Boundary Degeneracy of Topological Order

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Topological Degeneracy

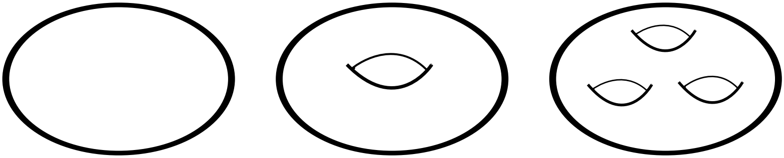
- Bulk and Boundary Degeneracy
- Z_2 **toric code** v.s. **doubled semions model**
- Gapping Rules

Application

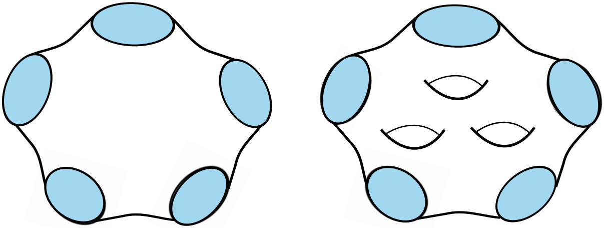
- Lattice model: Toric Code and String-net
- Flux Insertion

What is Topological Degeneracy?

Bulk Degeneracy:

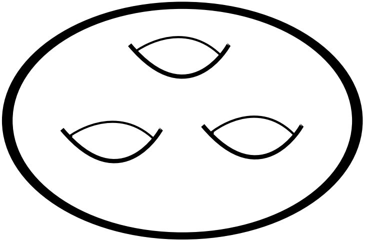


Boundary Degeneracy:



Bulk Ground State Degeneracy (GSD)

on genus-g spatial 2D Riemann surface.

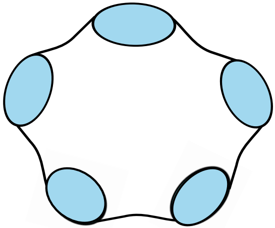


Abelian Chern-Simons: $$S_{bulk} = \frac{K_{IJ}}{4\pi} \int_{\mathcal{M}} dt\ d^2x\ \epsilon^{\mu\nu\rho} a^I_\mu \partial_\nu a^J_\rho \quad (1)$$

$$\boxed{\mathrm{GSD} = |\det K|^g} \quad (2)$$

X. G. Wen, Phys. Rev. B 40, 7387 (1989)

Boundary GSD (arXiv:1212.4863)



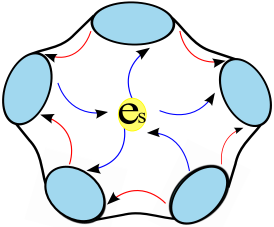
Chiral bosons Φ_I on the 1D boundary come from $a \to a + d\Phi$.

$$S_\partial = \frac{1}{4\pi} \int_{\partial \mathcal{M}} dt\, dx\, K_{IJ} \partial_t \Phi_I \partial_x \Phi_J + V_{IJ} \partial_x \Phi_I \partial_x \Phi_J \qquad (3)$$

There are gapless edge modes on the boundary, what are the gapping conditions to gap out all the edge modes? Gapping terms:

$$\int_{\partial \mathcal{M}} dt\, dx \sum_a g_a \cos(\ell_{a,I} \cdot \Phi_I) \qquad (4)$$

Boundary GSD - Physical Notions

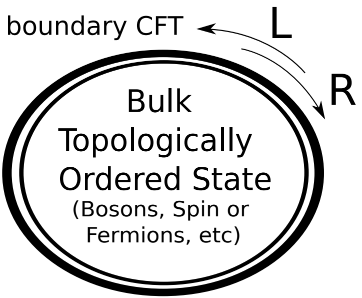


Scattered electrons condense on the boundary → open up mass gap of edge states. *Condensed electrons* (physical non-fractionalized particles) have relative zero Aharonov-Bohm (A-B or charge-flux) phase, no relative quantum fluctuation. *Compatible anyons* do not produce flux effect to *condensed electron* charge, their A-B phases are zero.

$$\text{GSD} = \text{the number of ways to transport fractionalized anyons} \tag{5}$$

Boundary GSD - Physical Notions:

Condensation of non-chiral bosons on the edge
defines the **boundary types**.



$$\text{correlators:}\langle \phi_I^\dagger(x, i\tau)\phi_J(x', i\tau')\rangle = (z - z')^{-h_{IJ}}(\bar{z} - \bar{z}')^{-\bar{h}_{IJ}} \quad (6)$$

$$\pi(h_{IJ} - \bar{h}_{IJ}) = \theta_{statistic} \quad (7)$$

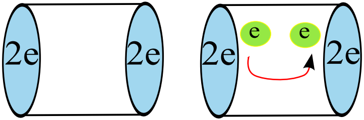
To *condense*: $(h_{IJ} - \bar{h}_{IJ}) = 0 \Rightarrow \theta_{statistic} = 0 \Rightarrow$
nonchiral bosonic electrons$(L\text{-}R=0)$

Boundary GSD - Example 1: Z_2 toric code.

Bulk anyons: $1, e, m, e+m=\varepsilon$.

$2e$ boundary type on two sides of a cylinder:

$\boxed{\text{GSD} = 2}$

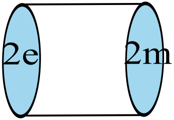


$2m$ boundary type on two sides of a cylinder:

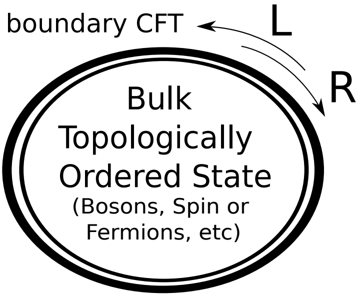
$\boxed{\text{GSD} = 2}$, too.

$2e$, $2m$ boundary type on each side of a cylinder.

$\boxed{\text{GSD} = 1}$



Boundary GSD - Physical Notions: Condensation of non-chiral bosons on the edge define the **boundary types**.



$$\text{correlators:}\langle \phi_I^\dagger(x, i\tau)\phi_J(x', i\tau')\rangle = (z-z')^{-h_{IJ}}(\bar{z}-\bar{z}')^{-\bar{h}_{IJ}} \quad (8)$$

$$\pi(h_{IJ} - \bar{h}_{IJ}) = \theta_{statistic} \quad (9)$$

To *condense*: $(h_{IJ} - \bar{h}_{IJ}) = 0 \Rightarrow \theta_{statistic} = 0 \Rightarrow$ *nonchiral bosonic electrons*(L-R=0)

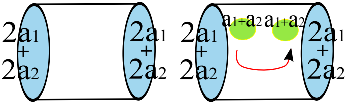
Boundary GSD - Example 2: Z_2 doubled semions.

Bulk anyons: $1, a_1, a_2, a_1 + a_2 = a_1 - a_2$.

However, $2a_1 + 2a_2$ and $2a_1 - 2a_2$ are different boundary types due to $\theta_{statistic} \neq 0$ (nonchiral).

$2a_1 + 2a_2$ boundary type on two sides of a cylinder:

$\boxed{\text{GSD} = 2}$

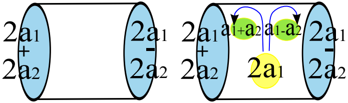


$2a_1 - 2a_2$ boundary type on two sides of a cylinder:

$\boxed{\text{GSD} = 2}$, too.

$2a_1 + 2a_2$, $2a_1 - 2a_2$ boundary type on each side of a cylinder.

$\boxed{\text{GSD} = 2!}$



Beyond Fusion Algebra:

Z_k gauge theory(K_{Z_k}) v.s. non-chiral $\mathrm{U}(1)_k \times \mathrm{U}(1)_k$FQH($K_{\mathrm{diag},k}$).
Z_2 toric code v.s. Z_2 doubled semions model.

Boundary GSD on a **cylinder**, for boundary types are different on two sides:

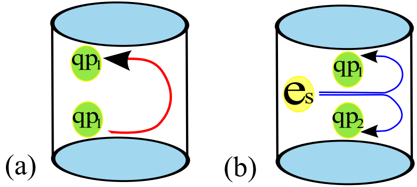
Z_2 toric code has **GSD=1**(Z_2 spin liquids, Z_2 gauge theory).

Z_2 doubled semions has **GSD=2**!

Remarkably these twos have the same **bulk GSD=4** on a **torus**, and the same fusion algebra!

Beyond Fusion Algebra:

Z_k gauge theory(K_{Z_k}) v.s. non-chiral $\text{U}(1)_k \times \text{U}(1)_k$FQH($K_{\text{diag},k}$).
Z_2 toric code v.s. Z_2 doubled semions model.



Boundary GSD on a cylinder, for boundary types are different on two sides:
Z_2 toric code(Z_2 spin liquids, gauge theory) has $\text{GSD} = 1$.
Z_2 doubled semions has $\text{GSD} = 2$.

Boundary GSD - In terms of ℓ in gapping term $\sum_a g_a \cos(\ell_{a,I} \cdot \Phi_I)$

Boundary Gapping Rules:

(**1**) Zeo self or mutual quantum fluctuations.
Zero A-B phase. Local and Bosonic - Null and mutual null.

(**2**) 'Physical' excitation: $\ell_a \in$ excitations of electron degree of freedom since it lives on the 'physical' boundary.

(**3**) Completeness of a set of condensed electrons.

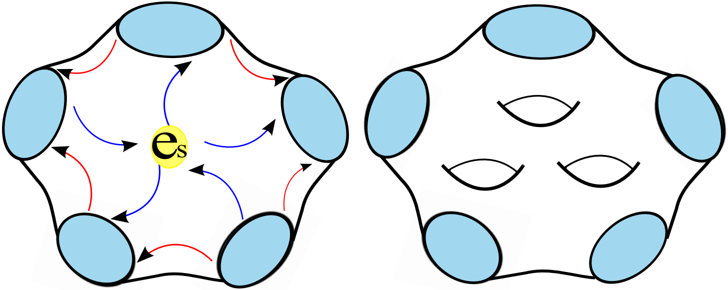
(**4**) The system is non-chiral, and $\sqrt{|\det K|} \in \mathbb{N}$.

(**5**) Total neutrality: Net charges of bulk and boundary balance to zero.

$$\boxed{\mathrm{GSD} = \text{the number of ways to transport fractionalized anyons}} \tag{10}$$

ℓ^∂: *condensed electrons* , ℓ^∂_{qp} : *compatible anyons*

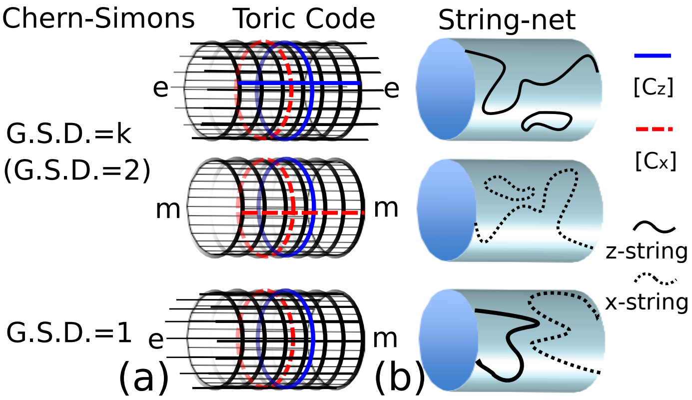
Boundary GSD -



ℓ^∂: *condensed electrons* , ℓ^∂_{qp} : *compatible anyons*

GSD = the number of ways to transport fractionalized anyons

Kitaev Toric Code and Levin-Wen string-net



$$H_0 = -\sum_v A_v - \sum_p B_p$$

S. B. Bravyi, A. Y. Kitaev, arXiv: quant-ph/9811052

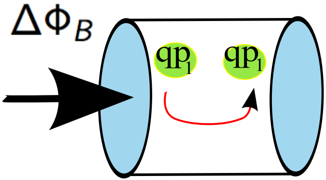
Use Flux Insertion arguments to distinguish boundary types:

$$q_I \Delta\Phi_B/(\frac{h}{e}) = \Delta P_{\phi,I} \qquad (11)$$

Num of unit flux insertion = Num of unit anyon transportation

for same boundary types on twos sides of a cylinder.

What dynamical effects can be detected for different boundary types? A method to distinguish boundary types. $\Delta\Phi_B$



Take-Home Messages(arXiv:1212.4863):

(1). We **introduce** the notion of **boundary GSD**, which depends on boundary types. Beyond Bulk-Edge Correspondence.

(2). For gappable non-chiral states, we provide another definition of Trivial Order or Symmetric Protect Topological Order: The boundary GSD on a cylinder must be 1 $\Leftrightarrow$ Abelian K matrix Chern-Simons theory(C-S) with $|\det K| = 1$. For Intrinsic Topological Order: boundary GSD on a cylinder ≥ 1

(3) Distinguish Z_2 toric code v.s. Z_2 doubled semions model by measuring boundary GSD on a cylinder.

Open questions(arXiv:1212.4863):

(1). Boundary GSD for non-Abelian topological order.

(2). Use Boundary GSD to classify intrinsic topological order or symmetric enriched topological order.

Thank you for your attention. arXiv:1212.4863

Contact: juven@mit.edu for discussions.

Related works:

arXiv:1205.3156; Theory and classification of interacting integer topological phases in two dimensions: A Chern-Simons approach; Yuan-Ming Lu, Ashvin Vishwanath

arXiv:1301.7355; Protected edge modes without symmetry; Michael Levin

arXiv:1302.2634; Classification and Properties of Symmetry Enriched Topological Phases: A Chern-Simons approach with applications to Z2 spin liquids; Yuan-Ming Lu, Ashvin Vishwanath

arXiv:1302.2951; A K matrix Construction of Symmetry Enriched Phases of Matter; Ling-Yan Hung, Yidun Wan

arXiv:1302.4803: Topological Response Theory of Abelian Symmetry-Protected Topological Phases in Two Dimensions Meng Cheng, Zheng-Cheng Gu, etc.