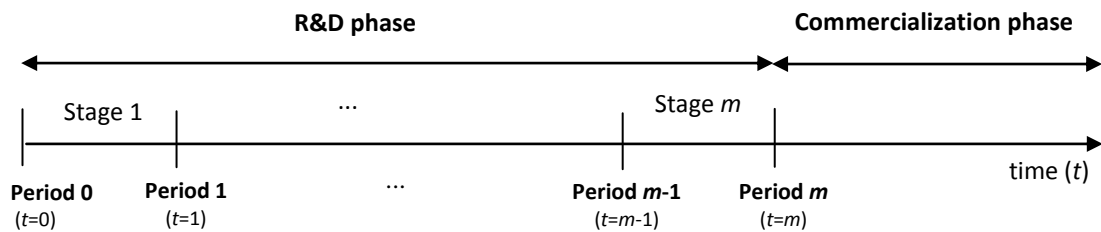


Fig. 2 Scheme of the R&D project relating to the evaluation model

The value of the R&D project depends on several factors of uncertainty, which will be listed in the next section.

3.2 Uncertainty factors considered in the evaluation model

A R&D project is characterized by the time associated to R&D phase, cost over time and by the performance of the resulting product. In turn, the market is characterized by the return from the project (caused by the size and attractiveness of the market) and by the performance requirements (which indicate how the return increases with the performance of the product). The interaction of project characteristics with the market characteristics contributes decisively to the value of the R&D project. Therefore, the value of a R&D project is function of the factors: time, cost, performance, market return and market requirements.

In the evaluation model developed in Huchzermeier and Loch (2001) is assumed that the R&D project can be defined in terms of the expected performance along a corridor or cone of performance states during the time of development, a fixed range of market

returns (for example, market returns that belong to the range $[u, U]$, $u, U \in \mathbb{R}$) and a range of market requirements.

Although the evaluation model only considers three factors of uncertainty, in Huchzermeier and Loch (2001) is studied the influence of each of the five risk factors in the value of the R&D project.

The following sections provide the modeling of each of the uncertainty factors that define the value of the R&D project.

3.2.1 Modeling of the performance associated to the product resulting

Given the uncertainty, the product performance it changes between periods of project review. In each review period, the R&D teams perform corrections to the project plan, where the expected performance is estimated from tests, simulations or prototypes (Huchzermeier and Loch 2001). Thus, for each period $t=0, \dots, m$, we define the system of product performance states by (t, i) , where i is a level of performance for the product under development.

Huchzermeier and Loch (2001) admit that the performance in each period $t=0, \dots, m-1$, follows a Binomial distribution, regardless of the history associated with the development of the project. Specifically, the product performance of period $t=0, \dots, m-1$ for the period $t+1$

- may improve in $1/2$, with probability p , or may decrease by $1/2$, with probability $1-p$, with a continuity action; or
- may improve in $3/2$, with probability p , or may improve by $1/2$, with probability $1-p$, with an improvement action.

In this paper, we consider this scheme of evolution for the performance of the product, however the authors generalize the binomial distribution, allowing that the improvement performance and the regression performance, respectively, are expanded by the next N (which represents a natural number) performance states.

3.2.2 Modeling of the market return and of the market requirements

After the R&D phase (that is, in period m), the product is launched on the market with a performance level i , which will produce an expected market return $\pi(i) = u + F(i) \times (U - u)$. Where $F(i)$ denotes the probability distribution function of the random variable D , which

represents the level of performance required by the market. Huchzermeier and Loch (2001) assumes that D follows the Normal distribution of parameters (μ, σ) .

At this point, it is possible present the model that allows determine the value of the R&D project.

3.3 Evaluation model

The evaluation of a R&D project corresponds to a sequential decision problem, which can be formulated as a stochastic dynamic program.

Let P_t be the set of feasible performance levels in period $t=0, \dots, m$. Also, let r be the risk-free interest rate. Assuming that c_t and d_t represent respectively the costs of continuity and of improvement in period $t=0, \dots, m$, and considering $V(t, i)$ as the value of the R&D project in period $t=0, \dots, m$, for the performance level $i \in P_t$, the backward recursion procedure for determining the value of the project at the present moment is the following:

begin

$$V(m, i) \leftarrow \pi(i), i \in P_m;$$

for $t=m-1$ **to** 0 **do**

for $i \in P_t$ **do**

$$V(t, i) \leftarrow \text{Max} \begin{cases} 0, & \text{(decision: abandon)} \\ -c_t + \frac{p \times V(t+1, i+0.5) + (1-p) \times V(t+1, i-0.5)}{1+r}, & \text{(decision: continue)} \\ -c_t - d_t + \frac{p \times V(t+1, i+1.5) + (1-p) \times V(t+1, i+0.5)}{1+r} & \text{(decision: improve)} \end{cases}$$

end

end

end.

The output of the procedure is $V(0, i)$, $i \in P_0$, which represents the value of the R&D project at the present moment before being subtracted from the initial investment, I . Therefore, the expression $V(0, i) - I$, $i \in P_0$, represents the optimal value of the R&D project.

4. Evaluation procedure R&D1

To evaluate an investment portfolio composed of n R&D projects, which is subject to a budget constraint for improvement actions, we propose the approach R&D1 that generalizes the Huchzermeier and Loch model (2001). The R&D projects considered in the portfolio have the characteristics mentioned in Sect. 3.1. Specifically, we define the following parameters:

- l_j , the initial capital outlay for project $j=1, \dots, n$;
- $[u_j, U_j]$, the range of market returns for project $j=1, \dots, n$;
- $F_j(i)$, the probability distribution function of the random variable D_j , which represents the level of performance required by the market for project j and follows the Normal distribution with parameters (μ_j, σ_j) , $j=1, \dots, n$;
- p_j , the probability of the Binomial distribution related to the performance of project $j=1, \dots, n$;
- $P_{t,j}$, the set of feasible performance levels in period $t=0, \dots, m$, for project $j=1, \dots, n$;
- $c_{t,j}$ and $d_{t,j}$, the costs of continuity and of improvement, respectively, in period $t=0, \dots, m-1$, for project $j=1, \dots, n$; and
- $\pi_j(i)$, the expected market return when the product relating to project j is launched on the market, at period m , with the performance level $i \in P_{m,j}$, $j=1, \dots, n$.

Considering the risk-free interest rate, r , and the existence of a limited budget, B_t , in period $t=0, \dots, m-1$ for improvement actions, the developed procedure allows evaluating simultaneously n R&D projects and respect the budget. Such evaluation requires the definition of a criterion to select the projects that should be improved when the budget B_t , $t=0, \dots, m-1$, is not sufficient for the volume of improvement decisions. We establish that B_t , $t=0, \dots, m-1$, must be allocated to projects that have the highest single value.

Assuming that in each period $t=0, \dots, m$, the performance level of a project is independent of the level achieved for other projects, the R&D1 procedure can be described as follows:

begin

A - Individual value of project $j=1, \dots, n$, in period m , for the performance level $i \in P_{m,j}$:

$$V_j(m, i) \leftarrow \pi_j(i) = u_j + F_j(i) \times (U_j - u_j), \quad i \in P_{m,j}, \quad j=1, \dots, n;$$

B - Value of project $j=1, \dots, n$, in period m , for the level of performance $i \in P_{m,j}$, considering the simultaneous evaluation:

$$VP_j(m, i) \leftarrow V_j(m, i), \quad i \in P_{m,j}, \quad j=1, \dots, n;$$

for $t=m-1$ **to** 0 **do**

C – Individual value of project $j=1, \dots, n$, in period t , for the performance level $i \in P_{t,j}$:

$$V_j(t, i) \leftarrow \text{Max} \begin{cases} 0 \\ -c_{t,j} + [p_j \times VP_j(t+1, i+0.5) + (1-p_j) \times VP_j(t+1, i-0.5)] / (1+r) \\ -c_{t,j} - d_{t,j} + [p_j \times VP_j(t+1, i+1.5) + (1-p_j) \times VP_j(t+1, i+0.5)] / (1+r) \end{cases} ;$$

D – Value of project $j=1, \dots, n$, in period t , for the level of performance $i \in P_{t,j}$, considering the simultaneous evaluation: $VP_j(t, i)$

if individual decision at $i \in P_{t,j}$ is abandon or continue **then** $VP_j(t, i) \leftarrow V_j(t, i)$;

if individual decision at $i \in P_{t,j}$ is improve **then**

D1 – Generation of combinations of performance levels, assuming that the performance level of the project j is $i: s_j(t, i) = (i_1, i_2, \dots, i_n)$, where $i_k \in P_{t,k}$, $k=1, \dots, n$ and $k \neq j$;

D2 – Calculation of the combined value of project $j=1, \dots, n$, in period t , for each combination of performance levels $s_j(t, i)$ (generated in **D1**): $VC_j(s_j(t, i))$

Create the set $W_j(s_j(t, i)) = \{k=1, \dots, n: V_k(t, i_k) > V_j(t, i)\}$;

Sort the projects $k \in W_j(s_j(t, i))$ in descending order of their individual values;

for $k \in W_j(s_j(t, i))$ **do**

if individual decision relating to project k is improve **then**

if $B_t \geq d_{t,k}$ **then** $B_t \leftarrow B_t - d_{t,k}$

end;

end;

if $B_t \geq d_{t,j}$ **then** $VC_j(s_j(t, i)) \leftarrow V_j(t, i)$

else

$$VC_j(s_j(t, i)) \leftarrow \text{Max} \begin{cases} 0 \\ -c_{t,j} + [p_j \times VP_j(t+1, i+0.5) + (1-p_j) \times VP_j(t+1, i-0.5)] / (1+r) \end{cases}$$

end; {End of component **D2**}

$VP_j(t, i) \leftarrow$ combined value of project j (calculated in **D2**) most frequently;

end; {End of component **D**}

end;

end. {End of Procedure R&D1}

According to the procedure R&D1 developed, the expression $VP_j(0,i)-I_j$, $i \in P_{0,j}$, represents an estimate for the value of the project $j=1,\dots,n$, at the present moment, considering the simultaneous evaluation.

5. Example

The following example illustrates the approach R&D1 proposed in this paper to evaluate an investment portfolio composed of n R&D projects, which is subject to a budget constraint for improvement actions. Let us consider a portfolio with three R&D projects described in Table 1 and a risk-free interest rate $r=8\%$.

Table 1 Parameters for the three R&D projects

Project j	Initial capital outlay I_j	Parameters of Normal distribution (μ_j, σ_j)	Range of market returns $[u_j, U_j]$	Probability p_j
1	87	(0.0,2.0)	[5, 490]	0.5
2	98	(0.0,2.0)	[8, 478]	0.5
3	77	(0.0,2.8)	[0, 490]	0.5

The R&D phases of the three projects are composed by $m=3$ stages. Finally, the costs of continuity and of improvement as well as the budget for improvement actions in each period $t=0,1,2$ are presented in Table 2.

Table 2 Costs of continuity, improvement and budget for improvement actions in period $t=0,1,2$

Period t	$c_{t,1}$	$d_{t,1}$	$c_{t,2}$	$d_{t,2}$	$c_{t,3}$	$d_{t,3}$	B_t
0	10.0	17.0	14.0	17.0	9.0	22.0	35
1	26.0	32.0	23.0	28.0	20.0	45.0	58
2	39.0	64.0	34.0	53.0	36.0	70.0	80

Assuming that in period $t=0$ the performance level of each of the three projects is 0, we start by evaluating each project individually.

The evaluation of each project is presented in Fig. 3, which enables to state that the values of the projects 1, 2 and 3, in the period $t=0$ are, respectively, $V_1(0,0)-I_1=206.1-87=119.1$, $V_2(0,0)-I_2=207.4-98=109.4$ and $V_3(0,0)-I_3=170.6-77=93.6$. Also, we can affirm that, for all the projects, the optimal decision in period $t=0$ (that is, at the beginning of stage 1) is improve. According to this evaluation, each project adds value to the company and, if desired, the projects can be developed.

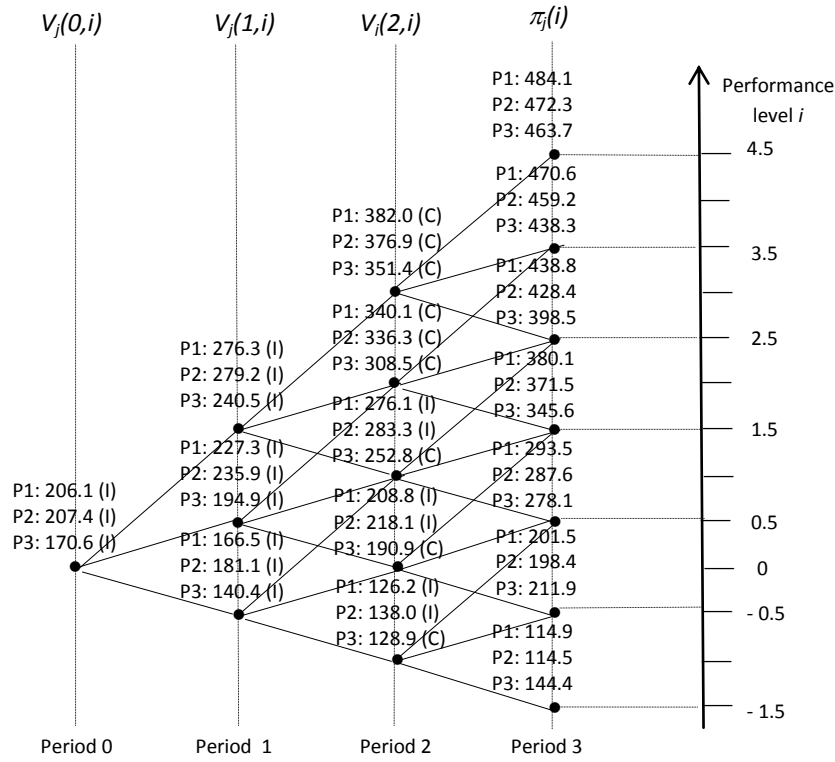


Fig. 3 Recombined tree which translates the possible levels of performance for the $n=3$ projects in each period $t=0,1,2,3$. In each state (t, i) of the tree, we present the value of project j , when it is evaluated individually, at each performance level i , $V_j(t,i)$, $t=0,1,2$, $j=1,2,3$, $i \in P_{t,j}$, and the respective decision (A: abandon; C: continue; I: improve)

Now we will evaluate the three projects simultaneously by applying the procedure R&D1. To exemplify the evaluation procedure, we calculate the estimate for the value of the project 1 at the period $t=2$ for the level of performance $i=-1$. According to the component **C** of the procedure, the individual value of the project 1 at the period $t=2$ for the level of performance $i=-1$ is equal to

$$V_1(2,-1) \leftarrow 126.2 = \text{Max} \begin{cases} 0 \\ -39+(0.5 \times 114.9+0.5 \times 201.5)/1.08 \\ -39-64+(0.5 \times 201.5+0.5 \times 293.5)/1.08 \end{cases} \quad \text{and} \quad \text{the associated}$$

decision is improve (which confirm the information presented in Fig. 3). To determine de value of the project 1 at the period $t=2$ for the level of performance $i=-1$, considering the simultaneous evaluation, we must generate the combinations of performance levels assuming that the performance level of project 1 is $i=-1$: $s_1(2,-1)=(-1, i_2, i_3)$, $i_2,$

$i_3 \in \{-1, 0, 1, 2, 3\}$. Then, for each of the 25 combinations of performance levels, we must determine the combined value of project 1. For example, in the combination of levels of performance $(-1, -1, -1)$, the project 2 has the higher individual value and is relating to an improvement decision (see Fig. 3), therefore the budget $B_2=80$ should be assigned to project 2. Since the remaining budget, $B_2-d_{2,2}=27$, is not sufficient to improve project 1, the value of project 1 for the combination $(-1, -1, -1)$ is given by

$$VC_1(-1, -1, -1) \leftarrow 107.5 = \text{Max} \begin{cases} 0 \\ -39 + (0.5 \times 114.9 + 0.5 \times 201.5) / 1.08 \end{cases} \quad \text{and the associated}$$

decision is continue. For the others combinations of performance levels, the process is analogous. In table 3, we resumed the 25 combined values for project 1 at the period $t=2$ for the level of performance $i=-1$. As the combined value most frequently is 107.5, the estimate for the value of project 1 at the period $t=2$ for the level of performance $i=-1$ is $VP_1(2, -1)=107.5$.

Table 3 Combined values for project 1 at the period $t=2$ for the level of performance $i=-1$

Combinations of performances levels: $s_1(2, -1) = (-1, i_2, i_3), i_2, i_3 \in \{-1, 0, 1, 2, 3\}$	Combined value of project 1 for the combination $s_1(2, -1) = (-1, i_2, i_3): VC_1(-1, i_2, i_3)$
$(-1, -1, i_3), i_3 \in \{-1, 0, 1, 2, 3\}$	107.5 (decision: continue)
$(-1, 0, i_3), i_3 \in \{-1, 0, 1, 2, 3\}$	107.5 (decision: continue)
$(-1, 1, i_3), i_3 \in \{-1, 0, 1, 2, 3\}$	107.5 (decision: continue)
$(-1, 2, i_3), i_3 \in \{-1, 0, 1, 2, 3\}$	126.2 (decision: improve)
$(-1, 3, i_3), i_3 \in \{-1, 0, 1, 2, 3\}$	126.2 (decision: improve)

After applying this procedure to all the projects and levels of performance, we obtain the evaluation results shown in Fig. 4.

According to the procedure R&D1 developed, the estimates for the values of the projects 1, 2 and 3, in period $t=0$ are, respectively, $VP_1(0,0) - I_1 = 192.8 - 87 = 105.8$, $VP_2(0,0) - I_2 = 195.1 - 98 = 97.1$ and $VP_3(0,0) - I_3 = 136.1 - 77 = 59.1$. With respect to the decision policy in period $t=0$, we can say that, for projects 1 and 2 the decision proposed is improve, while for project 3 the decision proposed is continue. In comparison with the individual values obtained for the period $t=0$, we can say that the respective estimated values of the projects, when subjected to a simultaneous evaluation, suffer reductions, being the highest recorded in the third project. Moreover, the decision in period $t=0$ associated with the project 3 is reviewed: the improvement decision of the individual evaluation is replaced by a continuity decision. These conclusions were predictable, since the budget $B_t, t=0,1,2$, is not enough to keep all the improvement decisions established on individual evaluation. However, each project continues to add value to the company, that is, if desired, the three projects can be developed simultaneously.

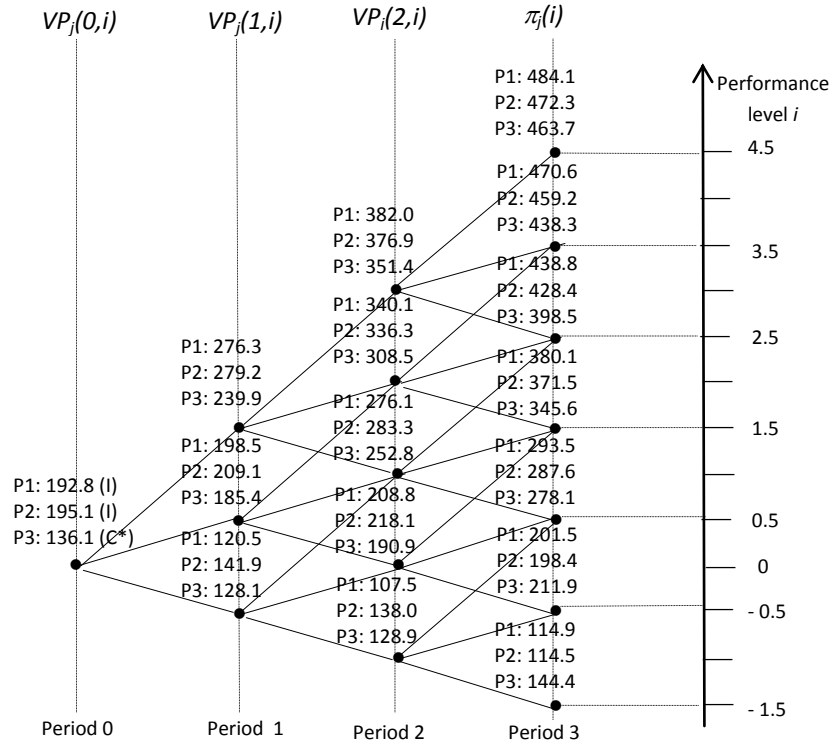


Fig. 4 Application of the procedure R&D1: in each state (t, i) of the tree, we present an estimate for the value of project j , when it is evaluated simultaneously with the remaining projects, at each performance level i , $VP_j(t, i)$, $t=0,1,2$, $j=1,2,3$, $i \in P_{t,i}$, and the decision proposed at the time period $t=0$ (A: abandon; C: continue; I: improve; C* or A*: continue or abandon, when in the simultaneous evaluation is not possible keep the improvement decision determined in the individual evaluation)

6. Evaluation Procedure R&D2

In order to assess the quality of the estimates determined by the procedure R&D1, we developed an alternative procedure, R&D2, based on Monte Carlo simulation, which simulates the joint evolution of the projects of the portfolio, taking into account the individual assessment carried out in accordance with the model Huchzermeier and Loch (2001).

Given the parameters defined in Sect. 4 and considering $i_j(t)$ the level of performance of the project $j=1, \dots, n$, at time period $t=0, \dots, m$, and M the number of simulation runs, the R&D2 procedure can be described as follows:

begin

for $k=1$ to M **do**

A – Determination of the k -th joint decision policy for the portfolio of n projects

for $j=1$ to n **do** $i_j(0) \leftarrow i, i \in P_{0,j}$;

for $t=0$ to $m-1$ **do**

Sort the n projects in descending order of their individual values in the respective levels of performance $i_j(t), j=1, \dots, n$;

for $j=1$ to n active (that is, which has not been abandoned) **do**

A1 – Decision relating to project j , in the simulation run k , for the level of performance $i_j(t)$, considering the simultaneous evaluation: $DP_j^k(i_j(t))$

if individual decision at $i_j(t)$ is abandon or continue **then** $DP_j^k(i_j(t)) \leftarrow$ individual decision;

if individual decision at $i_j(t)$ is improve **then**

if it is feasible improve the project j in the simultaneous valuation **then**

$DP_j^k(i_j(t)) \leftarrow$ improve

else

$DP_j^k(i_j(t)) \leftarrow$ abandon or continue depending on the value of $Max\{0, -c_{t,j} + [p_j \times V_j(t+1, i_j(t)+0.5) + (1-p_j) \times V_j(t+1, i_j(t)-0.5)] / (1+r)\} = 0$

end;

end; {End of component **A1**}

A2 – Generation of the performance level of project j for the period $t+1$ considering the joint decision $DP_j^k(i_j(t))$: $i_j(t+1)$

Generate a random observation of the Uniform distribution $U[0,1]$: v ;

if $v \leq p_j$ **then**

if $DP_j^k(i_j(t)) = \text{improve}$ **then** $i_j(t+1) \leftarrow i_j(t) + 1.5$;

if $DP_j^k(i_j(t)) = \text{continue}$ **then** $i_j(t+1) \leftarrow i_j(t) + 0.5$;

else

if $DP_j^k(i_j(t)) = \text{improve}$ **then** $i_j(t+1) \leftarrow i_j(t) + 0.5$;

if $DP_j^k(i_j(t)) = \text{continue}$ **then** $i_j(t+1) \leftarrow i_j(t) - 0.5$;

end; {End of component **A2**}

end;

end; {End of component **A**}

B – Determining the value of project $j=1, \dots, n$ at the initial time, according to the k -th joint decision policy: $VP_j^k(0, i_j(0))$

for $j=1$ to n **do**

if $DP_j^k(i_j(m-1))=$ continue or improve **then** $VP_j^k(0, i_j(0)) \leftarrow \pi_t(i_j(m))/(1+r)^m$;

end;

for $t=m-1$ to 0 **do**

for $j=1$ to n **do**

if $DP_j^k(i_j(t))=$ continue **then** $VP_j^k(0, i_j(0)) \leftarrow VP_j^k(0, i_j(0)) - [c_{t,j}/(1+r)^t]$;

if $DP_j^k(i_j(t))=$ improve **then** $VP_j^k(0, i_j(0)) \leftarrow VP_j^k(0, i_j(0)) - [(c_{t,j} + d_{t,j})/(1+r)^t]$;

end;

end; {End of component **B**}

end; {End of k -th simulation run}

C – Calculating the estimated value of project $j=1, \dots, n$ at the initial time, according to the joint policies simulated: $VP_j(0, i_j(0))$

for $j=1$ to n **do** $VP_j(0, i_j(0)) \leftarrow \frac{\sum_{k=1}^M VP_j^k(0, i_j(0))}{M} - I_j$;

end. {End of Procedure R&D2}

According to the procedure R&D2 developed, the expression $VP_j(0, i_j(0))$, represents an estimate for the value of the project $j=1, \dots, n$, at the present moment, considering the simultaneous evaluation.

7. Example

Consider again the example of Sect. 5, which relates to a portfolio of $n=3$ R&D projects, in which the respective R&D phases are divided into $m=3$ stages.

In order to determine alternative estimates for the value (at the initial time) of each project, considering the simultaneous evaluation, we applied the procedure R&D2.

To exemplify the evaluation procedure, we determine a joint decision policy for the portfolio of three projects. Based on the individual evaluation of each project (see Fig. 3), a possible joint decision policy is presented in Table 4.

Table 4 A joint decision policy for the portfolio of three projects considering the simultaneous evaluation (Continue* means that, in the simultaneous evaluation, is not possible keep the improvement decision determined in the individual evaluation)

Project j		$j=1$	$j=2$	$j=3$
Period $t=0$	Level of performance $i_j(0)$	0	0	0
	Decision relating to project j : $DP_j^k(i_j(0))$	Improve	Improve	Continue*
Period $t=1$	Level of performance $i_j(1)$	1.5	0.5	0.5
	Decision relating to project j : $DP_j^k(i_j(1))$	Improve	Continue*	Continue*
Period $t=2$	Level of performance $i_j(2)$	3	0	1
	Decision relating to project j : $DP_j^k(i_j(2))$	Continue	Improve	Continue
Period $t=3$	Level of performance $i_j(3)$	3.5	1.5	1.5
	Expected market return for project j : $\pi_j(i_j(3))$	470.6	371.5	345.6

According to this joint decision policy, the value of each project at the present moment is calculated as follows:

$$\begin{aligned}
 - VP_1^k(0,0) &= \frac{470.6}{1.08^3} - \frac{39}{1.08^2} - \frac{(26+32)}{1.08} - (10+17) = 269.4 ; \\
 - VP_2^k(0,0) &= \frac{371.5}{1.08^3} - \frac{(34+53)}{1.08^2} - \frac{23}{1.08} - (14+17) = 168.0 ; \\
 - VP_3^k(0,0) &= \frac{345.6}{1.08^3} - \frac{36}{1.08^2} - \frac{20}{1.08} - 9 = 216.0 .
 \end{aligned}$$

After repeating M times the process described in Table 4, we obtain, for each project $j=1,\dots,n$, M values at the present moment: $VP_j^k(0,0)$, $k=1,\dots,M$. Finally, the arithmetic average of this simulated values corresponds to the estimated value of project $j=1,\dots,n$ at the initial time.

In Table 5, we present the results obtained by this procedure, considering $M=2000$ and $M=3000$. The choice of values for M was done in order to ensure that the estimated values for the projects do not depend on the number of trials performed.

According with the results, the estimates for the values of the three projects, at the period $t=0$, are similar to those obtained with the procedure R&D1 (which were equal to 105.8, 97.1 and 59.1, respectively, for projects 1, 2 and 3 – see page 12). Therefore, for this example both procedures suggest that the three projects can be developed simultaneously.

Table 5 Simultaneous evaluation of the three projects in accordance with the procedure R&D2

Number of simulation runs	Value at the initial time for each project of the portfolio		
	Project 1	Project 2	Project 3
2000	105.0	94.0	59.2
3000	105.2	93.9	59.2

8. Computational experience

We conducted computational experiments for a total of 252 test instances, with the number of projects varying from 5 to 10 and a number of stages in the R&D phase ranging between 2 and 7. Specifically for each test problem of dimension (n, m) , $n=5, \dots, 10$ and $m=2, \dots, 7$, we generate seven instances with different percentages of improvement decisions in the individual evaluation.

The proposed evaluation procedures were coded in Fortran 77 and the computational tests were performed on a Pentium Dual-Core 2GHz and 3,00GB of RAM.

According to our experience, the computational time required to evaluate concurrently the projects of a portfolio with the approach R&D1 depends on the size (n, m) of the test instance, as well as of the volume of improvement decisions in individual evaluation. This conclusion was expected, since, in step **D2** of the proposed procedure, in each period $t=0, \dots, m-1$, it is necessary to calculate the combined value of projects for each combination of levels of performance. Indeed, in each period $t=0, \dots, m-1$, the number of combinations of levels of performance to be analyzed can reach $(2t+1)^n$, whose value depends strongly of the values of the parameters (n, m) . This is illustrated by Table 6, where we see that as the parameters n and m increase, the average computational time increases markedly. Finally, due to the high number of combinations of levels of performance was not possible to evaluate, according to the procedure R&D1, the instances of dimensions: $(8, 7)$, $(9, m)$, $m=5, 6, 7$ and $(10, m)$, $m=4, 5, 6, 7$ (see Table 6).

Table 6 Average execution time associated to approach R&D1. For each pair (n, m) , $n=5, \dots, 10$ and $m=2, \dots, 7$, seven instances were evaluated

n	m	Average Time of execution (seconds)	n	m	Average Time of execution (seconds)	n	m	Average Time of execution (seconds)
5	2	0.29	6	2	0.71	7	2	2.26
5	3	2.07	6	3	10.13	7	3	64.10
5	4	9.28	6	4	57.74	7	4	429.14
5	5	17.57	6	5	217.75	7	5	1231.184
5	6	40.54	6	6	755.01	7	6	2048.452
5	7	48.99	6	7	833.98	7	7	5869.886
8	2	7.725	9	2	18.97	10	2	70.46
8	3	409.247	9	3	581.15	10	3	4474.40
8	4	2318.920	9	4	7026.78	10	4	---
8	5	10808.41	9	5	---	10	5	---
8	6	18917.11	9	6	---	10	6	---
8	7	---	9	7	---	10	7	---

Regarding the approach R&D2 developed, it was possible to evaluate, in a short time, all portfolios of R&D projects generated.

To illustrate the estimates obtained with the two approaches, we have selected six instances whose results summarize the types of assessments obtained. In Table 7, we present the estimates obtained for the values of projects, at the initial time, in accordance with the two approaches. As we can see, the values obtained for each project through the two evaluation procedures are distinct. However, should be noted that the values tend to be of the same order of grandeur, which allows us to affirm that, in the overwhelming majority of cases, the two approaches lead to the same set of projects that can be developed simultaneously. The exception is in portfolio 1, where the estimates obtained for the project 5 lead to a decision policy, for the initial period, inconsistent. Whereas the estimate obtained by the procedure R&D1 indicates that the project 5 can be developed, the estimate of approach R&D2 suggests that the project should not be developed.

Table 7 Estimates for the value of each project at the initial moment obtained with the two procedures developed

Project	Approach R&D1	Approach R&D2	Project	Approach R&D1	Approach R&D2
<i>j</i>		(<i>M</i> = 3000 trials)	<i>j</i>		(<i>M</i> = 3000 trials)
Portfolio 1 (<i>n</i> =5; <i>m</i> =7)			Portfolio 2 (<i>n</i> =6; <i>m</i> =7)		
1	177.5	158.10	1	21.2	139.26
2	639.4	625.44	2	1702.9	1750.27
3	-8.5	-19.05	3	297.7	254.66
4	415.8	406.17	4	153.7	321.87
5	11.4	-7.83	5	508.5	737.23
--	--	--	6	703.3	1045.42
Portfolio 3 (<i>n</i> =7; <i>m</i> =7)			Portfolio 4 (<i>n</i> =8; <i>m</i> =6)		
1	211.0	248.80	1	83.3	66.25
2	1324.8	1344.85	2	581.5	577.17
3	134.0	134.16	3	-0.3	-0.24
4	239.3	267.81	4	-30.9	-42.04
5	546.4	617.51	5	200.3	187.43
6	731.8	603.41	6	53.0	52.96
7	1552.1	1561.0	7	204.2	163.21
--	--	--	8	622.5	619.66
Portfolio 5 (<i>n</i> =9; <i>m</i> =4)			Portfolio 6 (<i>n</i> =10; <i>m</i> =3)		
1	429.3	384.31	1	-55.9	-56.82
2	2426.9	2437.99	2	-8.0	-20.64
3	559.2	568.0	3	-20.3	-23.59
4	495.9	452.34	4	-0.9	-11.13
5	1644.8	1652.85	5	87.0	87.03
6	1333.5	1352.95	6	27.6	27.83
7	1362.0	1375.40	7	-9.2	-6.71
8	1098.2	1119.40	8	3.3	3.28
9	1317.2	1273.10	9	141.9	137.86
--	--	--	10	347.7	353.35

9. Final remarks

In this paper, we address the problem of evaluating simultaneously n R&D projects of an investment portfolio, which is subjected to a budget constraint. As in Huchzermeier and Loch (2001), we assume that the R&D phase of each project consists of m stages, and at the beginning of each stage is possible to abandon, continue or improve the project. But now we consider a limited budget for the improvement actions at each stage of R&D phase.

In order to determine, at the initial instant, the estimated value of each project of the portfolio when the R&D phases of projects are developed in parallel, we propose two evaluation procedures, which were designated by R&D1 and R&D2. The approach R&D1 is a dynamic programming procedure that generalizes the stochastic dynamic program developed in Huchzermeier and Loch (2001), while the approach R&D2 is a procedure based on Monte Carlo simulation.

In this paper, we report computational experience indicating that the procedures developed can be applied to evaluate any portfolio of R&D projects with the characteristics defined in Sect. 3.1. Since, in the approach R&D1, we assume that the level of performance of a project, at each stage of the R&D phase, is independent of the level reached by other projects, the application of the procedure is only limited by the dimensions of the parameters (n, m) of the instance. Nevertheless, we consider that improvements can be obtained from further research, namely by setting different estimates for the value of the project in the simultaneous evaluation (see component **D** of R&D1 procedure). With respect to approach R&D2, the computational experience indicates that it was possible to assess all the generated test portfolios. Finally, a comparison of the evaluations obtained with the two procedures allows affirming that, in the overwhelming majority of cases, the values estimated, at the initial time, for the projects are close and lead to the same set of projects that can be developed simultaneously.

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