Phase Precoding with Integrated Turbo-Equalization for Packet Retransmissions

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Abstract—In this paper, we present an effective phase precoding diversity scheme to mitigate intersymbol interference from multiple transmissions in hybrid automatic repeat request (HARQ) protocols over slowly time varying frequency-selective channels. For each HARQ transmission, only the phases of modulated symbols are changed according to a periodic pattern. This diversity scheme results in a spectrum thinning of the Euclidean distance at the output of the noiseless ISI channel. Periodic phase precoding only requires a small increase in receiver complexity but provides a substantial gains in terms of frame error rate.

I. INTRODUCTION

To ensure data reliability in data packet communication systems, hybrid automatic repeat request (HARQ) protocols [1] are usually used to combat errors introduced by the communication channel. In data transmissions over frequency-selective channels, the received signal suffers from intersymbol interference (ISI) resulting from the limited bandwidth of the channel or the multipath propagation. In that context, equalization techniques are usually used at the receiver to mitigate the ISI from the received signal. In particular, turbo-equalization [2] is an efficient technique that combines signal detection and error correction in an iterative way leading to significant performance gains in comparison with systems using separated signal detection and decoding. The performance gains are obtained, of course, at the expense of an increased receiver complexity. In order to take advantage of the available time diversity in retransmission-based HARQ protocols, i.e. HARQ using Chase combining, many diversity schemes are proposed to enhance channel equalization performance by introducing some transmission diversity among subsequent ARQ transmissions. For example, in [3], a bit interleaving diversity scheme between HARQ transmissions was proposed using iterative equalization scheme of relatively high complexity. In [4], a mapping diversity scheme was proposed to increase the Euclidean distance separation between transmitted frames. The drawback of this method is to be limited to high order modulations. When channel state information (CSI) is available at the transmitter, precoding (pre-equalization) techniques [5], [6] can be used in order to transform the ISI channel into an ISI-free channel. In [7], the precoding filters are optimized for multiple transmissions.

In this paper, we present a novel diversity scheme based on phase precoding for packet retransmission HARQ protocols assuming that no CSI is available at the transmitter. The proposed scheme is suitable for slowly time varying quasistatic channels. The remaining of the paper is organized as follows. In Section II, we introduce the system model and the proposed phase precoding diversity scheme. In Section III, we introduce the phase precoding pattern we use and give some justifications based on a pairwise error analysis. In Section IV, we focus on the joint turbo-equalization receiver scheme for both equalization criteria: the maximum a posteriori (MAP) and the minimum mean square error (MMSE). In Section V, we give some simulation results showing the efficiency of the proposed precoding technique. Finally, conclusions are given in Section VI.

II. SYSTEM MODEL

We consider the model of a turbo-equalized system with retransmissions shown in Fig. 1. A frame $c = (c_1 \cdots c_{QN})$ of $QN$ encoded and interleaved bits are mapped into a frame of symbols $x = (x_1 \cdots x_N)$ using a complex constellation $S$ of size $|S| = 2^d$ symbols. For each HARQ transmission, the same symbol $x_n$ is multiplied by a complex-valued precoding coefficient $a_n^{(f)} = e^{j\theta_n^{(f)}}$ to obtain the precoded symbol $y_n^{(f)} = a_n^{(f)} x_n$, where $f$ is the index of the HARQ transmission. The precoded symbols $y_n^{(f)}$ are then transmitted through an ISI channel into an ISI-free channel. In [7], the precoding filters are optimized for multiple transmissions.

![Phase precoded Hybrid-ARQ system model with integrated turbo-equalization.](image-url)
channel modeled by its equivalent complex-valued discrete-time finite impulse response of length \( L \), denoted by \( h^{(f)} = (h_0^{(f)} \cdots h_{L-1}^{(f)}) \) assumed constant during each transmission. The received sequence samples \( r_n^{(f)} \) are modeled as

\[
r_n^{(f)} = \sum_{i=0}^{L-1} h_i^{(f)} y_{n-i} + w_n^{(f)}, \quad n = 1 \cdots N,
\]

where \( w_n^{(f)} \) is an independent additive white complex Gaussian noise with variance \( \sigma_w^2 \). At the receiver, we consider a joint turbo-equalizer assuming a perfect CSI. If the frame is still in error after a maximum number \( F_{\text{max}} \) of allowable transmissions, an error is declared and the frame is dropped out from the transmission buffer.

The question arising is how to select the precoding coefficients \( a_n^{(f)} \) to reduce the frame error rate (FER) assuming that the ISI channel changes slowly between subsequent ARQ transmissions, and how it impacts in HARQ system with integrated turbo-equalization.

### III. PHASE PRECODING PATTERN

In this section, we carry out a pairwise error analysis for the joint maximum likelihood (ML) receiver in order to determine a performance criterion suitable for the choice of precoding coefficients. Let \( x \) and \( \tilde{x} \) be the transmitted and the estimated sequence, respectively, and \( y^{(f)} \) and \( \tilde{y}^{(f)} \) be the corresponding precoded sequences at the \( f \)th transmission. We define the following useful error sequences \( e = \tilde{x} - x \) and \( \tilde{e}^{(f)} = \tilde{y}^{(f)} - y^{(f)} \). The pairwise error probability for the joint ML receiver between \( x \) and \( \tilde{x} \), denoted by \( P_2(x, \tilde{x}) \), is given in [8] by

\[
P_2(x, \tilde{x}) = Q \left( \frac{d_E(x, \tilde{x})}{2\sigma_w} \right),
\]

where \( Q(\cdot) \) is the Gaussian error probability function, and \( d_E \) is the Euclidean distance at the output of noiseless ISI channel. After \( F \) transmissions, \( d_E^2 \) can be evaluated as a function of the error sequence as

\[
d_E^2 = \sum_{f=1}^{F} \sum_{n=1}^{N+L-1} \left| h_i^{(f)} \tilde{e}_{n-i}^{(f)} \right|^2,
\]

with \( \tilde{e}_{n}^{(f)} = a_{n}^{(f)} e_{n} \). By developing the squared sum in (1) and performing some algebraic computations, we can rewrite \( d_E^2 \) as the sum of two variables as follows:

\[
d_E^2 = \Gamma_F + \Delta_F,
\]

with

\[
\Gamma_F = \sum_{f=1}^{F} R_0(h^{(f)}) R_0(\tilde{e}^{(f)}),
\]

\[
\Delta_F = 2 \Re \left[ \sum_{f=1}^{F} \sum_{i=1}^{L-1} R_i^*(h^{(f)}) R_i(\tilde{e}^{(f)}) \right],
\]

where \( \Re[\cdot] \) denotes the real part, the superscript * denotes the complex conjugate, and \( R(\cdot) \) is the aperiodic auto-correlation function at lag \( \ell \), defined for an arbitrary complex sequence \( x \) of length \( N \) by \( R_\ell(x) = \sum_{n=1}^{N} x_n x_{n-\ell}^* \) with \( x_n = 0 \) for \( n \notin [1, N] \). The variable \( \Gamma_F \) gives the effect of the channel gain on the squared Euclidean distance, whereas the second variable \( \Delta_F \) reflects the fluctuation of the Euclidean distance due to the presence of the ISI. Providing that \( \| a_n^{(1)} \|^2 = 1 \), we have \( R_0(\tilde{e}^{(f)}) = R_0(e) \), hence \( \Gamma_F \) is independent of the precoding coefficients. In order to minimize the effects of the ISI on the Euclidean distance, we intend to minimize \( \Delta_F \) with respect to the precoding coefficients for any error sequence. To simplify our analysis, we consider the case of long-term quasi-static channels where the channel does not change between subsequent ARQ transmissions of the same frame \( (h^{(1)} = \cdots = h^{(F)} = h) \), but may change between different frames. The generalization for slowly time varying channels is straightforward. In this case, the equation (4) reduces to

\[
\Delta_F = 2 \Re \left[ \sum_{\ell=1}^{L-1} R_\ell^*(h) \Sigma_{F,\ell} \right],
\]

with

\[
\Sigma_{F,\ell} = \sum_{f=1}^{F} R_\ell(\tilde{e}^{(f)}) = \sum_{n=1}^{N-\ell} \alpha_F(n, \ell) e_n^* e_{n+\ell},
\]

where \( \alpha_F(n, \ell) \equiv \sum_{n=1}^{F} (a_n^{(f)})^* a_{n+\ell}^{(f)} \), which is the inner product between the two vectors \( a_n^{(F)} \) and \( a_{n+\ell}^{(F)} \) where \( a_n^{(F)} \equiv [a^{(1)}_n \cdots a^{(F)}_n]^T \) denotes the precoding vector obtained by regrouping the precoding coefficients of the same symbol \( x_n \) during \( F \) transmissions. Because \( \Delta_F \) depends on CSI, we can only minimize an upper bound on \( \Delta_F^2 \). Applying the Cauchy-Schwartz inequality on (5) and (6) yields to

\[
\Delta_F^2 \leq 4 \sum_{\ell=1}^{L-1} |R_\ell(h)|^2 \sum_{\ell=1}^{L-1} |\Sigma_{F,\ell}|^2,
\]

and

\[
|\Sigma_{F,\ell}|^2 \leq \sum_{n=1}^{N-\ell} |\alpha_F(n, \ell)|^2 \sum_{n=1}^{N-\ell} |e_n e_{n+\ell}|^2.
\]

Combining (7) and (8) shows that the minimization of the upper bound on \( \Delta_F^2 \) is equivalent to the minimization of the following cost function

\[
J_F(a_1^{(F)} \cdots a_N^{(F)}) = \sum_{\ell=1}^{L-1} \sum_{n=1}^{N-\ell} |\alpha_F(n, \ell)|^2.
\]

This shows that the problem of the minimization of \( \Delta_F \) is reduced to an orthogonality problem between precoding vectors of interfering symbols. The joint minimization of the cost function for \( F = 1 \cdots F_{\text{max}} \) is computationally prohibitive in general, especially for large \( N \). However, to simplify the optimization problem and for equalization complexity reasons (see Section IV), we restrict ourselves to periodic precoding vectors with a period \( P \) equals to the channel memory \( P = L \), i.e. \( a_{n+P}^{(f)} = a_n^{(f)} \). The problem reduces to find the precoding vectors \( a_1^{(F)} \cdots a_P^{(F)} \) with minimal cross-correlation. This problem can be solved numerically and is similar to the
design of signature sequences in a synchronous CDMA system with \( L \) users, with the particularity that the sequence length increases with \( F \). As an example, for \( L = 5 \), \( F_{max} = 4 \), and bipolar precoding coefficients \( \phi_n^{(l)} \in \{0, \pi\} \), an optimal solution that jointly optimizes the cost functions \( J_F \) for all values of \( F \) is given in a matrix form as follows

\[
[a_1^{(4)}, \ldots, a_4^{(4)}] = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & -1 & -1 & -1 \\ +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & +1 \end{bmatrix}
\] (10)

where each column defines a precoding vector. We remark that the first 4 vectors form a Hadamard matrix of size 4.

IV. JOINT TURBO-RECEIVER STRUCTURE

Turbo-equalization is an attractive method to perform a joint detection and decoding and requires the use of soft-input soft-output (SISO) equalizer and decoder. In the context of packet retransmissions, we are interested in a joint turbo-equalization of the various received precoded frames to benefit from transmission diversity. To this end, we present in this section the modifications introduced to both the joint MAP equalizer and the joint filter-based MMSE equalizer with a priori to handle the operation of precoding.

A. Joint MAP equalization

In order to keep the structure of the trellis diagram of the channel unchanged compared to the non-precoding case, the precoding operation is processed as a part of the ISI channel. Therefore, the equivalent ISI channel becomes time-variant whose impulse response during the \( f \)th transmission of the symbol \( x_n \) is

\[
\hat{h}_n^{(f)} = (a_n^{(f)}h_0^{(f)} \ldots a_{n-L-1}^{(f)}h_{L-1}^{(f)}).
\] (11)

In that case, the Forward-Backward algorithm [9] can be generalized for the joint equalization of multiple transmissions in the same way of [10] taking into account the equivalent time variant channel. The modification to the algorithm due to the introduced precoding diversity concerns only the definition of the transitional probabilities between channel states. On the trellis representation of the channel, each state \( S_n \) is identified by the previous \( L-1 \) transmitted symbols \( S_n = (x_{n-1} \ldots x_{n-L+1}) \). The output of the channel corresponding to the transmission of a symbol \( x_n \in S \) having the channel state \( S_n \) is given by

\[
\psi(x_n, S_n) = \hat{h}_n^{(f)}x_n + \sum_{k=1}^{L-1} \hat{h}_n^{(f)}x_{n-k}. \]

The transitional probabilities \( \gamma_n \) between two stats \( S_{n-1} = m' \) and \( S_n = m \) at the \( n^\text{th} \) trellis section are updated at each retransmission using the recursion

\[
\gamma_n(r_n^{(F)}, m', m) = \gamma_n(r_n^{(F-1)}, m', m)p(r^{(F)}|x_n = x, m', m)
\]

where \( r_n^{(F)} = [r_n^{(1)} \ldots r_n^{(F)}]^T \) and

\[
p(r^{(F)}|x_n = x, m', m) = \frac{1}{\sigma_n^2} e^{-|r_n^{(F)} - \psi(x, S_n)|^2}/\sigma_n^2.
\]

B. Joint MMSE equalization

We generalize the finite length MMSE equalizer with a priori proposed in [11] to the case of precoded system. The structure of the joint SISO equalizer is shown in Fig. 2 including multiple forward linear filters \( p_n^{(f)} \) and an interference canceler filter \( q_n \). The linear estimate \( \hat{x}_n \) of the transmitted symbol \( x_n \) after \( F \) transmissions is given by

\[
\hat{x}_n = \sum_{f=1}^{F} (p_n^{(f)})^H r_n^{(f)} - q_n^H S_n,
\] (12)

where \( H \) denotes the hermitian transpose, \( r_n^{(f)} = [r_n^{(f-1)} \ldots r_n^{(f+n+1)}]^T \) are the required observations samples around the estimated symbol. The forward filters \( p_n^{(f)} \) are implemented using \( N_p = n_1 + n_2 + 1 \) taps, where the parameters \( n_1 \) and \( n_2 \) specify the length of the non-causal and the causal part of the estimator filter, respectively. Note that we allow the filter coefficients to vary with \( n \) because of the time varying equivalent channel model defined in (11), and not because we are looking for a time varying solution.

The problem of the joint equalization can be turned back to the case of a single transmission by considering the equivalent multiple-input single-output (MISO) channel model given by

\[
r_n = \hat{H}_n x_n + w_n,
\]
where
\[
x_n = [x_{n-n_2-L+1} \cdots x_{n+n_1}]^T
\]
\[
w_n = [w_{n-n_2}^{(1)} \cdots w_n^{(F)} \cdots w_{n+n_1}^{(1)} \cdots w_{n+n_1}^{(F)}]^T
\]
\[
r_n = [r_{n-n_2}^{(1)} \cdots r_n^{(F)} \cdots r_{n+n_1}^{(1)} \cdots r_{n+n_1}^{(F)}]^T,
\]
and \( \hat{H}_n \) is the equivalent channel matrix of dimensions \( FN_p \times (N_p + L - 1) \) given by
\[
\hat{H}_n = A_n^{(F)} \otimes H^{(F)},
\]
where \( \otimes \) denotes the term-by-term matrix product, and the matrices \( A_n^{(F)} \) and \( H^{(F)} \) are of the same dimensions and defined as
\[
A_n^{(F)} = \begin{bmatrix}
    a_{n-n_2-L-1}^{(F)} & \cdots & a_{n+n_1}^{(F)} \\
    \vdots & & \vdots \\
    a_{n-n_2-L-1}^{(F)} & \cdots & a_{n+n_1}^{(F)}
\end{bmatrix},
\]
\[
H^{(F)} = \begin{bmatrix}
    h_{L-1}^{(F)} & \cdots & h_0^{(F)} & 0 & \cdots & 0 \\
    0 & h_{L-1}^{(F)} & \cdots & h_0^{(F)} & \cdots & \vdots \\
    \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\
    0 & \cdots & 0 & h_{L-1}^{(F)} & \cdots & h_0^{(F)}
\end{bmatrix},
\]
where \( h_n^{(F)} = [h_0^{(F)} \cdots h_L^{(F)}]^T \). Using the MISO model, the estimated symbol in (12) can be rewritten as
\[
\hat{x}_n = p_n^H r_n - q_n^H s_n.
\]
(13)

Following the same analysis in [12], the derivation of the expression of the filters that minimize the mean square error \( E[|\hat{x} - x|^2] \) is straightforward and leads to the following solution
\[
p_n = (\sigma_s^2 I + v^2 \hat{H}_n H_n^{(F)})^{-1} \hat{H}_n u
\]
\[
q_n = \hat{H}_n^T p_n - \mu_n u
\]
\[
u = [0_{1 \times (n_2+L-1)} 1] 1_{1 \times n_1}^T,
\]
with \( v^2 = \frac{1}{N} \sum_{n=1}^{N} \text{var}(\hat{x}_n) \) and \( \mu_n = p_n^H H_n u \). The output extrinsic a posteriori probabilities (APPs) are then calculated using the Gaussian model for the estimated symbols \( \hat{x}_n = \mu_n x_n + \eta_n \), where \( \eta_n \) is a complex Gaussian noise with variance \( \nu^2 = \mu_n (1 - \mu_n v^2) \).

APP(\( \hat{x}_n = x \)) \( = K_n \exp(-\frac{|\hat{x}_n - \mu_n x|^2}{\nu^2}), \)
where \( K_n \) is a normalization constant chosen to have a true probability mass function \( \sum_{x \in S} \text{APP}(x_n = x) = 1 \) at the output of the estimator.

As for the MAP equalizer, the same remark is made here about the additional complexity. For periodic precoding with a period \( P \) we have \( \hat{H}_{n+P} = \hat{H}_n \) and all MMSE equalizer parameters follow this property. Consequently, the complexity of the equalizer is reduced to only \( P \) matrix inversions instead of \( N \) in the general case. Moreover, there are low complexity methods to calculate the inverse of the \( P \) matrices recursively from the first matrix using the particular structure of the channel matrix as in [12].

\[\begin{array}{c}
\text{Fig. 3. FER performance in the precoded HARQ system for } L = 3 \text{ and rate-3/4 RSC code.}
\end{array}\]

\[\begin{array}{c}
\text{Fig. 4. FER performance in the precoded HARQ system for } L = 5 \text{ and rate-1/2 RSC code.}
\end{array}\]

V. RESULTS

In this section, we present some simulation results that show the effectiveness of the proposed phase precoding diversity scheme. Each channel tap is a complex-valued Gaussian random variable, with zero mean and unit variance, that varies per frame, but remains constant for retransmissions [13]. The channel is normalized to unit energy to maintain the desired \( E_s/N_0 = 1/2\sigma_s^2 \) as in [4].

First, we consider a frame of 1200 encoded bits by a rate-3/4 recursive systematic convolutional code (RSC) obtained by puncturing the RSC(1,27/31)8 code using the bit puncturing pattern [0 0 1 1 1 1] which is optimized for serial concatenated systems [14]. After a random interleaver, the encoded bits are mapped to \( N = 300 \) symbols using a 16-QAM constellation with Gray mapping. We assume \( L = 3, F_{\text{max}} = 4 \) and the precoding vectors set is the first 3 vectors of the set given in (10). These 3 vectors are assigned periodically to the transmitted symbols. At the receiver, we use a joint MMSE turbo-equalizer implemented using forward filters of 16-taps (\( n_1 = 9, n_2 = 5 \)). We evaluate the FER versus \( E_s/N_0 \).
over 10000 frames. Fig. 3 shows simulation results for the precoded HARQ system after 5 turbo iterations. We note that a gain of about 6 dB is obtained at FER=10^{-2}. We observe for the non-precoded system that only an energetic gain is obtained for each retransmission, whereas the slopes of the FER curves for the precoded system increase with $F$. This is a consequence of the increasing minimum Euclidean distance due to the precoding.

In Fig. 4, we show simulation results for channel length $L = 5$ using a rate-1/2 RSC(1, 27/31) without puncturing and the precoding set (10). The other simulation parameters are the same as used for Fig. 3. Using a lower code rate, the precoding gain is less important than for a high code rate, because the minimum Euclidean distance is enhanced by a higher free distance of the low code rate.

To show the convergence behavior of the precoded system, we have traced in Fig. 5 the FER performance of the joint turbo-equalizer for two transmissions $F = 2$ regardless if the first transmission was in error or not. We note that the precoding enhances the channel quality which is traduced by a good FER performance from the first turbo-iteration. The number of iterations required by the turbo-equalizer to converge is smaller for the precoded system. This allows simplifying the receiver complexity and enhancing the system performance simultaneously.

Finally, Fig. 6 shows the corresponding throughput efficiency for the same settings for Fig. 4. We note that for low to medium SNR values, the throughput in the precoded system are very close to the throughput in the coded system over an AWGN channel. For high SNR values, the throughput is dominated by the FER at the first transmission, hence there is no significant improvement expected in comparison with non-precoded system.

VI. CONCLUSIONS

We have presented in this paper an efficient phase precoding diversity scheme to mitigate ISI in HARQ systems when no CSI is available. The proposed scheme is suitable for slowly time-varying quasi-static ISI channels. Integrating phase precoding in a low complexity turbo-equalizer leads to a significant FER gain and fast convergence of the turbo-equalizer resulting in a higher throughput and a reduced receiver complexity.

REFERENCES