Iterative learning control of Hamiltonian systems

Kenji Fujimoto[a], Hiroki Kakiuchi[a] and Toshiharu Sugie[a]

[a] Department of Systems Science, Graduate School of Informatics
Kyoto University, Uji, Kyoto 611-0011, Japan
fujimoto@i.kyoto-u.ac.jp
sugie@i.kyoto-u.ac.jp

Abstract

This paper is concerned with iterative learning control of Hamiltonian systems, which is applicable to electro-mechanical systems. A novel iterative learning control scheme is proposed based the self-adjoint structure of the variational of those systems. This method does not require either the physical parameters of the target system nor the time derivatives of output signals. A concrete and effective learning algorithm for mechanical systems is also derived. Furthermore, experiments of a robot manipulator demonstrates the effectiveness of the proposed method.

1 Introduction

Hamiltonian control systems are the systems described by well known Hamilton's canonical equations with controlled Hamiltonians [2]. They are introduced mainly to characterize variational properties of dynamical systems and is used for optimal control, see also [13]. Those systems were also utilized to describe physical systems, and the related geometric methods of controlling this class of systems supplied fruitful results in control engineering [10, 7]. Furthermore, this control framework was generalized in order to handle electro-mechanical systems as well as conventional mechanical ones [8], and several control methods are proposed for them [8, 4, 11, 9]. Thus a scope of this paper contains control of physical systems such as mechanical and electrical systems.

In our former results we have investigated the self-adjoint structure of Hamiltonian systems [5]. It was revealed that the variational systems of Hamiltonian systems have self-adjoint structures. This fact implies that the input-output mappings of the adjoints of the variational of Hamiltonian systems can be obtained without using precise knowledge of the target system. This fact is useful for system identification and adjoint based learning control [12].

This paper is devoted to iterative learning control of Hamiltonian systems based on the above property the self-adjoint structure of the variational of Hamiltonian systems. After briefly referring to the preliminary results, a basic framework for iterative learning control of Hamiltonian systems will be proposed. This control scheme is very simple in the sense that it does not require any physical parameter of the target system. Also it does not require any time derivative either, whereas existing well-known simple learning scheme by Arimoto [1] does require high order time derivatives. Furthermore, we will show a concrete control system synthesis method for mechanical systems. Moreover experiments of a direct drive robot manipulator demonstrates the effectiveness of the proposed method.

2 Self-adjoint structure of Hamiltonian systems

This section refers to some preliminary results on the self-adjoint structure of the variational of Hamiltonian systems as preparation for the main results.

Consider an operator $\Sigma : X \times U \to X \times Y$ with Hilbert spaces X, U and Y with a state-space realization

$$(x^1, y) = \Sigma(x^0, u) : \left\{ \begin{array}{rcl} \dot{x} & = & f(x,u,t), \quad x(t^0) = x^0 \\ y & = & h(x,u,t) \\ x^1 & = & x(t^1) \end{array} \right. \tag{1}$$

defined on a time interval $t \in [t^0, t^1]$. Typically, $X = \mathbb{R}^n$, $U = L_2^m[t^0, t^1]$ and $Y = L_2^r[t^0, t^1]$. A simpler notation $\Sigma^{x^0} : U \to Y$ with

$$y = \Sigma^{x^0}(u) : \left\{ \begin{array}{rcl} \dot{x} & = & f(x,u,t) \quad x(t^0) = x^0 \\ y & = & h(x,u,t) \end{array} \right.$$

is also employed. Here let us recall Fréchet derivative of nonlinear operators.

Definition 1 Consider an operator $\Sigma : X \to Y$ with Banach spaces X and Y. Σ is said to be *Fréchet differentiable* at $x \in X$ if there exists an operator $\mathrm{d}\Sigma : X \times X \to Y$ such that $\mathrm{d}\Sigma(x, \xi)$ is linear in ξ and that

$$\lim_{\|\xi\|_X \to 0} \frac{\|\Sigma(x+\zeta) - \Sigma(x) - \mathrm{d}\Sigma(x,\xi)\|_Y}{\|\xi\|_X} = 0.$$

Under these circumstances, $\mathrm{d}\Sigma(x, \xi)$ is called the *Fréchet derivative* of Σ at x.

The Fréchet derivative $d\Sigma^{x^0}(u)(du)$ of $\Sigma^{x^0}(x)$ is given by [2, 3]

$$y_v = d\Sigma^{x^0}((u),(u_v)):$$
$$\begin{cases} \dot{x} &= f(x,u,t), \qquad\qquad x(0)=x^0 \\ \begin{pmatrix} \dot{x}_v \\ y_v \end{pmatrix} &= \dfrac{\partial}{\partial(x,u)}\begin{pmatrix} f(x,u,t) \\ h(x,u,t)\end{pmatrix}\begin{pmatrix} x_v \\ u_v\end{pmatrix}, \quad x_v(0)=0 \end{cases}.$$

By its construction in Definition 1, the Fréchet derivative $d\Sigma(x, dx)$ is a locally linear approximation to $\Sigma(x)$, that is

$$d\Sigma(u,v) \approx \Sigma(u+v) - \Sigma(u) \tag{2}$$

holds when v is small.

Next we consider a Hamiltonian system Σ_H with a controlled Hamiltonian $H(x,u,t)$ with dissipation

$$(x^1, y) = \Sigma_H(x^0, u): \tag{3}$$
$$\begin{cases} \dot{x} &= (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, \quad x(t^0)=x^0 \\ y &= -\dfrac{\partial H(x,u,t)}{\partial u}^{\mathrm{T}} \\ x^1 &= x(t^1) \end{cases}. \tag{4}$$

Here the structure matrices $J \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{n\times n}$ are skew-symmetric and symmetric positive semi-definite, respectively. The matrix R represents dissipative elements such as friction of mechanical systems and resistance of electric circuits. For this system, the following theorem holds.

Theorem 1 *[5] Consider the Hamiltonian system with dissipation and the controlled Hamiltonian Σ_H in (4). Suppose that J and R are constant and that there exist nonsingular matrices $T_x \in \mathbb{R}^{n\times n}$ and $T_u \in \mathbb{R}^{m\times m}$ satisfying*

$$\begin{aligned} J &= -T_x J\, T_x^{-1} \\ R &= T_x R\, T_x^{-1} \\ \frac{\partial^2 H(x,u,t)}{\partial(x,u)^2} &= \begin{pmatrix} T_x & 0 \\ 0 & T_u\end{pmatrix}\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}\begin{pmatrix} T_x & 0 \\ 0 & T_u\end{pmatrix}^{-1}. \end{aligned} \tag{5}$$

Then the Fréchet derivative of Σ_H is described by another linear Hamiltonian system

$$(x_v^1, y_v) = d\Sigma_H((x^0,u),(x_v^0,u_v)):$$
$$\begin{cases} \dot{x} &= (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}}, \qquad x(t^0)=x^0 \\ \dot{x}_v &= (J-R)\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial x_v}^{\mathrm{T}}, \quad x_v(t^0)=x_v^0 \\ y_v &= -\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial u_v}^{\mathrm{T}} \\ x_v^1 &= x_v(t^1) \end{cases} \tag{6}$$

with a controlled Hamiltonian $H_v(x,u,x_v,u_v,t)$

$$H_v(x,u,x_v,u_v,t) = \frac{1}{2}\begin{pmatrix} x_v \\ u_v\end{pmatrix}^{\mathrm{T}}\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}\begin{pmatrix} x_v \\ u_v\end{pmatrix}.$$

Furthermore, the adjoint of the variational system with zero initial state $u_a \mapsto y_a = (d\Sigma^{x^0}(u))^*(u_a)$ is given by

$$y_a = (d\Sigma_H^{x^0}(u))^*(u_a):$$
$$\begin{cases} \dot{x} &= (J-R)\dfrac{\partial H(x,u,t)}{\partial x}^{\mathrm{T}} \\ \dot{x}_v &= -(J-R)\left.\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial x_v}^{\mathrm{T}}\right|_{u_v=T_u u_a} \\ y_a &= -T_u\left.\dfrac{\partial H_v(x,u,x_v,u_v,t)}{\partial u_v}^{\mathrm{T}}\right|_{u_v=T_u u_a} \end{cases} \tag{7}$$

with the terminal states $x(t^0)=x^0$ and $x_v(t^1)=0$. Suppose moreover that $J-R$ is nonsingular. Then the adjoint $(x_a^1, u_a) \mapsto (x_a^0, y_a) = (d\Sigma(x^0,u))^*(x_a^1,u_a)$ is given by the same state-space realization (7) with the terminal states $x(t^0)=x^0$, $x_v(t^1) = -(J-R)T_x\, x_a^1$ and $x_a^0 = -T_x^{-1}(J-R)^{-1}x_v(t^0)$.

Remark 1 Note that the dynamics of x_a in (7) is the time reversal version of that of x_v in (6). Suppose the input u is given such that the time history of the Hessian of the Hamiltonian with respect to (x,u) is symmetrical with respect to the middle of the time interval $[t^0,t^1]$, i.e.,

$$\frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}(t-t^0) = \frac{\partial^2 H(x,u,t)}{\partial(x,u)^2}(t^1-t), \quad \forall t \in [t^0,t^1].$$

Then $d\Sigma_H$ has a self-adjoint state-space realization. This condition often occurs in a PTP control of robot manipulators.

Under the circumstances in Remark 1, Theorem 1 implies that the time reversal system of the adjoint $(d\Sigma_H)^*$ coincide with the variational $d\Sigma_H$, that is,

$$\mathcal{R}(d\Sigma_H(u))^*\mathcal{R}(du) \approx d\Sigma_H(u,du) \tag{8}$$

where $\mathcal{R}$ is a time reversal operator defined by

$$\mathcal{R}(u)(t-t^0) = u(t^1-t), \quad \forall t\in[t^0,t^1]. \tag{9}$$

Namely, the variational of the Hamiltonian system (4) has self-adjoint structure. Combined with the property of the variational system (2), we can calculate the input-output mapping of the adjoint by only using the input-output data of the original system.

Example 1 Consider an LCR-circuit depicted in Figure 1. Let φ_1 and φ_2 denote the flux linkages, H_L denotes the inductance energy (a nonlinear function of φ_1 and φ_2), R_1 denotes the resistance, H_C denotes the stored energy of capacitance (a nonlinear function of Q), Q denotes the charge, and V denote the input voltage. Let us definite the input $u=V$ and the state $x=(Q,\varphi_1,\varphi_2)$. Then we obtain the Hamiltonian system (4) with

$$H(Q,\varphi_1,\varphi_2,u) = H_C(Q) + H_L(\varphi_1,\varphi_2) + Q\,u$$
$$J = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0\end{pmatrix}, \quad R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & R_1\end{pmatrix}.$$

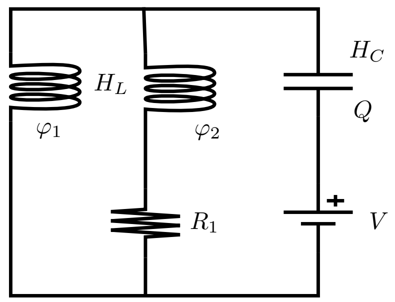

Figure 1: LCR-circuit

This system satisfies the matching condition (5) with

$$T_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \ T_u = 1.$$

Therefore, we can calculate the adjoint of the variational system by using the input-output mapping of the original system provided the assumptions in Remark 1 hold.

3 Iterative learning control

This section briefly explains how to apply the results in Section 2 to iterative learning control. The simplest problem setting of iterative learning control is as follows. Consider the nonlinear system (1) and a prescribed desired output y^d. The main purpose of learning control is to find an input $u = u^d$ which achieves $\Sigma(u^d) = y^d$. To this end, the iteration law

$$u_{(i+1)} = u_{(i)} + k(y^d - y_{(i)})$$

is adopted. Here $u_{(i)}$ and $y_{(i)}$ denote the input and output at the i-th operation. The objective is to find an appropriate iteration law $k(\cdot)$ such that

$$y_{(i)} \to y^d \quad \text{as} \quad i \to \infty.$$

Arimoto et al. [1] adopted the iteration law $k(\cdot)$ in a PD like controller form without using the precise knowledge of the target system (1) under mild assumptions. Yamakita and Furuta [12] proposed to use the adjoint of the target system as the iteration law $k(\cdot)$ based on optimization theory. Though this approach brings fast and numerically stable convergence, it needs precise knowledge of the target system. There are some other results adopting in-between approaches, e.g. [6], which give faster convergence and require less information of the target system. The main strategy taken here is similar to the Furuta's approach. But our result does not require the precise knowledge of the target system. Here we are going to utilize qualitative properties of physical systems rather than quantitative ones.

For iterative learning control problem, it is natural to consider a cost function

$$\Gamma(y) = \int_{t^0}^{t^1} (y(t) - y^d(t))^{\mathrm{T}} \Gamma_y (y(t) - y^d(t)) \mathrm{d}t \qquad (10)$$

with a positive definite matrix $\Gamma_y \in \mathbb{R}^{m \times m}$ and to try reducing it. The Fréchet derivative of Γ is given by

$$\begin{aligned} \mathrm{d}\Gamma(y)(\mathrm{d}y) &= -2\langle \Gamma_y(y^d - y),\ \mathrm{d}y\rangle_{L_2} \\ &= -2\langle \Gamma_y(y^d - y),\ \mathrm{d}\Sigma_H^{x^0}(u)(\mathrm{d}u)\rangle_{L_2} \\ &= -2\langle (\mathrm{d}\Sigma_H^{x^0}(u))^* \Gamma_y(y^d - y),\ \mathrm{d}u\rangle_{L_2}. \end{aligned}$$

Hence the steepest decent method implies that the law

$$u_{(i+1)} = u_{(i)} + K_{(i)} (\mathrm{d}\Sigma_H^{x^0}(u_{(i)}))^* \Gamma_y (y^d - y_{(i)})$$

with an appropriate positive gain $K_{(i)} > 0$ reduces the cost function Γ efficiently. The input-output mapping of the adjoint $(\mathrm{d}\Sigma_H^{x^0}(u_{(i)}))^*$ can be obtained by that of the original operator Σ_H using (2) and (8).

Thus iterative learning control with respect to the cost function (10) can be executed. Of course this procedure can be performed with any cost functional $\Gamma(x^0, u, x^1, y)$, provided $\Sigma = \Sigma_H$ as in (4) (under the circumstances in Remark 1). Here we formally adopt the following assumptions according to Remark 1 (in order to use the self-adjoint property (8)).

Assumption A1 It is assumed that the desired trajectory $x^d(t)$ and input $u^d(t)$ satisfy

$$\left.\frac{\partial^2 H(x,u)}{\partial (x,u)^2}\right|_{\substack{x = x^d(t-t^0)\\ u = u^d(t-t^0)}} = \left.\frac{\partial^2 H(x,u)}{\partial (x,u)^2}\right|_{\substack{x = x^d(t^1-t)\\ u = u^d(t^1-t)}}, \ \forall t \in [t^0, t^1].$$

Procedure 1 Consider the Hamiltonian system (4) with a given desired trajectory $x^d(t)$. Suppose the assumptions in Theorem 1 and Assumption A1 hold. Then the iterative learning control law is given by

$$\begin{aligned} u_{(2i+1)} &= u_{(2i)} + T_u \mathcal{R}\ (\kappa_{(i)} \Gamma_y (y^d - y_{(2i)})) &(11)\\ u_{(2i+2)} &= u_{(2i)} + K_{(i)}\ \mathcal{R}\ (T_u (y_{(2i+1)} - y_{(2i)})) &(12) \end{aligned}$$

for $i = 0, 1, 2, \cdots$. Here Γ_y defines the cost function Γ in (10) and T_u is the parameter defined in Theorem 1. The parameters $\kappa_{(i)} > 0 \in \mathbb{R}$ and $K_{(i)} > 0 \in \mathbb{R}^{m \times m}$ are small enough design parameters. $\mathcal{R}$ denotes the time reversal operator defined in (9).

This result will provide a basis of a new iterative learning control for a class of physical systems. Unfortunately, this iteration procedure only guarantees the convergence to a local minimum of the cost function (10), that is, the convergence to an optimal input u^d is not ensured in general.

4 Iterative learning control of mechanical systems

A typical mechanical system can be described by a Hamiltonian system

$$\Sigma_H : \begin{cases} \begin{pmatrix} \dot q \\ \dot p \end{pmatrix} = \begin{pmatrix} 0 & I \\ -I & -R_p \end{pmatrix} \begin{pmatrix} \frac{\partial H(q,p,u)}{\partial q}^{\mathrm T} \\ \frac{\partial H(q,p,u)}{\partial p}^{\mathrm T} \end{pmatrix} \\ y = \frac{\partial H(q,p,u)}{\partial u}^{\mathrm T} = q \end{cases} \tag{13}$$

with the Hamiltonian

$$H(q,p,u) = H_0(q,p) - u^{\mathrm T} q = \underbrace{\frac{1}{2} p^{\mathrm T} M(q)^{-1} p + V(q)}_{H_0(q,p)} - u^{\mathrm T} q$$

where a positive definite matrix $M(q) > 0 \in \mathbb{R}^{m\times m}$ denotes the inertia matrix, a scalar function $V(q)$ denotes the potential energy of the system and H_0 denotes the total physical energy.

Unfortunately, however, this system does not satisfy the assumptions in Theorem 1 since there do not exist the matrices T_x and T_u satisfying the matching condition (5). The procedure in the sequel enables the system to satisfy this condition approximately.

Typically, feedback controllers are employed to control the system (13) even when the iterative learning control is applied, since it is marginally stable. This subsection discusses feedback system design for iterative learning control. It was shown in [10] that a simple PD feedback preserves the structure of the Hamiltonian system (13). Further discussions on controller design preserving the structure of general Hamiltonian systems can be found in [11, 4, 9]. Let us consider a PD controller

$$u = \bar u - K_q\, q - K_p\, \dot q \tag{14}$$

where $\bar u$ is a new input and $K_q, K_p > 0 \in \mathbb{R}^{m\times m}$ are symmetric positive definite matrices. Applying a coordinate transformation

$$q = \varepsilon \bar q$$

with a positive constant $\varepsilon > 0$ converts the system into another Hamiltonian system

$$\begin{cases} \begin{pmatrix} \dot{\bar q} \\ \dot p \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{\varepsilon} I \\ -\frac{1}{\varepsilon} I & -(R_p + K_p) \end{pmatrix} \begin{pmatrix} \frac{\partial \bar H(\bar q,p,\bar u)}{\partial \bar q}^{\mathrm T} \\ \frac{\partial \bar H(\bar q,p,\bar u)}{\partial p}^{\mathrm T} \end{pmatrix} \\ y = -\frac{\partial \bar H(\bar q,p,\bar u)}{\partial \bar u}^{\mathrm T} = \varepsilon \bar q = q \end{cases} \tag{15}$$

with a new Hamiltonian

$$\bar H(\bar q, p, \bar u) = \underbrace{\frac{1}{2} p^{\mathrm T} M(\varepsilon \bar q)^{-1} p + V(\varepsilon \bar q)}_{H_0(\varepsilon \bar q, p)} + \frac{\varepsilon^2}{2} \bar q^{\mathrm T} K_q\, \bar q - \varepsilon\, \bar u^{\mathrm T} \bar q.$$

Let us choose the parameter matrices in Theorem 1 as

$$T_x = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad T_u = I \tag{16}$$

and check the matching condition (5). The former two equations hold straightforwardly and the left and right hands of the last equation become

$$\frac{\partial^2 \bar H(\bar q,p,\bar u)}{\partial(\bar q,p,\bar u)^2} = \begin{pmatrix} \varepsilon^2 \left(\frac{\partial^2 H_0(q,p)}{\partial q^2} + K_q\right) & \varepsilon \frac{\partial M(q)^{-1} p}{\partial q}^{\mathrm T} & -I \\ \varepsilon \frac{\partial M(q)^{-1} p}{\partial q} & M(q)^{-1} & 0 \\ -I & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} T_x & 0 \\ 0 & T_u \end{pmatrix} \frac{\partial^2 \bar H(\bar q,p,\bar u)}{\partial(\bar q,p,\bar u)^2} \begin{pmatrix} T_x & 0 \\ 0 & T_u \end{pmatrix}^{-1} = \begin{pmatrix} \varepsilon^2 \left(\frac{\partial^2 H_0(q,p)}{\partial q^2} + K_q\right) & -\varepsilon \frac{\partial M(q)^{-1} p}{\partial q}^{\mathrm T} & -I \\ -\varepsilon \frac{\partial M(q)^{-1} p}{\partial q} & M(q)^{-1} & 0 \\ -I & 0 & 0 \end{pmatrix}.$$

Hence, if the "P gain" K_q is chosen large enough and the parameter ε is taken small enough accordingly, then the relation

$$\frac{\partial^2 \bar H(\bar q,p,\bar u)}{\partial(\bar q,p,\bar u)^2} \approx \begin{pmatrix} T_x & 0 \\ 0 & T_u \end{pmatrix} \frac{\partial^2 \bar H(\bar q,p,\bar u)}{\partial(\bar q,p,\bar u)^2} \begin{pmatrix} T_x & 0 \\ 0 & T_u \end{pmatrix}^{-1}$$

holds, that is, the assumption (5) in Theorem 1 is satisfied approximately. Note that the "D gain" K_p should also be chosen large enough to let the matrix $R_q + K_q$, which describes the dissipation behavior of the system (15) in the coordinate $(\bar q, p)$, sufficiently large compared with the matrix I/ε, which denotes the oscillation behavior. This should be done for numerical stability of the iterative learning procedure. Here we adopt the following assumptions corresponding to Assumption A1.

Assumption B1 It is assumed that the desired trajectory $x^d(t) = (q^d(t), p^d(t))$ satisfies

$$\left.\frac{\partial^2 H_0(q,p)}{\partial(q,p)^2}\right|_{x=x^d(t-t^0)} = \left.\frac{\partial^2 H_0(q,p)}{\partial(q,p)^2}\right|_{x=x^d(t^1-t)}, \quad \forall t \in [t^0, t^1].$$

Assumption B2 PD gains K_q and K_p are large enough.

Remark 2 When the desired trajectory $x^d(t)$, $t \in [t^0, t^1]$ does not satisfy Assumption B1, we can produce a desired trajectory fulfilling B1 by simply reproducing the same trajectory in the time domain $t \in [t^1, 2t^1 - t^0]$ as

$$x^d_{\mathrm{new}}(t) = \begin{cases} x^d(t) & t \in [t^0, t^1] \\ x^d(2t^1 - t^0 - t) & t \in [t^1, 2t^1 - t^0] \end{cases}.$$

The iterative learning procedure is given below on the assumptions B1 and B2.

Procedure 2 Consider the mechanical Hamiltonian system (13) with the PD feedback (14) and a prescribed desired trajectory $q^d(t)$. Suppose Assumptions B1 and B2 hold. Then the iterative learning control law is given by

$$\begin{array}{rcl} \bar{u}_{(2i+1)} & = & \bar{u}_{(2i)} + \mathcal{R}\ (\kappa_{(i)}\Gamma_y(q^d - q_{(2i)})) \\ \bar{u}_{(2i+2)} & = & \bar{u}_{(2i)} + K_{(i)}\ \mathcal{R}\ (q_{(2i+1)} - q_{(2i)}) \end{array} \quad (17)$$

for $i = 0, 1, 2, \cdots$. Here Γ_y defines the cost function Γ in (10). The parameters $\kappa_{(i)} > 0 \in \mathbb{R}$ and $K_{(i)} > 0 \in \mathbb{R}^{m\times m}$ are small enough design parameters. $\mathcal{R}$ denotes the time reversal operator defined in (9).

This iterative learning control scheme is very simple in the sense that it does not employ any physical parameters of the target system. Compared with Arimoto's method [1] which is also simple, the proposed method is expected to be numerically more stable because our approach does not employ time derivative whereas Arimoto's method requires second order time derivative of q for mechanical systems (13).

Furthermore, we can prove the convergence to the global minimum, i.e., the convergence to the optimal input $\bar{u}^d$, of this iteration procedure, though the general version of this procedure given in Procedure 1 only guarantees the convergence to a local minimum.

Proposition 1 *Consider the Hamiltonian system (13). Suppose Assumptions B1 and B2 hold and there exists a positive constant ϵ satisfying*

$$\kappa_{(i)}K_{(i)} \geq \epsilon I > 0, \quad \forall i. \quad (18)$$

Then, for any initial input $\bar{u}_{(0)}$, the iterative learning control law (17) in Procedure 2 converges to an optimal input $\bar{u}^d$.

Proof: The variational system $d\Sigma_H^{x^0}$ of the mechanical Hamiltonian system (13) can be described by

$$\left\{ \begin{array}{rcl} \begin{pmatrix} \dot{q}_v \\ \dot{p}_v \end{pmatrix} & = & \begin{pmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{pmatrix} \begin{pmatrix} q_v \\ p_v \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} u_v \\ y_v & = & q_v \end{array} \right.$$

with appropriate matrices A_{ij}'s. Let us now calculate the zero-dynamics of this system. Take $y_v \equiv 0$. Then it follows that

$$0 \equiv \dot{q}_v = A_{11}q_v + A_{12}p_v = A_{12}p_v = M(q)^{-1}p_v.$$

Therefore we prove $p_v \equiv 0$. Finally we obtain

$$0 \equiv \dot{p}_v = A_{21}p_v + A_{22}q_v + u_v = u_v.$$

This suggests that the variational system has no zero-dynamics. Therefore, the iteration law (17) and the assumption (18) imply

$$\bar{u}_{(2i)} \to \bar{u}_{(2i+2)} \quad \Rightarrow \quad q_{(2i)} \to q^d,$$

that is, the control law converges to an optimal input $\bar{u}^d$. This completes the proof. ∎

5 Experimental results

The procedure given in the previous section is now applied to 2-link robot manipulator depicted in Figure 2, whose height is 0.55 [m].

Figure 2: 2-link manipulator

Each joint is driven by a direct drive motor, and each link rotates on the horizontal plane. This system is a typical example of Hamiltonian systems and its dynamics can be described by (13). Here $q = (q_1, q_2)$ and $u = (u_1, u_2)$ and the generalized momentum p is defined by $p = M(q)\dot{q}$ with the inertia matrix $M(q)$ given by

$$M(q) = \begin{pmatrix} \rho_1 + \rho_2 + 2l_1\rho_3\cos\theta_2 & \rho_2 + l_1\rho_3\cos\theta_2 \\ \rho_2 + l_1\rho_3\cos\theta_2 & \rho_2 \end{pmatrix}$$

$$\rho_1 := I_1 + m_1l_{g1}^2 + m_2l_1^2,\ \rho_2 := I_2 + m_2l_{g2}^2,\ \rho_3 := m_2l_{g2}.$$

The friction matrix R_p is given by $R_p = \mathrm{diag}(r_1, r_2)$ and the potential energy is $V = 0$ because the links moves on the horizontal plane. The parameters are defined as follows: u_i [Nm] denotes the input torque for joint i, m_i [kg] denotes the mass of link i, l_i [m] denotes the length of link i, l_{gi} [m] denotes the length from the center to joint of link i, I_i [kgm^2] denotes the inertia of link i, r_i [Nms/rad] denotes the friction coefficient of joint i and q_i [rad] denotes the rotation angle of link i. The concrete value of parameters are $l_1 = 0.25$, $l_2 = 0.30$, $\rho_1 = 2.55$, $\rho_2 = 0.72$, $\rho_3 = 2.60$, $r_1 = 0.2415$ and $r_2 = 0.2457$.

The design parameters of the iterative learning control scheme in Procedure 2 are chosen as follows:

$$\Gamma_y = I,\ \kappa_{(i)} \equiv 1,\ K_{(i)} \equiv 1400I,\ K_q = 30I,\ K_p = 20I.$$

The desired trajectory $y^d(t) = q^d(t)$, $t \in [0, 3]$ is given by

$$q^d(t) = \begin{pmatrix} -0.473451\sin(0.01\pi t) \\ 0.463212\sin(0.01\pi t) + 0.4 \end{pmatrix}.$$

The experimental results of 10 times iteration are given in Figures 3-5. Figure 3 shows the responses of the angle q_1 of link 1. In the figure, the solid (thick) line denotes the desired trajectory q_1^d, the dotted (thin) lines denote the responses of the angle $q_{1(i)}$ at the i-th operation, and the dashed (thick) line denotes the response of the angle $q_{1(10)}$ at the 10-th iteration. Figure 4 shows the responses with respect to the angle q_2 in the way similar to Figure 3. Figure 5 depicts the value of the cost function Γ in (10) in the log scale at each iteration.

The figures shows that the responses converge to the desired trajectory smoothly. In particular, Figure 5 shows that the convergence is sufficiently fast. These experimental results show that the proposed method works quite well. Utilizing the qualitative property of physical systems intensively, we can thus obtain a simple and effective iterative learning control scheme in this paper.

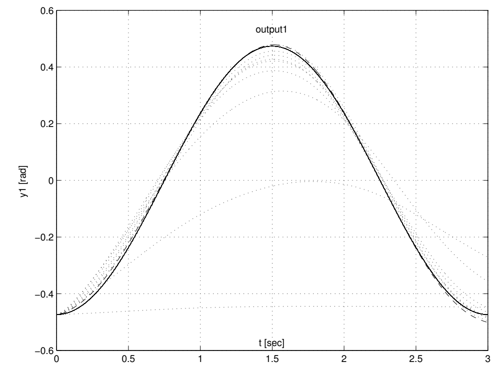

Figure 3: Responses of the angle q_1 of link 1

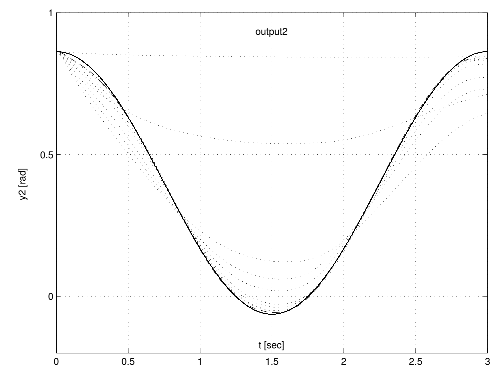

Figure 4: Responses of the angle q_2 of link 2

6 Conclusion

This paper has discussed iterative learning control of Hamiltonian systems. A novel iterative learning control scheme has been proposed based the self-adjoint structure of the variational of those systems. This method does not require either the physical parameters of the target system nor the time derivatives of output signals. A concrete and effective learning algorithm for mechanical systems is also derived. Furthermore, experiments of a robot manipulator demonstrates the effectiveness of the proposed method.

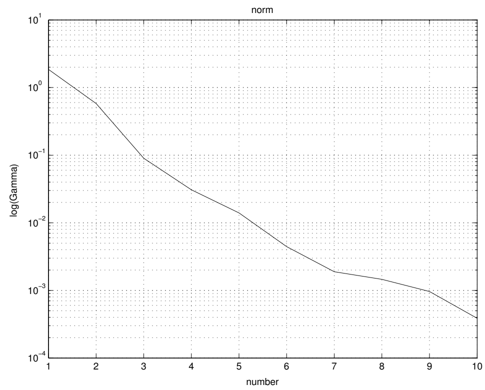

Figure 5: The cost function Γ

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