Multiple Equilibria Regulation Model in Cellular Automata Topology

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"Human beings, viewed as behaving systems, are quite simple. The apparent complexity of our behavior over time, is largely a reflection of the complexity of the environment in which we found ourselves"

Introduction
An important survival skill for humans is an ability to predict future events and to react correspondingly. In many real life situations such as social interactions, the anticipation of our own and others behavior is vital. According to the statistical models employed in psychological data analyses, such as multiple regression, prediction is possible if the appropriate predictors can be determined. Following this rationale, prediction error depends on the accuracy with which the predictors and the predicted behavior is measured, so it should be rather time independent. Therefore, our ability to predict future events should depend almost entirely on measurement error. Positivism as a place to start has exercised enormous influence over sociology's understanding of its explanatory task (Halfpenny, 1982). Certain limits seem to be placed over the capacity of socio-behavioral sciences to accumulate knowledge in the traditional sense (Gergen, 1973, 1982). Patterns of human conduct are subject to continuous alteration across time. Therefore, many complex environmental systems are not as stable as linear statistical models might suggest. Complexity seems to be ubiquitous in every social or individual action. It occurs everywhere and, it's a common statement that we have not agreed-upon a general scientific definition of complexity, probably because it manifests itself in many different ways. Complexity is also pervasive, so it is necessary to have a theoretical basis for its understanding.

In our former work (Katerelos & Koulouris, submitted), we presented a new model of opinion dynamics called Multiple Equilibria Regulation (MER) Model, in which we show that, under certain parameters' setting, the system becomes unpredictable.

Cellular Automata Topology applied in Multiple Equilibria Regulation Model
In order to approach empirical reality, we introduce a certain topology: we suppose that unlimited communication between agents, in real life, is being compromised due to a proximity-locality criterion. This means that two agents, even if they find themselves having similar opinions (within the bound of confidence $\varepsilon$), they do not exchange opinions if the criterion of "proximity" is not fulfilled. As mentioned before, one weakness$^1$ of MER model is that every agent is potentially in position to communicate with every other agent if and only if, their opinions differ in a degree less or equal than the bound of confidence. This functional postulate assumes that there is not any geographical (or any other kind of) obstacle excluding the magnitude of difference $\varepsilon$. Nevertheless, in real life, two (or more) individuals may never communicate just because they will never meet each other. Consequently, in a more realistic scope, we must consider first a topology of the agents in question and then take into account their opinions. The idea is not new: applying a topology in form of cellular automata or neural networks in a set of "entities" usually signifies that we impose one more specific rule of interaction between them. Thus, one can hypothesise that (in simple terms because we do not refer in restrained groups of experts but rather in large groups of individuals or entire "artificial societies") a geographical pattern of interaction between agents must be taken into account.

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$^1$ Inherited by BC Model…
The main feature of CA is locality. This means that all interactions take place only within well-defined spatial neighborhoods. Low dimensional, especially two-dimensional, CA is believed to be a promising modeling approach for understanding social dynamics (Hegselmann & Flache, 1998). All agents (N=100) are posed on a different cell of a square cellular automaton 10x10. Each agent is assigned a pair of integers $(i,j)$ that denote the coordinates of its position in the Cellular Automaton.

In figure 1, as an example, one can see the initial topology of opinion 1 situated on a cellular automaton (CA) with dimensions 10x10. Each color represents a different zone of values. Concerning the color coding, we follow an approximate pattern: we assign pink color to opinions which belong to the interval $[0,0.1]$, red to the interval $[0.1,0.2]$ and we continue the same procedure with orange, yellow, green, blue, and finally, black for opinions near 1. We use light and dark tones of these colors whenever is necessary to distinguish opinions with slight differences. By making these figures, we intend to show group formation (the degree of "self-organization") on the cellular automaton for a given iteration in a more apparent way. In figure 1, we note that there is a mosaic of colors distributed randomly on the CA: this means that there isn't any formed group, as we expected, due to random initial opinion profile.

![Figure 1: Initial opinion profile (for opinion 1) on a 10x10 cellular automaton (the same initial opinion profile will be used for all simulations in this paper)](image)

Each agent is aware of the opinions of the other agents that lie on his neighborhood (a Moore Neighborhood 3x3). Every agent $(i,j)$ is aware of both the opinions of his neighbors. For opinion 1, agent $(i,j)$ is influenced by his neighbors that their opinion 1 differs from his own less or equal the threshold distance $\varepsilon$. He calculates the average of these opinions 1. The same procedure is followed concerning opinion 2.

Although, for all simulations we have done up to now, the system reaches a final steady state (either it is a fixed point or periodic, Katerelos & Koulouris, submitted) and thus remains bounded (no opinion escapes to infinity), after iterating, some opinions become bigger than 1 or lesser than 0. In order to keep opinions into the original interval $[0,1]$ and, at the same time, not to change the dynamical behavior of the system we apply a procedure we call “rescale” (see Katerelos & Koulouris, submitted). In figures 2 and 3, one can see opinions 1 and 2 respectively for 10,000 iterations and in figures 4 and 5 the opinion profiles of the agents at the 10,000th iteration, posed on a cellular automaton.
Figure 2: MER Model, Moore Neighbourhood 3x3, N=100, $\varepsilon = 0.1$, $\psi = 1$, Opinion 1, Rescaled, iterations 1-10,000.

Figure 3: MER Model, Moore Neighbourhood 3x3, N=100, $\varepsilon = 0.1$, $\psi = 1$, Opinion 2, Rescaled, iterations 1-10,000.

Figure 4: MER Model, Iteration 10,000, opinion 1 on a 10x10 cellular automaton

Figure 5: MER Model, Iteration 10,000 opinion 2 on a 10x10 cellular automaton
In figures 4 and 5, we see that in both opinions a group formation is taking place. This means that the system exhibits self-organization. Since the system involves in a high dimensional space ($\mathbb{R}^{200}$), we run the simulation for 300,000 iterations. In figure 6, the system seems to stabilize into a rest state but, if we look closely (figure 7, a detail of figure 6), we can see that the dynamic behavior of the system is still strong and far from equilibrium. We also note that the agents have formed 7 groups for opinion 1 (figure 8) and 5 groups for opinion 2 (Figure 9).

\[ \text{Figure 6: } N=100, \varepsilon = 0.1, \psi = 1, \text{Opinion 1, Iterations 299,000-300,000} \]

\[ \text{Figure 7: } N=100, \varepsilon = 0.1, \psi = 1, \text{Opinion 1, Iterations 299,950-300,000} \]

\[ \text{Figure 8: MER Model, Iteration 300,000, opinion 1 on a cellular automaton} \]

\[ \text{Figure 9: MER Model, Iteration 300,000, opinion 2 on a cellular automaton} \]
Since the system does not attain equilibrium, we cannot examine it by means of linear statistical modeling.

**Sensitivity to Initial Conditions**
In this section we are going to examine if the system derived from MER Model exhibits Sensitivity to Initial Conditions. The phenomenon of sensitivity means that even the smallest "error" in the initial conditions will be magnified after a number of iterations.

In order to test this we do the following: we run the simulation for the MER Model that we can see in figures 2 and 3 (we call this simulation 1) with the same parameter values \((\varepsilon = 0.1, \psi = 1, 100 \text{ agents on a } 10\times10 \text{ cellular automaton, a } 3\times3 \text{ Moore Neighborhood})\) and with "almost" the same initial profile. This means that all agents have exactly the same initial opinion 1 and 2 except agent \((1,1)\) whose opinion 1 in simulation 1 is 0.7055475 and in simulation 2 is 0.7055475001. As we can see in figure 10, in simulation 2 this slight difference of \(10^{-10}\) is magnified and after about 200 iterations it becomes as large as the opinions themselves.

![Figure 10: Opinion 1 of agent 1 for two slightly different initial values \((10^{-10})\).](image)

The blue line is the trajectory of agent \((1,1)\) in simulation 1 and the red line in simulation 2 (opinion 1). We notice that the slightest perturbation in initial conditions causes a completely different outcome. The difference of their opinion in these two simulations is given in figure 11. The same is true for all other agents.

![Figure 11: Differences of agent \((1,1)\) in simulations 1 and 2 (opinion 1)](image)

Sensitivity to initial conditions is not limited to one agent who experienced the slight difference. As we know in such a system, each component of the system is highly interrelated with all others. This means that if we shift slightly one of them, all the others will be affected. For example, as presented on figure 12, the trajectories of agent \((10,10)\) (who is posed on the down right cell of the cellular automaton, the most "distant" agent), seems to be completely different according to our intervention in initial opinion 1 of agent \((1,1)\).
Figure 12: A minimal change on Agent 1 signifies a major change of Agent (10,10) - “Personal” history of Agent (10,10) according to two slightly different (in Agent (1,1), opinion 1) versions of the same model. Agent (10,10) had exactly the same initial opinion 1 and 2 in both simulations but we changed slightly the initial opinion of Agent (1,1). Table 1 shows agent (10,10)’s opinion 1 for a number of iterations. In the first 6 iterations, the perturbation of the system caused by the change in the initial opinion 1 of agent (1,1) has not yet reached agent (10,10). But, when this “disturbance” comes at iteration 7, is magnified continuously and becomes as big as the opinions themselves. This is a strong indication of sensitivity to initial conditions and unpredictability of the whole system.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Agent(10,10), opinion 1, simulation 1</th>
<th>Agent(10,10), opinion 1, simulation 2</th>
<th>Opinion’s Difference in the simulations</th>
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<tbody>
<tr>
<td>0</td>
<td>0.06091624</td>
<td>0.06091624</td>
<td>0</td>
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<tr>
<td>1</td>
<td>0.0711072</td>
<td>0.0711072</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
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<td>0.08664885</td>
<td>0</td>
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<td>3</td>
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<td>0.117523374</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
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<td>0.13505326</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.160337596</td>
<td>0.1603376</td>
<td>0</td>
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<td>0</td>
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<td>-3.3417E-07</td>
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<tr>
<td>5422</td>
<td>0.148271223</td>
<td>0.39029671</td>
<td>-0.24202549</td>
</tr>
</tbody>
</table>

Table 1: Differentiated opinions of agent (10,10) according to two simulations with slightly different initial opinion of agent (1,1).

This demonstration indicates clearly that in an “empirical” way our system seems to be unpredictable.

Computation of Lyapunov Exponent
In order to quantify this error propagation and examine if the MER model exhibits strong sensitivity to initial conditions and thus is unpredictable, we compute the Largest Lyapunov Exponent (from now on we will call it Lyapunov Exponent) for the three models (for the parameters values used in the previous paragraphs). In figure 14 we give the running average of the Lyapunov Exponent for 300,000 iterations for our Model. We note that the value of the running average of the Lyapunov Exponent converges to
the number 0.11985 and is clearly positive. This means that a tiny mistake in the initial conditions will get bigger $e^{0.11985} = 1.13$ times on the average for each iteration.

![Figure 13: Running average of the Lyapunov Exponent of MER Model + CA.](image)

The system is deterministic and exhibits sensitive dependence on initial conditions, i.e. it is unpredictable and thus chaotic.

**Discussion**

Our system is complex, self-organizing and disorganizing with emerging dynamics. Cellular Automata contribution has been proven crucial, since the introduced topology converts the behavior of our system in a noteworthy way: from transient chaos to "pure" chaos. This is to say that the system is not only unpredictable on the long run\(^2\) but, in addition, it will never rest in a final steady state. Always on the move, "snapshots" seem meaningless, since they can not capture the dynamics of the system. The initial position of the agents is extremely important in a precision which makes the applied errors useless in socio-psychological view: it is important however to say that such microscopic errors in opinion assessment can magnify themselves in a disproportional degree. The sensitivity in initial conditions is rather a characteristic of the system itself than a characteristic of the applied measurement tool. Thus, a "public opinion" approach has every reason to be unstable in such manner that many experts accused the concept of being useless because of its instability and lack of prediction...

**Prediction and methodology in the social sciences**

« Savoir pour prévoir et prévoir pour pouvoir »

*Auguste Compte*

No doubt, social sciences were meant to be predictive sciences. This was a key component of theirs constitution as sciences, in contrast to religious mythology and metaphysics (Aldridge, 1999). According to Wright Mills, prediction had a pre-eminent position in the social sciences even as late as the 1950s. He does not hesitate to claim that "the purpose of social science is the prediction and control of human behaviour" (Mills, 1970, p.127). The positivist vision of social sciences is now characterised as a rather "naïve" project: we all know that there is profound problems about prediction (for example see Fiske & Sweder 1986).

Although prediction should not been seen as something confined to the caste of scientists\(^3\), as Giddens claims (1979, p.248), "orthodox" social scientists had an "oversimple revelatory model of social science" in which they produced startling insights which supposedly left common people in awe. However, the

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\(^2\) prediction is only possible for a short predictability horizon

\(^3\) As we have already mentioned in introduction, prediction is a common practice which is necessary to survival in everyday activity. In some cases, naïve subjects can predict chaotic number series better than linear statistical models (Health, 2002).
debate about prediction in social sciences is, more or less, based on their perceived "utility" in society: if they cannot predict, what are they good for? On the other hand, if they can do valuable predictions, social sciences can become some kind of common foreteller useful as much to individuals as to societies. Let us not forget that during the 1950s and 1960s, especially psychology but all other social sciences also, were perceived like "intimidating magic" which should explain human social behaviour in terms of predicting and controlling even in an "unconscious" way. What is demanded is unmistakably a positivistic conception of social sciences: implicit in this conception is the assumption that "scientific" knowledge of social conditions can contribute, positively, to interventions that assist the course of human development. It is therefore a conception of social science that entails a particular relationship between the acquisition of knowledge and its potential use. Hence Comte's famous pronouncement that 'from science comes prevision and from prevision comes control' can be argued in the sense that the outcome of social sciences had to be, in the best case, a political argument. Therefore, the central task of social sciences is to 'discover' the knowledge required by politicians to enact the necessary 'rational' interventions. However, the fact that social science and policy are not simply or contingently related can be shown by looking at the nature of the relationship between causal explanation and prediction. According to Fay (1975), causal explanation necessarily implies prediction and, by extension, the desire to control, because they share a 'structural identity'. For Mills (1970), accepting prediction as the central element for social sciences coincides with an anti-democratic (bureaucratic) ethos. The scientific innovation would be mutilated in a modern "cline of Procrustes" where a "grand theory with predictive power" should grant ideological legitimacy to political choices while abstracted empiricism supplies them with technology of social control. Even so, recognizing the limits to prediction and abandoning prediction as THE grand project, does not mean that no predictions should be attempted (Aldridge 1999).

Chaos: a Micro-Macro contrast?

Much of the cognitive social psychology has been concerned to reveal the "rules of social thinking". For some social psychologists this involves the attempt to discover the rules that should be followed, if thinking is to proceed successfully. Other social psychologists are not so much concerned with the rules, which ought to be followed, but with those which actually do follow (Guimeli, 1999; Papastamou 2001). It will be suggested that there is something missing in both these cognitive accounts of thinking. It is not that social psychologists are relying on theories which make erroneous predictions, nor that their experimental results have arisen from faulty methodological procedures. What is missing is a feel of the contentious and dynamic nature of social thinking. Although, both approaches accept conventional theory testing in statistical terms, they both fail to present an understandable aspect of social action in a combined micro and macro way.

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4 The subliminal influence (advertisement) was recognized like a threat to common wealth (see rigorous legislation). A dreadful perspective indeed: someone can influence me to do things without even knowing it. Nevertheless, the experiments conducted under this title were strongly questioned in terms of replicability.
References:


