A new approach to solve multi-objective multi-choice multi-item Atanassov’s intuitionistic fuzzy transportation problem using chance operator

Dipankar Chakrabortya, Dipak Kumar Jana, and Tapan Kumar Roy
Department of Mathematics, Heritage Institute of Technology, East Kolkata Township, Chowbaga Road, Anandapur, Kolkata, West Bengal, India
Department of Applied Science, Haldia Institute of Technology, Haldia, Purba Midnapur, West Bengal, India
Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah, West Bengal, India

Abstract. In some practical situations the decision maker is interested in setting multi aspiration levels for objectives that may not be expressed in a specific manner. So in this paper, Atanassov’s intuitionistic fuzzy transportation problem with multi-item, multi-objective function assuming multiple choices is considered. We have modeled multi-objective multi-choice multi-item Atanassov’s intuitionistic fuzzy transportation problem (MMMFITP), and its several special cases. Possibility, necessity and credibility measures for Atanassov’s intuitionistic fuzzy numbers for the first time have been developed here. Solution methodology of those models using chance operator has been discussed. A real life example is presented to illustrate proposed models numerically and the results are compared. The optimal results are obtained by using three different soft computing techniques (i) Interactive satisfied method, (ii) Global criteria method and (iii) Goal programming method.

Keywords: Atanassov’s intuitionistic fuzzy, chance operator, possibility, necessity, credibility, multi-objective, multi-choice, multi-item, transportation problem

1. Introduction

Transportation problem is a well known problem of operations research that can be formulated and solved as a linear programme. The classical transportation problem (TP) refers to a special class of linear programming problems. Transportation problem was first introduced by Hitchcock [27]. Several researchers have shown their interest in transportation problem in fuzzy environment (cf. Pramanik et al. [41, 42]). Chanas et al. [20] analysed the transportation problem with fuzzy supply values of the deliverers and with fuzzy demand values of the receivers. Ojha et al. [15] proposed transportation problem with fuzzy-stochastic cost. Tang and Gong [17] focused on hybrid two-stage transportation and batch scheduling problem. Chanas and Kuchta [18] has developed the the concept of the optimal solution of the transportation problem with fuzzy cost coefficients.

Recently multi objective, multi-choice and multi-item optimization have gained more importance for decision making problems in many areas like industry, health care, transportation etc. Biswal and Acharya [5]

Atanassov’s intuitionistic fuzzy set (A-IFS) is one of generalization of fuzzy set theory. A-IFS was first introduced by Atanassov [21]. Fuzzy sets is totally characterized by the membership function but A-IFS is described by not only membership function but also a non-membership function so that the sum of both values lies between zero and one [22]. Many researchers have shown their interest in the study of Atanassov’s intuitionistic fuzzy sets/numbers and its application in optimization problems [1], [2], [3], [4], [14], [24], [26], [31], [37]. Mahapatra and Roy [19] have discussed about Atanassov’s intuitionistic Fuzzy Number and its application in transportation problems under fuzziness. Two-warehouse supply-chain model under possibility/necessity/credibility measures have been developed by Das et al. [35]. Panda et al. [34] proposed imprecise constraints single period inventory model with imperfect production and stochastic demand. Measurement theory has been used by Ban [30] to developed Atanassov’s intuitionistic fuzzy-valued possibility and necessity measures. Table-1 represents the summary of related literature for transportation problem.

The rest of this paper is organized as follows. In section 2, we recall some preliminary knowledge about Atanassov’s intuitionistic fuzzy. Section 3 provides possibility, necessity and credibility measures in Atanassov’s intuitionistic fuzzy environment. In section 4, we have proposed MMMIFTP and its several special cases have been discussed in this section. The solution methodology of the proposed models using chance operator has been discussed in section 5. In section 6, a real life example is solved using three different soft computing techniques (i) Interactive satisfied method, (ii) Global criteria method and (iii) Goal programming method. Results of different models and methods are also discussed and compared in this section. Section 7 summarizes the paper as well as some directions for future research.

2. Preliminaries

Definition 2.1. Atanassov’s Intuitionistic Fuzzy Set (A-IFS) [21], [22]). Let \( E \) be a given set and let \( A \subseteq E \) be a set. An A-IFS \( A^* \in E \) is given by \( A^* = \{< x, \mu_A(x), \nu_A(x) > \mid x \in E \} \) where \( \mu_A : E \rightarrow [0, 1] \) and \( \nu_A : E \rightarrow [0, 1] \) define the degree of membership and the degree of non-membership of the element \( x \in E \) to \( A \subseteq E \) satisfy the condition \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \).

Definition 2.2. Atanassov’s Intuitionistic Fuzzy Number (A-IFN) [7]: An Atanassov’s intuitionistic fuzzy subset \( A^* = \{< x, \mu_A(x), \nu_A(x) > \mid x \in A \} \) of the real line \( R \) is called A-IFN, if the following holds

(i) there exist \( m \in R \), \( \mu_A^*(m) = 1 \), \( \nu_A^*(m) = 0 \).
respectively and 

\[ f(x) = \begin{cases} 
1, & x = m; \\
0, & \text{otherwise.} 
\end{cases} \]

Non-membership function

\[ \nu_{A^I}(x) = \begin{cases} 
0, & f_1(x) + f_2(x) > 4; \\
0, & h_1(x) + h_2(x) > 4; \\
1, & \text{otherwise.} 
\end{cases} \]

Here \( f_1(x) \) and \( h_1(x) \) are strictly increasing and decreasing function in \([m-a, m]\) and \([m, m+b]\) respectively, and \( f_2(x) \) and \( h_2(x) \) are strictly decreasing and increasing function in \([m-a, m]\) and \([m, m+b]\) respectively, where \( m \) is the mean value of \( A^I \), \( a \) and \( b \) are called the left and right spreads of membership function \( \mu_{A^I}(x) \) respectively. \( a \) and \( b \) are called the left and right spreads of non-membership function \( \nu_{A^I}(x) \) respectively.

**Definition 2.3. Trapezoidal Atanassov’s Intuitionistic Fuzzy Number (TIFN):** Let \( a_1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \). A TIFN \( A^I \) in \( \mathbb{R} \) written as \((a_1, a_2, a_3, a_4, a'_1, a'_2, a'_3, a'_4)\) has membership function (c.f. fig-1)

\[ \mu_{A^I}(x) = \begin{cases} 
\frac{x-a_1}{a_1-a_2}, & a_1 \leq x \leq a_2; \\
\frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3; \\
\frac{x-a_3}{a_4-a_3}, & a_3 \leq x \leq a_4; \\
0, & \text{otherwise.} 
\end{cases} \]

and non-membership function

\[ \nu_{A^I}(x) = \begin{cases} 
\frac{x-a_1}{a_4-a_1}, & a_1 \leq x \leq a_2; \\
\frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3; \\
\frac{x-a_3}{a_4-a_3}, & a_3 \leq x \leq a_4; \\
1, & \text{otherwise.} 
\end{cases} \]

**Definition 2.4. Triangular Atanassov’s Intuitionistic Fuzzy Number (TrIFN):** Let \( a_1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \). A TrIFN \( A^I \) in \( \mathbb{R} \) written as \((a_1, a_2, a_3, a_4)\) has membership function (c.f. fig-2)
Definition 2.5.

A positive TrIFN $\tilde{A}$ is denoted as $\tilde{A} = (a_1, a_2, a_3)$ where $a_i > 0$.

Definition 2.6. [36] Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two TrIFN then

(i) $\tilde{A} \otimes \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
(ii) $k\tilde{A} = (ka_1, ka_2, ka_3)$ if $k \geq 0$
(iii) $\tilde{A} \ominus \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
(iv) $\tilde{A} \oplus \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$
(v) $\tilde{A} \oplus \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$

Fig. 2. Membership and non-membership function of TrFN.

3. Possibility, Necessity and Credibility

Measures of Atanassov’s Intuitionistic Fuzzy Number

Definition 3.1. Let $\tilde{A}$ and $\tilde{B}$ be two A-IFN with membership function $\mu_{\tilde{A}}, \mu_{\tilde{B}}$ and non-membership function $\nu_{\tilde{A}}, \nu_{\tilde{B}}$ respectively and $R$ is the set of real numbers. Then

$\text{Pos}_{\tilde{A}}(\tilde{A} \ast \tilde{B}) = \sup \{\min(\mu_{\tilde{A}}, \mu_{\tilde{B}}), x, y \in R, x \ast y\}$
$\text{Pos}_{\tilde{A}}(\tilde{A} \ast \tilde{B}) = \sup \{\min(\nu_{\tilde{A}}, \nu_{\tilde{B}}), x, y \in R, x \ast y\}$

and

$\text{Nec}_{\tilde{A}}(\tilde{A} \ast \tilde{B}) = \inf \{\max(\mu_{\tilde{A}}, \mu_{\tilde{B}}), x, y \in R, x \ast y\}$
$\text{Nec}_{\tilde{A}}(\tilde{A} \ast \tilde{B}) = \inf \{\max(\nu_{\tilde{A}}, \nu_{\tilde{B}}), x, y \in R, x \ast y\}$

where the abbreviation $\text{Pos}_{\tilde{A}}$, $\text{Pos}_{\tilde{B}}$, represents possibility of membership and non-membership function, and $\text{Nec}_{\tilde{A}}$, $\text{Nec}_{\tilde{B}}$ represents necessity of membership and non-membership function. $\ast$ is any of the relations $<, \leq, \geq, \leq, =$.

The dual relationship of possibility and necessity gives that

$\text{Nec}_{\tilde{A}}(\tilde{A} \ast \tilde{B}) = 1 - \text{Pos}_{\tilde{A}}(\tilde{A} \ast \tilde{B})$
$\text{Nec}_{\tilde{A}}(\tilde{A} \ast \tilde{B}) = 1 - \text{Pos}_{\tilde{A}}(\tilde{A} \ast \tilde{B})$

where $\tilde{A} \ast \tilde{B}$ represents complement of the event $\tilde{A} \ast \tilde{B}$.

Definition 3.2. Measure of $\tilde{A}$ Let $\tilde{A}$ be a A-IFN.

Then the fuzzy measure of $\tilde{A}$ for membership and non-membership function are

$\text{Me}_{\tilde{A}} = \lambda \text{Pos}_{\tilde{A}} + (1 - \lambda)\text{Nec}_{\tilde{A}}$
$\text{Me}_{\tilde{A}} = \lambda \text{Pos}_{\tilde{A}} + (1 - \lambda)\text{Nec}_{\tilde{A}}$

where $\lambda (0 \leq \lambda \leq 1)$ is the optimistic-pessimistic parameter to determine the combined attitude of a decision maker.

If $\lambda = 1$ then $\text{Me}_{\tilde{A}} = \text{Pos}_{\tilde{A}}, \text{Me}_{\tilde{B}} = \text{Pos}_{\tilde{B}}$.
If $\lambda = 0$ then $\text{Me}_{\tilde{A}} = \text{Nec}_{\tilde{A}}, \text{Me}_{\tilde{B}} = \text{Nec}_{\tilde{B}}$.
If $\lambda = 0.5$ then $\text{Me}_{\tilde{A}} = \text{Cr}_{\tilde{A}}, \text{Me}_{\tilde{B}} = \text{Cr}_{\tilde{B}}$, where $\text{Cr}$ is the credibility measure.
3.1. Measures of triangular Atanassov’s intuitionistic fuzzy number

Let $\tilde{A}^l = (a_1, a_2, a_3)(a_1', a_2', a_3')$ and $\tilde{B}^l = (b_1, b_2, b_3)(b_1', b_2', b_3')$ are two TrIFNs. From the definition-3 possibility of $(\tilde{A}^l \leq \tilde{B}^l)$ for membership and non-membership function (c.f. fig. 3 and 4) are as follows

\[
\text{Pos}_{\mu}(\tilde{A}^l \leq \tilde{B}^l) = \begin{cases} 
1, & a_2 \leq b_2; \\
\frac{a_2 - b_2}{a_3 - b_2 - a_1'} & a_2 > b_2, a_1 < b_1; \\
0, & a_1 \geq b_3.
\end{cases}
\]

\[
\text{Pos}_{\nu}(\tilde{A}^l \leq \tilde{B}^l) = \begin{cases} 
0, & a_2 \leq b_2; \\
\frac{b_3 - a_3}{b_2 - a_2 + b_1'} & a_2 > b_2, b_3 > a_1'; \\
1, & a_1 \geq b_1.
\end{cases}
\]

From the definition-3.1 possibility of $(\tilde{A}^l \geq \tilde{B}^l)$ for membership and non-membership function (c.f. fig. 5 and 6) are as follows

\[
\text{Pos}_{\mu}(\tilde{A}^l \geq \tilde{B}^l) = \begin{cases} 
1, & a_2 \geq b_2; \\
\frac{a_2 - b_2}{a_3 - b_2 - a_1} & a_2 > b_2, a_1 > b_1; \\
0, & a_1 \leq b_3.
\end{cases}
\]

\[
\text{Pos}_{\nu}(\tilde{A}^l \geq \tilde{B}^l) = \begin{cases} 
0, & a_2 \geq b_2; \\
\frac{b_3 - a_3}{b_2 - a_2 + b_1} & a_2 < b_2, a_3 > b_1'; \\
1, & a_3 \leq b_1.
\end{cases}
\]

Now by definition-3.1 necessity of $(\tilde{A}^l \leq \tilde{B}^l)$ are as follows

\[
\text{Nes}_{\mu}(\tilde{A}^l \leq \tilde{B}^l) = 1 - \text{Pos}_{\mu}(\tilde{A}^l > \tilde{B}^l) = \begin{cases} 
0, & b_1 \leq a_2; \\
\frac{b_2 - a_2}{a_3 - a_2 + b_1} & b_1 > a_2, b_2 < a_3; \\
1, & b_2 \geq a_3.
\end{cases}
\]
Now by definition 3.1 necessity of $\tilde{\mathbf{A}}^I \leq \tilde{\mathbf{B}}^I$ are as follows

$$N_{\text{E} \mathbf{N}}(\tilde{\mathbf{A}}^I \leq \tilde{\mathbf{B}}^I) = 1 - \text{Pr}_{\text{E} \mathbf{N}}(\tilde{\mathbf{A}}^I > \tilde{\mathbf{B}}^I)$$

$$= \left\{ \begin{array}{ll}
0, & b_2 \geq a_1; \\
\frac{a_1 - b_3}{a_2 - a_3 + b_3}, & b_2 < a_3, b_1 > a_2; \\
1, & a_2 \leq b_1.
\end{array} \right. \quad (4)$$

Now by definition 3.2 measure of $\tilde{\mathbf{A}}^I \leq \tilde{\mathbf{B}}^I$ are as follows

$$N_{\text{E} \mathbf{N}}(\tilde{\mathbf{A}}^I \geq \tilde{\mathbf{B}}^I) = 1 - \text{Pr}_{\text{E} \mathbf{N}}(\tilde{\mathbf{A}}^I < \tilde{\mathbf{B}}^I)$$

$$= \left\{ \begin{array}{ll}
0, & b_3 \geq a_2; \\
\frac{a_2 - b_3}{a_3 - a_2 + b_3}, & a_2 > b_3, b_2 > a_1; \\
1, & b_2 \leq a_1.
\end{array} \right.$$

$$N_{\text{E} \mathbf{N}}(\tilde{\mathbf{A}}^I \geq \tilde{\mathbf{B}}^I) = 1 - \text{Pr}_{\text{E} \mathbf{N}}(\tilde{\mathbf{A}}^I \leq \tilde{\mathbf{B}}^I)$$

$$= \left\{ \begin{array}{ll}
0, & b_2 \leq a_2; \\
\frac{b_2 - a_2}{a_2 - a_3 + b_3}, & b_2 > a_3, b_2 > a_1; \\
1, & a_2 \leq b_1.
\end{array} \right.$$

Now by definition 3.2 measure of $\tilde{\mathbf{A}}^I \leq \tilde{\mathbf{B}}^I$ are as follows

$$M_{\text{E} \mathbf{N}}(\tilde{\mathbf{A}}^I \leq \tilde{\mathbf{B}}^I) = \lambda \text{Pr}_{\text{E} \mathbf{N}}(\tilde{\mathbf{A}}^I \leq \tilde{\mathbf{B}}^I) + (1 - \lambda)N_{\text{E} \mathbf{N}}(\tilde{\mathbf{A}}^I \leq \tilde{\mathbf{B}}^I)$$

$$= \left\{ \begin{array}{ll}
0, & b_1 \leq a_1; \\
\lambda \frac{b_1 - a_2}{a_2 - a_3 + b_3}, & a_2 > b_1, a_1 < b_1; \\
(1 - \lambda) \frac{b_1 - a_2}{a_2 - a_3 + b_3}, & b_1 > a_2, b_2 < a_1; \\
1, & b_2 \geq a_1.
\end{array} \right.$$
Note: \( Pos_{\alpha}(\tilde{A}^l \leq \tilde{B}^l) \geq \alpha \) and \( Pos_{\beta}(\tilde{A}^l \leq \tilde{B}^l) \leq \beta \)
and

\[ Cr_1(A^1) \leq \beta \]

\[ \Rightarrow 2a_2 - 2b_2 + b_1 - a_1 \]

\[ \leq \beta, \]

\[ \frac{a_1 - b_2}{2(a_2 - a_1 - b_1 + b_2)} \leq \beta. \]

Note:

\[ Cr_1(A^1) \leq x, Cr_2(A^1) \leq \beta \]

\[ \Rightarrow \frac{x - a_1}{2(a_2 - a_1)} \geq x - a_2 + x \geq \frac{x - a_1}{2(a_2 - a_1)} \]

\[ and \frac{a_1 - x - a_2}{2(a_2 - a_1)} \leq \beta, \frac{a_1 - x - a_2}{2(a_2 - a_1)} \leq \beta. \]

4. Atanassov’s intuitionistic fuzzy transportation problem

Let \( p = (1, 2, \ldots, T) \) items are to be transported from \( M \) origins (or sources) \( a^p_I \) \((i = 1, 2, \ldots, M)\), \( N \) destinations (i.e. demands) \( b^p_J \) \((j = 1, 2, \ldots, N)\). Let \( a^p_I \) be the product available at \( i \)th origin for items \( p = 1, 2, \ldots, T \), \( b^p_J \) be the demand at \( j \)th destination for items \( p = 1, 2, \ldots, T \). The variable \( x_{ij}^p \) represents the unknown quantity to be transported from origin \( a^p_I \) to destination \( b^p_J \) for item \( p = 1, 2, \ldots, T \).

Here \( a^p_I \) and \( b^p_J \) are Atanassov’s intuitionistic fuzzy numbers.

4.1. Model-1: Multi objective multi choice multi item Atanassov’s intuitionistic fuzzy transportation problem (MMMFPT)

We propose the mathematical model of \( t \times t \) items Atanassov’s intuitionistic fuzzy transportation problem as follows

\[ \begin{align*}
    \text{min } F_1 = \sum_{p=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{T} \left[ \alpha_i x_{ij}^p + \beta_j x_{ij}^p \right], \\
    t = 1, 2, \ldots, R \\
    \text{subject to } \sum_{j=1}^{T} x_{ij}^p = a^p_i, i = 1, 2, \ldots, M \\
    \text{and } \sum_{i=1}^{M} x_{ij}^p = b^p_j, j = 1, 2, \ldots, N \quad \text{for all } p = 1, 2, \ldots, T \\
    x_{ij}^p \geq 0 \quad \forall i, j, p.
\end{align*} \]

4.2. Model-2: Single objective multi choice single item Atanassov’s intuitionistic fuzzy transportation problem (SMIFTP)

We propose the mathematical model of single objective \((t = 1), k(>1)\) choices and single item \((p = 1)\) Atanassov’s intuitionistic fuzzy transportation problem as follows

\[ \begin{align*}
    \text{min } F = \sum_{p=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{T} \left[ \alpha_i x_{ij}^p + \beta_j x_{ij}^p \right] x_{ij}^p, \\
    i = 1, 2, \ldots, M \\
    \text{subject to } \sum_{j=1}^{T} x_{ij}^p \leq a^p_i, i = 1, 2, \ldots, M \quad \text{and } \forall p \\
    \sum_{i=1}^{M} x_{ij}^p \geq b^p_j, j = 1, 2, \ldots, N \quad \text{and } \forall p \\
    x_{ij}^p \geq 0 \quad \forall i, j, p.
\end{align*} \]

4.3. Model-3: Single objective multi choice multi item Atanassov’s intuitionistic fuzzy transportation problem (SMIMFTP)

We propose the mathematical model of single objective \((t = 1), k(>1)\) choices and \(p = 1, 2, \ldots, T\) items Atanassov’s intuitionistic fuzzy transportation problem as follows

\[ \begin{align*}
    \text{min } F = \sum_{p=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{T} \left[ \alpha_i x_{ij}^p + \beta_j x_{ij}^p \right] x_{ij}^p, \\
    i = 1, 2, \ldots, M \\
    \text{subject to } \sum_{j=1}^{T} x_{ij}^p \leq a^p_i, i = 1, 2, \ldots, M \quad \text{and } \forall p \\
    \sum_{i=1}^{M} x_{ij}^p \geq b^p_j, j = 1, 2, \ldots, N \quad \text{and } \forall p \\
    x_{ij}^p \geq 0 \quad \forall i, j, p.
\end{align*} \]
4.4. Model-4: Multi objective multi choice single item Atanassov’s intuitionistic fuzzy transportation problem (MMSIFTP)

We propose the mathematical model of \( t(t = 1, 2, \ldots, R) \) objectives, \( k(>1) \) choices and and single item \( (p = 1) \) Atanassov’s intuitionistic fuzzy transportation problem as follows

\[
\min F_4 = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij}^{(p)} x_{ij},
\]

subject to
\[
\sum_{i=1}^{M} x_{ij} \leq a_i^p \quad i = 1, 2, 3, \ldots, M
\]
\[
\sum_{j=1}^{N} x_{ij} \geq b_j^p \quad j = 1, 2, 3, \ldots, N
\]
\[
x_{ij} \geq 0 \quad \forall i, j.
\]

4.5. Model-5: Multi objective single choice multi item Atanassov’s intuitionistic fuzzy transportation problem (MSMIFTP)

We propose the mathematical model of \( t(t = 1, 2, \ldots, R) \) objectives, single choice \( (k = 1) \) and \( p(=1) \) items Atanassov’s intuitionistic fuzzy transportation problem as follows

\[
\min F_5 = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij}^{(p)} x_{ij},
\]

subject to
\[
\sum_{i=1}^{M} x_{ij} \leq a_i^p \quad i = 1, 2, 3, \ldots, M
\]
\[
\sum_{j=1}^{N} x_{ij} \geq b_j^p \quad j = 1, 2, 3, \ldots, N
\]
\[
x_{ij} \geq 0 \quad \forall i, j.
\]

4.6. Model-6: Single objective single choice single item Atanassov’s intuitionistic fuzzy transportation problem (SSSIFTP)

We propose the mathematical model of single objective, single choice \( (k = 1) \) and single item \( (p = 1) \) Atanassov’s intuitionistic fuzzy transportation problem as follows

\[
\min F_6 = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij}^{(p)} x_{ij},
\]

subject to
\[
\sum_{i=1}^{M} x_{ij} \leq a_i^p \quad i = 1, 2, 3, \ldots, M
\]
\[
\sum_{j=1}^{N} x_{ij} \geq b_j^p \quad j = 1, 2, 3, \ldots, N
\]
\[
x_{ij} \geq 0 \quad \forall i, j.
\]

4.7. Model-7: Single objective single choice multi item Atanassov’s intuitionistic fuzzy transportation problem (SSMIFTP)

We propose the mathematical model of single objective, single choice \( (k = 1) \) and \( p(=1) \) items Atanassov’s intuitionistic fuzzy transportation problem as follows

\[
\min F_7 = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij}^{(p)} x_{ij},
\]

subject to
\[
\sum_{i=1}^{M} x_{ij} \leq a_i^p \quad i = 1, 2, 3, \ldots, M \text{ and } \forall p
\]
\[
\sum_{j=1}^{N} x_{ij} \geq b_j^p \quad j = 1, 2, 3, \ldots, N \text{ and } \forall p
\]
\[
x_{ij}^{(p)} \geq 0 \quad \forall i, j, p.
\]

4.8. Model-8: Multi objective single choice single item Atanassov’s intuitionistic fuzzy transportation problem (MSSIFTP)

We propose the mathematical model of \( t(t = 1, 2, \ldots, R) \) objectives, single choice \( (k = 1) \) and and single item \( (p = 1) \) Atanassov’s intuitionistic fuzzy transportation problem as follows

\[
\min F_8 = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij}^{(p)} x_{ij},
\]

subject to
\[
\sum_{i=1}^{M} x_{ij} \leq a_i^p \quad i = 1, 2, 3, \ldots, M
\]
The chance operator is taken as possibility or necessity or credibility measures. We use chance operator to transform the Atanassov’s intuitionistic fuzzy transportation problem into crisp equivalent transportation problem.

5.1. Model-1 based on chance operator

Applying Chance operator on (7), model-1 transform to

\[
\begin{align*}
\text{Min} \left\{ f_1^{(1)} + f_2^{(1)} + f_3^{(2)} + f_4^{(2)} + f_5^{(R)} + f_6^{(R)} \right\} \\
\text{Subject to} \\
C_{\text{Ch}_1} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} \left[ \alpha_t x_{ij}^{(p)} + \beta_t x_{ij}^{(R)} \right] \leq \lambda_t \right\} \geq \alpha_t \\
C_{\text{Ch}_2} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} \left[ \alpha_t x_{ij}^{(p)} + \beta_t x_{ij}^{(R)} \right] \geq \lambda_t \right\} \leq \beta_t \\
C_{\text{Ch}_3} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} \left[ \alpha_t x_{ij}^{(p)} + \beta_t x_{ij}^{(R)} \right] \leq \lambda_t \right\} \leq \psi_t \\
C_{\text{Ch}_4} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} \left[ \alpha_t x_{ij}^{(p)} + \beta_t x_{ij}^{(R)} \right] \geq \lambda_t \right\} \leq \tau_t \\
C_{\text{Ch}_5} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} \left[ \alpha_t x_{ij}^{(R)} \right] \leq \eta_j \right\} \leq \eta_j \\
\end{align*}
\]

Now for the transformation of the objective function involving multi-choice cost parameter under imprecise environments, using [33] and lemma-1, the above model can transform to

\[
\begin{align*}
\text{Min} \left\{ f_1^{(1)} + f_2^{(1)} + f_3^{(2)} + f_4^{(2)} + f_5^{(R)} + f_6^{(R)} \right\} \\
\text{Subject to} \\
Z_t \leq f_1^{(1)} + f_2^{(1)}, \quad t = 1, 2, \ldots, R \\
C_{\text{Ch}_1} \left\{ \sum_{i=1}^{N} \sum_{j=1}^{M} \left[ \alpha_t x_{ij}^{(p)} \right] \leq \lambda_t \right\} \geq \lambda_t \\
C_{\text{Ch}_2} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} \left[ \alpha_t x_{ij}^{(p)} \right] \leq \psi_t \right\} \leq \psi_t \\
C_{\text{Ch}_3} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} \left[ \alpha_t x_{ij}^{(p)} \right] \geq \tau_t \right\} \leq \tau_t \\
C_{\text{Ch}_4} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} \left[ \alpha_t x_{ij}^{(R)} \right] \leq \eta_j \right\} \leq \eta_j \\
\end{align*}
\]

The abbreviation \( C_{\text{Ch}_1} \) and \( C_{\text{Ch}_2} \) represents chance operator for membership and non-membership function. \( \alpha_t, \beta_t, \lambda_t, \psi_t, \tau_t \) and \( \eta_j \) are the predetermined confidence levels such that \( 0 \leq \lambda_t + \psi_t \leq 1 \) for \( i = 1, 2, \ldots, M, 0 \leq \alpha_t + \beta_t \leq 1 \) \( (t = 1, 2, \ldots, R) \) and \( 0 \leq \tau_t + \eta_j \leq 1 \) for \( j = 1, 2, \ldots, N \).
6.1. Input data for model-1,2,3,4:

\[
\begin{bmatrix}
\tilde{a}_{11}^{(11)} & \tilde{a}_{12}^{(11)} & \tilde{a}_{13}^{(11)} & \tilde{a}_{14}^{(11)} \\
\tilde{a}_{21}^{(12)} & \tilde{a}_{22}^{(12)} & \tilde{a}_{23}^{(12)} & \tilde{a}_{24}^{(12)} \\
\tilde{a}_{31}^{(13)} & \tilde{a}_{32}^{(13)} & \tilde{a}_{33}^{(13)} & \tilde{a}_{34}^{(13)} \\
\tilde{a}_{41}^{(14)} & \tilde{a}_{42}^{(14)} & \tilde{a}_{43}^{(14)} & \tilde{a}_{44}^{(14)}
\end{bmatrix}
\begin{bmatrix}
(6, 7, 9, 5, 7, 11) (4, 5, 7)(3, 5, 8) (2, 3, 5)(1, 5, 3, 6) (1, 2, 3)(0, 5, 2, 4) \\
(4, 6, 8)(3, 6, 8) (3, 6, 8)(2, 6, 9) (1, 3, 6)(0, 5, 3, 6) (0, 1, 3)(0, 1, 4).
\end{bmatrix}
\]

\[
\text{Min}\ \left\{ Z_1, Z_2, \ldots, Z_8 \right\}
\]

Subject to

\[
(16) - (20)
\]

which is a multi objective crisp transportation problem and can be solved using Interactive satisfied method, Goal programming method and Global criteria method.

Similarly Model-2, 3, 4, 5, 6, 7 and 8 can be transformed into corresponding equivalent crisp model by above explain method.

6. A real life example

A fruit supplier company of West Bengal, India supply two types of fruits namely mango and orange from four source points namely Darjeeling, Siliguri and Malda of West Bengal, India by lorry or train to the three destinations situated at Kolkata, Ranchi, Durgapur and Kharagpur through twelve routes. The main aims of the proposed models are to minimize the transportation cost (first objective) and packaging cost (second objective), so as to maximize the profit against the market price at different market. Due to fluctuation of fuel price, road tax for different route, political issues, different type of packaging for different route, fluctuation of price packaging material the transportation cost and packaging cost in each route is not fixed. The transportation cost and packaging cost of carrying one unit (500 gm) of mango and orange from source to destination is treated as a multi objective multi choice multi item transportation problem. Here we have considered the parameters of transportation problem in Atanassov’s intuitionistic fuzzy environment. The input data are given as follows:

6.1. Input data for model-1,2,3,4:

Amount of items available at origin \([\tilde{a}_{ij}^{(mp)}]^{(1)}\):

\[
\begin{bmatrix}
(4, 6, 9)(2, 6, 10) (0, 5, 1, 3)(0, 1, 5) (8, 5, 10, 12)(8, 10, 14) \\
(5, 8, 9)(4, 8, 10) (0, 2, 4)(0, 2, 6) (6, 8, 10, 5, 8, 11)
\end{bmatrix}
\]

The demand amount of items at destination \([\tilde{b}_{ij}^{(np)}]^{(1)}\):

\[
\begin{bmatrix}
(6, 7, 9)(5, 7, 11) (4, 5, 7)(3, 5, 8) (2, 3, 5)(1, 5, 3, 6) (1, 2, 3)(0, 5, 2, 4) \\
(4, 6, 8)(3, 6, 8) (3, 6, 8)(2, 6, 9) (1, 3, 6)(0, 5, 3, 6) (0, 1, 3)(0, 1, 4).
\end{bmatrix}
\]

We formulate a MMMIFTP using the above data, where the objective function and constraints are as in Model-1. Now using possibility measure as a chance operator and method explain in solution methodology we get the following crisp transportation model

\[
\text{min} \{Z_1, Z_2, \ldots, Z_8\}
\]

Subject to

\[
(21) - (29)
\]

\[
\begin{align*}
\psi_{ij}^{(m)} & \geq 0, \ j, t, p. & (36) \\
0 & \leq \omega_{ij}^{(m)} + m^{(n)} & (37) \\
0 & \leq \lambda_{ij}^{(m)} + m^{(n)} & (38)
\end{align*}
\]

\[
p = 1, 2 \text{ and } i = 1, 2, \ldots, 7.
\]
where \( Z_t \) for \( t = 1, 2 \) has been given in Appendix B.

For possibility levels \( (\alpha^{(t)}) = 0.6, \beta^{(t)} = 0.4 \) for \( t = 1, 2 \) and \( (\lambda^{(t)}) = 0.5, \psi^{(t)} = 0.4 \) for \( i = 1, 2, \ldots, 8 \) and \( p = 1, 2 \), \( Z_1, Z_2 \) are calculated using GRG (LINGO-11.0) and we get

\[
Z_{1t}^i = 150.92, Z_{2t}^i = 49.20, Z_{1t}^0 = 126.90, Z_{2t}^0 = 49.12.
\]

Then we compute the following model to get the interactive satisfied solution,

\[
\min_{x \in \mathbb{R}^n} Z(x) = 150.92 - (\mu_2 - \lambda)(150.92 - 49.20) + \sum_{i=1}^{8} \sum_{p=1}^{2} \left( 150.92 - 49.20 \right) \left( \frac{\beta_t}{\alpha_t} \right) \left( \frac{\psi_t}{\lambda} \right) \left( x_i - x_p \right)
\]

To get the satisfied solution, we solve the above problem which are listed in the Table-2. The first line of Table-2 lists initial reference of membership function as 1, the value of objective function \( Z(x) \) and

\[
\begin{array}{c|c|c|c|c}
 i & j & p & Z_{1t}^i & Z_{2t}^i \\
\hline
 1 & 1 & 1 & 2.34(1.35,1.23) & 0.51(0.57,0.52) \\
 1 & 2 & 1 & 3.45(2.46,1.34) & 0.51(0.57,0.52) \\
 1 & 3 & 1 & 0.1(1.31,2.31) & 0.51(0.57,0.52) \\
 1 & 4 & 1 & 6.78(5.79,2.34) & 0.51(0.57,0.52) \\
 2 & 1 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 2 & 2 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 2 & 3 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 2 & 4 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 3 & 1 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 3 & 2 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 3 & 3 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 3 & 4 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 4 & 1 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 4 & 2 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 4 & 3 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52) \\
 4 & 4 & 1 & 0.12(0.13,0.51) & 0.51(0.57,0.52)
\end{array}
\]
Table 2
Interactive satisfied solution of model-1 based on possibility measure

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>Value of the decision variables</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>74.97</td>
<td>68.82</td>
<td>$x_{11}^{(1)}=0.37, x_{12}^{(1)}=1.12, x_{21}^{(1)}=0.12, x_{22}^{(1)}=3.37, x_{31}^{(1)}=0.5$</td>
<td>0.253</td>
</tr>
<tr>
<td>1</td>
<td>0.98</td>
<td>76.61</td>
<td>69.34</td>
<td>$x_{11}^{(1)}=1.5, x_{12}^{(1)}=5, x_{21}^{(1)}=1, x_{22}^{(1)}=1, x_{31}^{(1)}=6.6, x_{32}^{(1)}=3.2, x_{33}^{(1)}=0.2$</td>
<td>0.240</td>
</tr>
<tr>
<td>1</td>
<td>0.95</td>
<td>71.69</td>
<td>70.21</td>
<td>$x_{11}^{(1)}=0.9, x_{12}^{(1)}=0.9, x_{21}^{(1)}=1, x_{22}^{(1)}=0.6, x_{31}^{(1)}=3.5, x_{32}^{(1)}=0.49, x_{33}^{(1)}=0.2$</td>
<td>0.221</td>
</tr>
<tr>
<td>1</td>
<td>0.92</td>
<td>69.75</td>
<td>71.07</td>
<td>$x_{11}^{(1)}=1.5, x_{12}^{(1)}=5, x_{21}^{(1)}=1, x_{22}^{(1)}=0.6, x_{31}^{(1)}=3.2, x_{32}^{(1)}=0.2$</td>
<td>0.202</td>
</tr>
<tr>
<td>1</td>
<td>0.90</td>
<td>68.49</td>
<td>71.65</td>
<td>$x_{11}^{(1)}=1.5, x_{12}^{(1)}=5, x_{21}^{(1)}=1, x_{22}^{(1)}=0.6, x_{31}^{(1)}=3.2, x_{32}^{(1)}=0.2$</td>
<td>0.197</td>
</tr>
<tr>
<td>0.98</td>
<td>1</td>
<td>76.32</td>
<td>68.30</td>
<td>$x_{11}^{(1)}=0.9, x_{12}^{(1)}=0.9, x_{21}^{(1)}=1, x_{22}^{(1)}=0.6, x_{31}^{(1)}=3.2, x_{32}^{(1)}=0.2$</td>
<td>0.246</td>
</tr>
<tr>
<td>0.95</td>
<td>1</td>
<td>78.36</td>
<td>67.53</td>
<td>$x_{11}^{(1)}=0.9, x_{12}^{(1)}=0.9, x_{21}^{(1)}=1, x_{22}^{(1)}=0.6, x_{31}^{(1)}=3.2, x_{32}^{(1)}=0.2$</td>
<td>0.236</td>
</tr>
<tr>
<td>0.92</td>
<td>1</td>
<td>80.40</td>
<td>66.75</td>
<td>$x_{11}^{(1)}=0.9, x_{12}^{(1)}=0.9, x_{21}^{(1)}=1, x_{22}^{(1)}=0.6, x_{31}^{(1)}=3.2, x_{32}^{(1)}=0.2$</td>
<td>0.226</td>
</tr>
<tr>
<td>0.90</td>
<td>1</td>
<td>81.76</td>
<td>66.24</td>
<td>$x_{11}^{(1)}=0.9, x_{12}^{(1)}=0.9, x_{21}^{(1)}=1, x_{22}^{(1)}=0.6, x_{31}^{(1)}=3.2, x_{32}^{(1)}=0.2$</td>
<td>0.220</td>
</tr>
</tbody>
</table>

$Z_2(x)$ and its corresponding solution $x$. If the decision maker hopes that improve $Z_2(x)$ on the basis of sacrifice $Z_1(x)$. We may consider reset the reference value of membership function $(\mu_1, \mu_2)$, e.g., we set $(\mu_1, \mu_2) = (1, 0.98)$, or $(\mu_1, \mu_2) = (0.98, 1)$. The corresponding result are listed in the second and third lines. Suppose that when the reference value of membership function is $(\mu_1, \mu_2) = (1, 0.9)$, the decision maker is satisfied, then the interactive process is stopped, so we obtain satisfied solution as $x^* = \{x_{11}^{(1)} = 6.5, x_{12}^{(1)} = 6.7, x_{11}^{(2)} = 0.32, x_{12}^{(2)} = 0.31, x_{21}^{(2)} = 1.9, x_{22}^{(2)} = 1.8, x_{31}^{(2)} = 0.17, x_{32}^{(2)} = 1.5, x_{41}^{(2)} = 5, x_{42}^{(2)} = 1, x_{43}^{(2)} = 2, x_{44}^{(2)} = 3.2, x_{51}^{(2)} = 0.2\}$ and $(Z_1(x), Z_2(x)) = (194.29, 150.18)$. Moreover the decision maker can modify $Z_1^*, Z_2^*$ for $i = 1, 2$ and build a new reference membership function to obtain his or her satisfactory solution.

Applying global criteria in $L_2$ norm in MMOIFTP based on possibility measure, we get the following result $Z_1^* = 67.04$ and $Z_2^* = 72.3$ and optimum solution is $x^* = \{x_{11}^{(1)} = 6.50, x_{12}^{(1)} = 0.58, x_{11}^{(2)} = 0.41, x_{12}^{(2)} = 1.41, x_{21}^{(2)} = 2.1, x_{22}^{(2)} = 0.08, x_{31}^{(2)} = 0.08, x_{32}^{(2)} = 0.08, x_{41}^{(2)} = 1.5, x_{42}^{(2)} = 1.5, x_{43}^{(2)} = 1, x_{44}^{(2)} = 0, x_{51}^{(2)} = 0.0, x_{52}^{(2)} = 0.0, x_{53}^{(2)} = 0.0, x_{54}^{(2)} = 0.0\}$. We can obtain the efficient solution as follows $x^* = \{x_{11}^{(1)} = 6.50, x_{12}^{(1)} = 0.76, x_{11}^{(2)} = 0.23, x_{12}^{(2)} = 0.23, x_{21}^{(2)} = 3.73, x_{22}^{(2)} = 0.26, x_{31}^{(2)} = 1.5, x_{32}^{(2)} = 0, x_{41}^{(2)} = 5, x_{42}^{(2)} = 1, x_{43}^{(2)} = 0, x_{44}^{(2)} = 0\}$.

Table 3
Optimum results of Model-2, Model-3, Model-4 based on possibility measure

<table>
<thead>
<tr>
<th>Models</th>
<th>Method</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>Value of the decision variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model-2</td>
<td>GRG</td>
<td>26.6</td>
<td>-</td>
<td>$x_{11}^{(1)} = 6.5, x_{12}^{(1)} = 0.3, x_{31}^{(1)} = 0.0, x_{32}^{(1)} = 0.83$</td>
</tr>
<tr>
<td>Model-3</td>
<td>GRG</td>
<td>49.2</td>
<td>-</td>
<td>$x_{11}^{(1)} = 6.5, x_{12}^{(1)} = 0.3, x_{31}^{(1)} = 8.5, x_{32}^{(1)} = 0.83, x_{33}^{(1)} = 4.5, x_{34}^{(1)} = 1$</td>
</tr>
<tr>
<td>Model-4</td>
<td>GLOBAL CRITERIA</td>
<td>55.85</td>
<td>53.61</td>
<td>$x_{11}^{(1)} = 4.79, x_{12}^{(1)} = 2.70, x_{31}^{(1)} = 2.70, x_{32}^{(1)} = 0.6, x_{33}^{(1)} = 1.5, x_{34}^{(1)} = 0.2$</td>
</tr>
</tbody>
</table>
Optimum results of Model-2, Model-3, Model-4 based on possibility measure using three soft computing technique has given in Table-3.

6.2. Input data for Model-5, 6, 7, 8:

Let us consider a multi-objective multi-item transportation problem with two types of item (i.e. $T = 2$), 3 origins (i.e. $M = 3$), 4 destinations (i.e. $N = 4$). The parameter are given as follows

Transportation cost for 1st objective and 1st item

$$
\begin{bmatrix}
0.1 & 0.2 \\
0.3 & 0.4 \\
0.5 & 0.6
\end{bmatrix}
$$

Transportation cost for 1st objective and 2nd item

$$
\begin{bmatrix}
0.7 & 0.8 \\
0.9 & 1.0 \\
1.1 & 1.2
\end{bmatrix}
$$

Transportation cost for 2st objective and 1st item

$$
\begin{bmatrix}
1.3 & 1.4 \\
1.5 & 1.6 \\
1.7 & 1.8
\end{bmatrix}
$$

Transportation cost for 2st objective and 2nd item

$$
\begin{bmatrix}
1.9 & 2.0 \\
2.1 & 2.2 \\
2.3 & 2.4
\end{bmatrix}
$$

Amount of items available at origin

$$
\begin{bmatrix}
2.5 & 2.6 \\
2.7 & 2.8 \\
2.9 & 3.0
\end{bmatrix}
$$
\[
\begin{align*}
\frac{\alpha_1}{\alpha_2} & = \left(\begin{array}{c}
(4, 6, 9)(2, 6, 10) \\
(5, 8, 9)(4, 8, 10)
\end{array}\right) \\
\frac{\alpha_1}{\alpha_2} & = \left(\begin{array}{c}
(0, 2, 4)(0, 2, 6) \\
(6, 8, 10)(5, 8, 11)
\end{array}\right)
\end{align*}
\]

The demand amount of items at destination \(b_{ij}^{(e)}\)

\[
\begin{align*}
\begin{bmatrix}
b_1^{(e)} & b_2^{(e)} & b_3^{(e)} & b_4^{(e)} \\
b_1^{(e)} & b_2^{(e)} & b_3^{(e)} & b_4^{(e)}
\end{bmatrix}
& = \left(\begin{array}{c}
(6, 7, 9, 5, 7, 11) \\
(4, 6, 8, 3, 6, 8)
\end{array}\right)
\end{align*}
\]

Using the above input data Model-5 can be formulate as

\[
\begin{align*}
\text{Min} & \quad \left\{f_1^{(1)} + f_2^{(1)}, f_3^{(2)}, f_4^{(2)}\right\}
\end{align*}
\]

Subject to (21) – (38)

and (41) – (43)

Which is equivalent to

\[
\begin{align*}
\min Z_I & = (1 - \alpha_1^{(1)}) \left(2a_1^{(1)} + 3a_2^{(1)} + 10a_3^{(1)} + 6a_4^{(1)} \\
& + 0a_5^{(1)} + 0a_6^{(1)} + 0a_7^{(1)} + 0a_8^{(1)} + 0a_9^{(1)} + 0a_{10}^{(1)} + 0a_{11}^{(1)} + 0a_{12}^{(1)} + 0a_{13}^{(1)} + 0a_{14}^{(1)} \\
& + 8a_1^{(2)} + 15a_2^{(2)} + 7a_3^{(2)} + 0a_4^{(2)} + 0a_5^{(2)} + 0a_6^{(2)} + 0a_7^{(2)} + 0a_8^{(2)} + 0a_9^{(2)} + 0a_{10}^{(2)} + 0a_{11}^{(2)} + 0a_{12}^{(2)} + 0a_{13}^{(2)} + 0a_{14}^{(2)} \\
& + 7a_1^{(3)} + 4a_2^{(3)} + 0a_3^{(3)} + 1a_4^{(3)} + 5a_5^{(3)} + 0.5a_6^{(3)} + 2a_7^{(3)} + 10a_8^{(3)} \\
& + 6a_9^{(3)} + 1a_10^{(3)} + 0a_11^{(3)} + 3a_12^{(3)} + 4a_13^{(3)} + 31a_14^{(3)} + 7a_{15}^{(3)} + 2a_{16}^{(3)} + 12a_{17}^{(3)} \\
& + 11a_1^{(4)} + 7a_2^{(4)} + 7a_3^{(4)} + 0a_4^{(4)} + 0a_5^{(4)} + 0a_6^{(4)} + 0a_7^{(4)} + 2a_8^{(4)} + 0a_9^{(4)} + 0a_{10}^{(4)} + 0a_{11}^{(4)} + 0a_{12}^{(4)} + 0a_{13}^{(4)} + 0a_{14}^{(4)} \\
& + 23^{(12)} + 6a_1^{(13)} + 4a_2^{(13)} + 6a_3^{(13)} + 8a_4^{(13)} + 4a_5^{(13)} + 0a_6^{(13)} + 0a_7^{(13)} + 0a_8^{(13)} + 0a_9^{(13)} \\
& + 0a_{10}^{(13)} + 2a_{11}^{(13)} + 0a_{12}^{(13)} + 0a_{13}^{(13)} + 0a_{14}^{(13)} + 0a_{15}^{(13)} + 0a_{16}^{(13)} + 0a_{17}^{(13)} \\
& + 13a_1^{(14)} + 7a_2^{(14)} + 0a_3^{(14)} + 2a_4^{(14)} + 0a_5^{(14)} + 0a_6^{(14)} + 0a_7^{(14)} + 0a_8^{(14)} \\
& + 9a_9^{(14)} + 0a_{10}^{(14)} + 0a_{11}^{(14)} + 0a_{12}^{(14)} + 0a_{13}^{(14)} + 0a_{14}^{(14)} \\
& + 13a_1^{(15)} + 7a_2^{(15)} + 0a_3^{(15)} + 2a_4^{(15)} + 0a_5^{(15)} + 0a_6^{(15)} + 0a_7^{(15)} + 0a_8^{(15)} \\
& + 9a_9^{(15)} + 0a_{10}^{(15)} + 0a_{11}^{(15)} + 0a_{12}^{(15)} + 0a_{13}^{(15)} + 0a_{14}^{(15)} \\
& + 9a_1^{(16)} + 6a_2^{(16)} + 0a_3^{(16)} + 0a_4^{(16)} + 0a_5^{(16)} + 0a_6^{(16)} + 0a_7^{(16)} + 0a_8^{(16)} + 0a_9^{(16)} + 0a_{10}^{(16)} + 0a_{11}^{(16)} + 0a_{12}^{(16)} + 0a_{13}^{(16)} + 0a_{14}^{(16)}
\end{align*}
\]

Subject to (21) – (38)

As the above problem is a multi objective problem we can use the Interactive satisfied method or global criteria method or goal programming method to solve the above transportation problem.
For possibility levels ($\omega^0 = 0.6, \theta^0 = 0.4$) for $t = 1, 2$ and $\psi^{(m)} = 0.5, \phi^{(m)} = 0.4$ for $t = 1, 2, \ldots, 8$ and $p = 1, 2$, $Z_1$, $Z_2$ are calculated using GRG and we get :

$$Z_1^1 = 218.4, Z_1^2 = 185.72, Z_2^1 = 223.02, Z_2^2 = 117.1$$

So we can get the membership function of $Z_1$ and $Z_2$ as follows,

$$\mu_1(Z_1(x)) = \begin{cases} 1 & \text{for } Z_1(x) < 185.72 \\ \frac{218.4 - Z_1(x)}{218.4 - 185.72} & \text{for } 185.72 < Z_1(x) < 218.4 \\ 0 & \text{for } Z_1(x) > 218.4 \end{cases}$$

$$\mu_2(Z_2(x)) = \begin{cases} 1 & \text{for } Z_2(x) < 117.1 \\ \frac{223.02 - Z_2(x)}{223.02 - 117.1} & \text{for } 117.1 < Z_2(x) < 223.02 \\ 0 & \text{for } Z_2(x) > 223.02 \end{cases}$$

Then we compute the following model to get the interactive satisfied solution,

$$\text{Min } Z(x) = \begin{cases} x_1 \text{ s.t. } & Z_1(x) \leq 218.4 - (\mu_1 - \lambda) (218.4 - 185.72) \\ & Z_2(x) \geq 223.02 - (\mu_2 - \lambda) (223.02 - 117.1) \end{cases}$$

The first line of Table 4 lists initial reference of membership function as 1, the value of objective function $Z(x)$ and $Z'(x)$, and its corresponding solutions $x$. If the decision maker hopes to improve $Z_1(x)$ on the basis of sacrifice $Z_2(x)$, we may consider reset the reference value of membership function ($\mu_1, \mu_2$), e.g., we set ($\mu_1, \mu_2$) = (1, 0.98), or ($\mu_1, \mu_2$) = (0.98, 1).

The corresponding result are listed in the second and third lines. Suppose that when the reference value of membership function is ($\mu_1, \mu_2$) = (0.95, 0.9), the decision maker is satisfied, then the interactive process is stopped, so we obtain satisfied solution as $x_1 = [x_1^1 = 2.5, x_1^2 = 2.5, x_1^3 = 0.5, x_1^4 = 2.3, x_1^5 = 2.4, x_1^{(1)} = 1.5, x_1^{(1)} = 4.5, x_1^{(2)} = 2.5, x_1^{(2)} = 4.5, x_1^{(2)} = 1.5, x_1^{(2)} = 4.3, x_1^{(2)} = 0.81, x_1^{(2)} = 0.2]$

### Table 4

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>Value of the decision variables</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$x_1^1 = 2.5, x_1^2 = 2.5, x_1^3 = 0.5, x_1^4 = 2.3, x_1^5 = 2.4$</td>
<td>195.13</td>
<td>147.60</td>
<td>0.28</td>
</tr>
<tr>
<td>1</td>
<td>0.98</td>
<td>$x_1^1 = 2.5, x_1^2 = 2.5, x_1^3 = 0.5, x_1^4 = 2.3, x_1^5 = 2.4$</td>
<td>194.79</td>
<td>148.63</td>
<td>0.27</td>
</tr>
<tr>
<td>0/8</td>
<td>1</td>
<td>$x_1^1 = 2.5, x_1^2 = 2.5, x_1^3 = 0.5, x_1^4 = 2.3, x_1^5 = 2.4$</td>
<td>195.46</td>
<td>146.57</td>
<td>0.28</td>
</tr>
<tr>
<td>0/8</td>
<td>0.95</td>
<td>$x_1^1 = 2.5, x_1^2 = 2.5, x_1^3 = 0.5, x_1^4 = 2.3, x_1^5 = 2.4$</td>
<td>196.62</td>
<td>149.15</td>
<td>0.29</td>
</tr>
<tr>
<td>0/5</td>
<td>0.95</td>
<td>$x_1^1 = 2.5, x_1^2 = 2.5, x_1^3 = 0.5, x_1^4 = 2.3, x_1^5 = 2.4$</td>
<td>196.63</td>
<td>146.06</td>
<td>0.23</td>
</tr>
<tr>
<td>0/5</td>
<td>0.9</td>
<td>$x_1^1 = 2.5, x_1^2 = 2.5, x_1^3 = 0.5, x_1^4 = 2.3, x_1^5 = 2.4$</td>
<td>196.29</td>
<td>150.18</td>
<td>0.21</td>
</tr>
<tr>
<td>0/9</td>
<td>0.95</td>
<td>$x_1^1 = 2.5, x_1^2 = 2.5, x_1^3 = 0.5, x_1^4 = 2.3, x_1^5 = 2.4$</td>
<td>195.97</td>
<td>145.03</td>
<td>0.23</td>
</tr>
</tbody>
</table>

### Table 5

Optimum results of Model-6, Model-7, Model-8 based on possibility measure.
and \((Z^1, Z^2) = (194.29, 150.18)\). Moreover the decision maker can modify \(Z^i\) for \(i = 1, 2\) and built a new reference membership function to obtain his or her satisfactory solution.

Applying global criteria method in \(L_2\) norm in MSMIFTP based on possibility measure, we get the following result \(Z^1 = 194.86\) and \(Z^2 = 148.42\) and optimum solution is \(x^* = \{x^*_{ij}\} = \{2.5, x^*_{12} = 2.5, x^*_{12} = 2.5, x^*_{12} = 2.5, x^*_{12} = 4.1, x^*_{12} = 1.5, x^*_{12} = 2.8, x^*_{12} = 4.2, x^*_{12} = 0.99, x^*_{12} = 0.6, x^*_{12} = 2.1, x^*_{12} = 1, x^*_{12} = 0.2\}.

Suppose the tolerance given by decision maker is \(Z^1 = 195\) and \(Z^2 = 148\). Then we can use the goal programming method to handle the problem (39) and get the following goal programming model

\[
\begin{align*}
\min & \sum_{i=1}^{2} w_i (d^+_i + d^-_i) \\
\text{subject to} & \quad Z_i(x) + d^+_i - d^-_i = Z^*_i, \quad i = 1, 2
\end{align*}
\]

We take \(w_1 = 0.5, w_2 = 0.5\) and solving the above model, we can obtain the efficient solution as follows \(x^* = \{x^*_{ij}\} = \{2.5, x^*_{12} = 2.5, x^*_{12} = 2.5, x^*_{12} = 2, x^*_{12} = 4.1, x^*_{12} = 1.5, x^*_{12} = 2.77, x^*_{12} = 4.2, x^*_{12} = 1.02, x^*_{12} = 0.6, x^*_{12} = 2.22, x^*_{12} = 0.97, x^*_{12} = 0.2\}.

Optimum results of Model-6, Model-7, Model-8 based on possibility measure using three soft computing techniques (i) Interactive satisfied method, (ii) Global criteria method and (iii) Goal programming method. The present idea can be extended for solid transportation problem for future research work.

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**References**


\[(1-a^{(1)}) \left( x_{11}^{(1)} + x_{12}^{(1)} + 10x_{13}^{(1)} + 6x_{14}^{(1)} + 0.4x_{21}^{(1)} + 6x_{22}^{(1)} + 0.3x_{23}^{(1)} + 5x_{24}^{(1)} + 8x_{31}^{(1)} + 15x_{32}^{(1)} + 7x_{33}^{(1)} \right)
+ 0.5x_{11}^{(2)} + 3x_{12}^{(2)} + 4x_{13}^{(2)} + 4x_{14}^{(2)} + 0.4x_{21}^{(2)} + 1x_{22}^{(2)} + 5x_{23}^{(2)} + 15x_{24}^{(2)} + 0.5x_{31}^{(2)} + 2x_{32}^{(2)} + 10x_{33}^{(2)} + 6x_{34}^{(2)} \right)
+ \alpha^{(1)} \left( 3x_{11}^{(1)} + 4x_{12}^{(1)} + 11x_{13}^{(1)} + 7x_{14}^{(1)} + 0.4x_{21}^{(1)} + 7x_{22}^{(1)} + 2.4x_{23}^{(1)} + 6x_{24}^{(1)} + 9x_{31}^{(1)} + 16x_{32}^{(1)} + 8x_{33}^{(1)} \right)
+ 0.5x_{11}^{(2)} + 2x_{12}^{(2)} + 9x_{13}^{(2)} + 5x_{14}^{(2)} + 4x_{21}^{(2)} + 2x_{22}^{(2)} + 5x_{23}^{(2)} + 17x_{24}^{(2)} + 11x_{31}^{(2)} + 3x_{32}^{(2)} + 11x_{33}^{(2)} + 7x_{34}^{(2)} \right) \leq 1^{(1)} \quad (40)
\]

\[(1-\beta^{(1)}) \left( 3x_{11}^{(1)} + 4x_{12}^{(1)} + 11x_{13}^{(1)} + 7x_{14}^{(1)} + 0.4x_{21}^{(1)} + 7x_{22}^{(1)} + 2.4x_{23}^{(1)} + 6x_{24}^{(1)} + 9x_{31}^{(1)} + 16x_{32}^{(1)} + 8x_{33}^{(1)} \right)
+ 0.5x_{11}^{(2)} + 2x_{12}^{(2)} + 9x_{13}^{(2)} + 5x_{14}^{(2)} + 4x_{21}^{(2)} + 2x_{22}^{(2)} + 5x_{23}^{(2)} + 17x_{24}^{(2)} + 11x_{31}^{(2)} + 3x_{32}^{(2)} + 11x_{33}^{(2)} + 7x_{34}^{(2)} \right) \leq 1^{(1)} \quad (41)
\]

\[(1-a^{(2)}) \left( 0.5x_{11}^{(1)} + 2x_{12}^{(1)} + 3x_{13}^{(1)} + 5x_{14}^{(1)} + 1x_{21}^{(1)} + 2x_{22}^{(1)} + 13x_{23}^{(1)} + 7x_{24}^{(1)} + 3x_{31}^{(1)} + 13x_{32}^{(1)} + 6x_{33}^{(1)} \right)
+ 0.4x_{11}^{(2)} + 2x_{12}^{(2)} + 5x_{13}^{(2)} + 5x_{14}^{(2)} + 0.4x_{21}^{(2)} + 13x_{22}^{(2)} + 9x_{23}^{(2)} + 11x_{24}^{(2)} + 14x_{31}^{(2)} + 9x_{32}^{(2)} + 6x_{33}^{(2)} \right) \leq 1^{(2)} \quad (42)
\]

\[(1-\beta^{(2)}) \left( 0.5x_{11}^{(1)} +es x_{12}^{(1)} + 3x_{13}^{(1)} + 5x_{14}^{(1)} + 1x_{21}^{(1)} + 2x_{22}^{(1)} + 13x_{23}^{(1)} + 7x_{24}^{(1)} + 3x_{31}^{(1)} + 13x_{32}^{(1)} + 6x_{33}^{(1)} \right)
+ 0.4x_{11}^{(2)} + 2x_{12}^{(2)} + 5x_{13}^{(2)} + 5x_{14}^{(2)} + 0.4x_{21}^{(2)} + 13x_{22}^{(2)} + 9x_{23}^{(2)} + 11x_{24}^{(2)} + 14x_{31}^{(2)} + 9x_{32}^{(2)} + 6x_{33}^{(2)} \right) \leq 1^{(2)} \quad (43)
\]
9. Appendix B

\[ Z_t = \left(1-a^{(i)}\right) \left(g^{(i)}_{11} + g^{(i)}_{12} + g^{(i)}_{13} + g^{(i)}_{14} + g^{(i)}_{15}\right) + \left(1-a^{(i)}\right) \left(g^{(i)}_{12} + g^{(i)}_{13} + g^{(i)}_{14} + g^{(i)}_{15}\right) + \left(1-a^{(i)}\right) \left(g^{(i)}_{13} + g^{(i)}_{14} + g^{(i)}_{15}\right) + \left(1-a^{(i)}\right) \left(g^{(i)}_{14} + g^{(i)}_{15}\right) + \left(1-a^{(i)}\right) \left(g^{(i)}_{15}\right) \]

\[ \left(\begin{array}{cc}
g^{(i)}_{11} & h^{(i)}_{11} + h^{(i)}_{12} + h^{(i)}_{13} + h^{(i)}_{14} + h^{(i)}_{15} \\
+g^{(i)}_{12} + h^{(i)}_{12} & +g^{(i)}_{13} + h^{(i)}_{13} + h^{(i)}_{14} + h^{(i)}_{15} \\
+g^{(i)}_{13} + h^{(i)}_{13} & +g^{(i)}_{14} + h^{(i)}_{14} + h^{(i)}_{15} \\
+g^{(i)}_{14} + h^{(i)}_{14} & +g^{(i)}_{15} \\
+g^{(i)}_{15} & & & & \end{array}\right) + \beta^{(i)} \]

for \( t = 1, 2 \).

where

\[ s^{(i)}_{t1} = 2(1-y^1_0)(1-y^2_0) + y^3(1-y^0_1) + S(1-y^2_0)y^0_1, \]

\[ s^{(i)}_{t2} = 3(1-y^1_0)(1-y^2_0) + y^3(1-y^0_1) + 2(1-y^2_0)y^0_1, \]

\[ s^{(i)}_{t3} = 10y^1_0 + 5(1-y^2_0)y^0_1, \]

\[ s^{(i)}_{t4} = 6y^2_0 + 2(1-y^2_0), \]

\[ s^{(i)}_{t5} = 0(1-y^1_0) + 0(1-y^1_0), \]

\[ s^{(i)}_{t6} = 0y^2_0 + 0(1-y^2_0), \]

\[ s^{(i)}_{t7} = 6y^2_0 + 0(1-y^2_0), \]

\[ s^{(i)}_{t8} = 0y^3_0 + 0(1-y^3_0), \]

\[ s^{(i)}_{t9} = 0y^4_0 + 0(1-y^4_0), \]

\[ s^{(i)}_{t10} = 0y^5_0 + 0(1-y^5_0), \]

\[ s^{(i)}_{t11} = 0y^6_0 + 0(1-y^6_0), \]

\[ s^{(i)}_{t12} = 0y^7_0 + 0(1-y^7_0), \]

\[ s^{(i)}_{t13} = 0y^8_0 + 0(1-y^8_0), \]

\[ s^{(i)}_{t14} = 0y^9_0 + 0(1-y^9_0), \]

\[ s^{(i)}_{t15} = 0y^{10}_0 + 0(1-y^{10}_0). \]
\[ +1(1 - y_{10})y_{10}^2. \]
\[ c_{11}^{(1)} = 16y_{11} + 14(1 - y_{11}). \]
\[ c_{12}^{(1)} = 8y_{12}^2 + 7(1 - y_{12}^2). \]
\[ c_{11}^{(2)} = 0.5y_{11} + 1(1 - y_{11}^2). \]
\[ c_{12}^{(2)} = 2(1 - y_{12})y_{12} + 4y_{12}(1 - y_{12}^2) \]
\[ +3(1 - y_{12})y_{12}^4. \]
\[ c_{11}^{(3)} = 9y_{11}^2 + 8(1 - y_{11}^2). \]
\[ c_{12}^{(3)} = 8(1 - y_{12})y_{12} + 4y_{12}(1 - y_{12}^2) \]
\[ +6(1 - y_{12})y_{12}^6. \]
\[ c_{11}^{(4)} = y_{11}^2 + 2(1 - y_{11}^2). \]
\[ c_{12}^{(4)} = 2y_{12} + 0.5(1 - y_{12}^2). \]
\[ c_{11}^{(5)} = 5y_{11}^2 + 5(1 - y_{11}^2). \]
\[ c_{12}^{(5)} = 17y_{12} + 16(1 - y_{12}^2). \]
\[ c_{11}^{(6)} = y_{11}^2 + 2(1 - y_{11}^2). \]
\[ c_{12}^{(6)} = 3(1 - y_{12})y_{12} + 2y_{12}(1 - y_{12}^2) \]
\[ +4(1 - y_{12})y_{12}^2. \]
\[ c_{11}^{(7)} = 11y_{11} + 10(1 - y_{11}^2). \]
\[ c_{12}^{(7)} = 7y_{12}^2 + 6(1 - y_{12}^2). \]
\[ h_{11}^{(1)}(1 - y_{11})y_{11} + 0.5y_{11}(1 - y_{11}^2) + 4(1 - y_{11})y_{11}^2. \]
\[ h_{12}^{(1)} = 2(1 - y_{12})y_{12} + 0.5y_{12}(1 - y_{12}^2) + 4(1 - y_{12})y_{12}^2. \]
\[ h_{11}^{(2)} = 9y_{11}^2 + 4(1 - y_{11}^2). \]
\[ h_{12}^{(2)} = 5y_{12}^2 + 4(1 - y_{12}^2). \]
\[ h_{11}^{(3)} = 0.5y_{11} + 0.5(1 - y_{11}^2). \]
\[ h_{12}^{(3)} = 5y_{12}^2 + 4(1 - y_{12}^2). \]
\[ h_{11}^{(4)} = 0.5y_{11}^2 + 0.5(1 - y_{11}^2). \]
\[ h_{12}^{(4)} = 5y_{12}^2 + 4(1 - y_{12}^2). \]
\[ h_{11}^{(5)} = 0.5y_{11}^2 + 0.5(1 - y_{11}^2). \]
\[ h_{12}^{(5)} = 5y_{12}^2 + 4(1 - y_{12}^2). \]
\[ h_{11}^{(6)} = 0.5y_{11}^2 + 0.5(1 - y_{11}^2). \]
\[ h_{12}^{(6)} = 5y_{12}^2 + 4(1 - y_{12}^2). \]
\[ h_{11}^{(7)} = 0.5y_{11}^2 + 0.5(1 - y_{11}^2). \]
\[ h_{12}^{(7)} = 5y_{12}^2 + 4(1 - y_{12}^2). \]
\[ h_{11}^{(8)} = 0.5y_{11}^2 + 0.5(1 - y_{11}^2). \]
\[ h_{12}^{(8)} = 5y_{12}^2 + 4(1 - y_{12}^2). \]
\[ h_{11}^{(9)} = 0.5y_{11}^2 + 0.5(1 - y_{11}^2). \]
\[ h_{12}^{(9)} = 5y_{12}^2 + 4(1 - y_{12}^2). \]
\[ h_{11}^{(10)} = 0.5y_{11}^2 + 0.5(1 - y_{11}^2). \]
\[ h_{12}^{(10)} = 5y_{12}^2 + 4(1 - y_{12}^2). \]
\[ \begin{align*}
\delta_{12}^{(22)} &= z_{12}^3 + 0.5(1 - z_{14}^3). \\
\delta_{13}^{(22)} &= 2(1 - z_{14}^3)(1 - z_{13}^3) + 3z_{15}^3(1 - z_{13}^3) \\
&+ (1 - z_{14}^3)z_{16}^3. \\
\delta_{14}^{(22)} &= 8(1 - z_{14}^3)(1 - z_{15}^3) + 4z_{14}^3(1 - z_{15}^3) \\
&+ 6(1 - z_{14}^3)z_{16}^3. \\
\delta_{11}^{(22)} &= 0.5z_{13} + 3(1 - z_{15}^3). \\
\delta_{12}^{(22)} &= 7(1 - z_{14}^3)(1 - z_{13}^3) + 5z_{14}^3(1 - z_{13}^3) \\
&+ 4(1 - z_{14}^3)z_{16}^3. \\
\delta_{13}^{(22)} &= 12z_{15} + 10(1 - z_{14}^3). \\
\delta_{14}^{(22)} &= 9z_{15} + 7(1 - z_{15}^3). \\
\delta_{11}^{(22)} &= 2z_{12}^3 + (1 - z_{13}^3). \\
\delta_{12}^{(22)} &= 2z_{13}^3 + 3(1 - z_{12}^3). \\
\delta_{13}^{(22)} &= 2(1 - z_{14}^3)(1 - z_{13}^3) + z_{15}^3(1 - z_{13}^3) \\
&+ 3(1 - z_{14}^3)z_{16}^3. \\
\delta_{14}^{(22)} &= 5z_{13}^3 + 4(1 - z_{14}^3). \\
\delta_{11}^{(22)} &= z_{12}^3 + 2(1 - z_{12}^3). \\
\delta_{12}^{(22)} &= 5z_{14}^3 + 2(1 - z_{12}^3). \\
\delta_{13}^{(22)} &= 5z_{13}^3 + 2(1 - z_{12}^3). \\
\delta_{14}^{(22)} &= 4(1 - z_{14}^3)(1 - z_{13}^3) + 3z_{15}^3(1 - z_{13}^3) + 4(1 - z_{14}^3)z_{16}^3. \\
\delta_{11}^{(22)} &= 6z_{14}^3 + 6(1 - z_{14}^3). \\
\delta_{12}^{(22)} &= 2z_{14}^3 + (1 - z_{14}^3). \\
\delta_{13}^{(22)} &= 2z_{15} + 2(1 - z_{15}^3). \\
\delta_{14}^{(22)} &= 14(1 - z_{14}^3)(1 - z_{13}^3) + 12z_{15}^3(1 - z_{13}^3) + 10z_{14}^3(1 - z_{13}^3) + 10z_{14}^3(1 - z_{15}^3) \\
&+ 8(1 - z_{14}^3)z_{16}^3. \\
\delta_{11}^{(22)} &= 8z_{15}^3 + 8(1 - z_{15}^3). \\
\delta_{12}^{(22)} &= 4z_{12}^3 + 3(1 - z_{12}^3). \\
\delta_{13}^{(22)} &= 7z_{12} + 6(1 - z_{12}^3). \\
\delta_{14}^{(22)} &= 14(1 - z_{14}^3)(1 - z_{13}^3) + 12z_{15}^3(1 - z_{13}^3) + 13(1 - z_{14}^3)z_{16}^3. \\
\delta_{11}^{(22)} &= 8z_{12} + 5(1 - z_{12}^3). \\
\delta_{12}^{(22)} &= c_{13} + 2(1 - c_{13}). \\
\delta_{13}^{(22)} &= 3z_{14} + 2(1 - z_{14}^3). \\
\delta_{12}^{(22)} &= 3z_{14} + 2(1 - z_{14}^3). \\
\delta_{11}^{(22)} &= 3z_{14} + 0.5(1 - z_{14}^3). \\
\end{align*} \]
\[ h_{i3}^{(22)} = (1 - z_{1i}^1)(1 - z_{1i}^2) + 2z_{1i}^3(1 - z_{1i}^3) + 0.5(1 - z_{1i}^1)z_{1i}^3, \]
\[ h_{i4}^{(22)} = 5(1 - z_{1i}^1)(1 - z_{1i}^2) + 3z_{1i}^3(1 - z_{1i}^3) + 0.5(1 - z_{1i}^1)z_{1i}^3, \]
\[ h_{i7}^{(22)} = 0.5z_{1i}^1 + 2(1 - z_{1i}^1), \]
\[ h_{i8}^{(22)} = 6(1 - z_{1i}^1)(1 - z_{1i}^2) + 4z_{1i}^3(1 - z_{1i}^3) + 3(1 - z_{1i}^1)z_{1i}^3, \]
\[ h_{i11}^{(22)} = 11z_{1i}^1 + 9(1 - z_{1i}^1) + 6(1 - z_{1i}^1), \]
\[ h_{i21}^{(22)} = c_{21}^1 + 0.5(1 - z_{1i}^1), \]
\[ h_{i31}^{(22)} = 0.5c_{21}^1 + 2(1 - z_{1i}^1), \]
\[ h_{i32}^{(22)} = (1 - z_{1i}^1)(1 - z_{1i}^2) + 0.5c_{21}^2(1 - z_{1i}^2) + 2(1 - z_{1i}^1)z_{1i}^2, \]
\[ h_{i33}^{(22)} = 4z_{1i}^2 + 3(1 - z_{1i}^2). \]

\( y_i^a \) and \( z_i^a \) = 0 or 1 for \( i = 1, 2, \ldots, 24 \) and \( a = 1, 2, \]
\( 1 \leq y_i^1 + y_i^2 \leq 2 \) for \( i = 1, 2, 5, 8, 10, 14, 16, 22, \) and
\( 1 \leq z_i^1 + z_i^2 \leq 2 \) for \( i = 3, 7, 11, 15, 16, 18, 23. \)