Assisted motor therapies play a critical role in enhancing functional musculoskeletal recovery and neurological rehabilitation. Our long term goal is to assist and automate the performance of repetitive motor-therapy of the human lower limbs. Hence, in this paper, we examine the viability of a light-weight and reconfigurable hybrid (articulated-multibody and cable) robotic system for assisting lower-extremity rehabilitation and analyze its performance. A hybrid cable-actuated articulated multibody system is formed when multiple cables are attached from a ground-frame to various locations on an articulated-linkage based orthosis. Our efforts initially focus on developing an analysis and simulation framework for the kinematics and dynamics of the cable-driven lower limb orthosis. A Monte Carlo approach is employed to select configuration parameters including cuff sizes, cuff locations, and the position of fixed winches. The desired motions for the rehabilitative exercises are prescribed based upon motion patterns from a normative subject cohort. We examine the viability of using two controllers—a joint-space feedback linearized PD controller and a task-space force-control strategy—to realize trajectory- and path-tracking of the desired motions within a simulation environment. In particular, we examine performance in terms of (i) coordinated control of the redundant system; (ii) reducing internal stresses within the lower-extremity joints; and (iii) continued satisfaction of the unilateral cable-tension constraints throughout the workspace.

1 INTRODUCTION

Several neurological disorders including stroke, spinal cord injury, cerebellar disorders, and neuromuscular diseases manifest themselves via generation of abnormal patterns of lower limb motion. Numerous studies have noted that systematic deployment of rehabilitation regimen (of adequate intensity, duration, and consistency) can help restore motor functionalities in such patients [1]. However, significant challenges exist for realization of an automated rehabilitation system that can function in close coordination with a patient’s musculoskeletal system.

Traditionally, lower limbs motor therapy is carried out manually, requiring multiple (often 3 or more) physiotherapists. The difficulty and inconsistency in therapy from one session to the next motivated researchers to develop gait training treadmills with body weight support to provide consistent gait motion [2–4]. Our interest is in rehabilitation systems which allow for significant greater flexibility of therapeutic motions/forces as well as customization of the training regimen. Volpe et al. [5] review the successes noted from several groups in reducing impairment and enhancing human motor control with task specific exercises delivered by robotic devices. It is worth noting that several exoskeleton systems such as BLEEX [6], MIT exoskeleton [7], HAL-3 [8] have been developed (in non-rehabilitation contexts) to augment human walking. Additionally, many active/passive orthoses such as gravity-balancing leg orthosis [9], pneumatic-muscle powered ankle-foot orthosis [10], elastic knee orthosis [11], ALEX [12], and LOPES [13] have been developed.

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Several of these architectures are now being evaluated for rehabilitation application potential to help patients to overcome muscle-weakeness and remedy abnormal motor control etc. The challenge arises both for design of robotic system with adequate degrees of freedom to not hinder natural gait kinematics/dynamics as well as coordinated control to achieve the normal physiological gait patterns. Among clinically-deployed rehabilitation systems, Lokomat [14] is an exoskeletal orthosis which is attached to a patient’s legs to assist the person to walk on the treadmill. The Haptic Walker [15] is a multidegree-of-freedom system intended to generates foot pedal motions to simulate regular and stair walking. However, in addition to performance limitations, these devices are expensive and only available in some clinical or rehabilitation centers.

Hence, in this paper, we examine the feasibility of a light-weight and reconfigurable hybrid (articulated multibody and cable) robotic system for assisting lower-extremity rehabilitation and analyze its performance. Many of the commonly prescribed rehabilitation exercises engender closed-loop ankle trajectories which can be realized by an appropriate cable-driven rehabilitation device. Such closed-loop ankle trajectories can serve to guide the other leg-members (shank/thigh) via the natural kinematics and dynamics of the limbs. Cable robots are well-known for their low inertia, relatively large workspace, low fabrication costs and reconfigurability. The use of cables allows the relatively heavy motors and gearboxes to be moved from the joints to the base. This reduces the mass and inertia of the moving bodies and can allow the robot to be designed with smaller, less costly motors.

Several recent exoskeleton designs have emerged to take advantage of the cable-based architecture. In [16] a cable-driven robotic gait training system called CaLT was designed for gait training of spinal cord injury patients, and it delivered a promising and acceptable experimental results. In this end-effector type apparatus cables are directly connected to the subject’s ankle, and only apply force at the ankle. In the C-ALEX [17] design, no rigid or joints are employed within the exoskeleton. Instead, three cuffs are connected to the waist, thigh, and shank, and four cables are routed through the cuffs to actuate two degrees-of-freedom of the human user lower limbs. The exoskeleton is controlled in force mode using assist-as-needed control paradigm. Our proposed design differs in seeking to take advantage of the hybrid cable-articulated architecture.

The main challenge in any cable-robot control scheme is satisfaction of the tensionability conditions, i.e., assuring that all cables are always in tension. The manipulation is performed via increasing and decreasing the lengths of the cables connected to the end-effector. One can increase the number of redundant cables to satisfy the tensionability of the cables. However, this increases the interference of the cables in the working space. The minimum sufficient number of cables in multibody systems that guarantees all cables are in tension have been investigated in [18].

In traditional cable robots, multiple cables are attached to single-payload/platform from multiple points on the ground. In contrast, in the proposed articulated multibody cable-robot system, cables are connected to different links of the multibody system from multiple points on the ground as illustrated in Fig. 1. In this work, we assume a simplified model wherein each human lower limb segment (i.e., foot, shank and thigh) and its corresponding orthosis-link are considered as one part. Thus, each human lower limbs maybe considered as a serial multibody system driven by the cables attached to them.

Limited literature has examined multi-body articulated cable-drive systems in the past. For example, the wrench closure workspace of multibody cable-driven mechanisms is determined based on Lagrange’s approach [18] and reciprocal screw theory [19]. The concept of generalized forces and Lagrange’s method have been employed to eliminate forces and moments from the equilibrium equations. It is noteworthy that the operational workspace of multibody cable-driven systems is largely impacted by the choice of cable placement and routing. Bryson and Agrawal [20] identified and analyzed cable configurations for serial robot driven by cables. Yan et al. [21] designed a 7-DOF humanoid arm driven with 14 cables. They employed the force closure method to evaluate the workspace of multi-finger grasping. In [22, 23], a hybrid articulated-cable parallel mechanism named PACER developed by authors for upper limb rehabilitation in 3D space in which with appropriate design, and proper selection of the position of cable winches and cable attachment points to the multibody system, the positive tension in the cables assured.

The key contribution of this paper is the development and feasibility/performance evaluation of a reconfigurable hybrid cable-articulated architecture that will work closely with the human musculoskeletal system to provide motor therapies. This system is an extension of traditional single-body cable-driven to multibody cable-driven system. The combined system features multiple holonomic cable-loop-closure constraints acting on a tree-structured multibody
system. The wrench feasible workspace \cite{24} of RObotic Physical Exercise and System (ROPES) is determined and improved by adding linear tensional and torsional springs to articulated-multibody system to cover whole operational workspace of the prescribed ankle trajectory. Such springs help in keeping cables taut, and result in larger workspace. There remains significant design-freedom in determining the location of base spooling motors as well as the sizing of the cable-attachment (cuffs) to the articulated-leg-frame. We highlight the opportunity to exploit this freedom by design-optimization to enhance the functional performance. Specifically, the focus of this paper is to assist the performance of repetitive therapy of human lower limbs in sagittal plane. The desired motion is prescribed based upon normative sub-jects’ motion patterns. The appropriate coordination of cable forces to each segment of the lower limbs is realized by an impedance/force-field and a feedback linearized PD controllers to (i) assure the tensionability in cables, and (ii) avoid increasing the internal forces at the lower extremity joints.

The remainder of the paper is organized as follows: In Section 2, the kinematic analysis of ROPES is presented. The dynamic equations of motion of ROPES using the Newton-Euler, and Lagrangian approach are derived in both joint-space and task-space in Section 3. In Section 4, the optimal cable configuration analysis of ROPES is identified, and then the tension distribution in the resulting optimal configuration is examined in Section 5, and Section 6 and 7 are devoted to designing feedback-linearized PD controller and impedance/force-field controller for ROPES, and finally Section 8 concludes the paper.

2 KINEMATIC ANALYSIS OF ROPES

In order to describe the motion of the lower limbs driven with cables, the coordinate systems \{F\}, \{0\}, \{1\}, \{2\} and \{3\} are attached to the trunk, hip, knee, ankle and the end of the foot, respectively, as illustrated in Fig. 2. The cable length \(l_i\), cable unit vector \(t_i\), and Jacobian matrix \(J_F\) can be found by writing the loop-closure equations for each cable. In these relations \(w_i\) denotes the winch position respect to the fixed frame \{F\}, and \(u_i\) denotes the position vector of cable attachment point to orthosis respect to the fixed frame \{F\}.

\[
\begin{align*}
\mathbf{u}_i - \mathbf{w}_i + i l_i &= 0, i \in \{1, 2, 3, 4\} \quad (1) \\
l_i &= \sqrt{\mathbf{(w}_i - \mathbf{u}_i)^T (\mathbf{w}_i - \mathbf{u}_i)} \quad (2) \\
\mathbf{t}_i &= (\mathbf{w}_i - \mathbf{u}_i)/l_i \quad (3)
\end{align*}
\]

By taking the derivative of both sides of Eqn. (1) with respect to time, and multiplying both sides of resulting equation to \(t_i^T\), it gives the Jacobian matrix as follows

\[
\dot{l}_i = -t_i^T \frac{\partial u_i}{\partial \mathbf{q}} \dot{\mathbf{q}} = -J_F \dot{\mathbf{q}} \quad (4)
\]

where \(\dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2, \dot{q}_3]^T\) (note that \(q_1 = \frac{3\pi}{2} - \theta_h\), \(q_2 = \theta_k\) and \(q_3 = \frac{3\pi}{2} - \theta_a\) which \(\theta_h, \theta_k, \theta_a\) are hip, knee and ankle joint angles as shown in Fig. 2), and \(J_F = t_i^T \frac{\partial u_i}{\partial \mathbf{q}}\) is a Jacobian matrix which maps joint angular velocities to the cable velocities.

The angular velocity and acceleration of each segment i.e., thigh, shank and foot in sagittal plane can be found such that for thigh is, \(\omega = \dot{q}_1, \omega = \dot{q}_1\), and linear acceleration of the mass center of thigh is \(\mathbf{a}_t = \dot{\mathbf{q}}_1 + \dot{\mathbf{q}}_1 \times (\mathbf{q}_1 \times \mathbf{a}_t)\), where \(\mathbf{a}_t = R_t \mathbf{a}_c\) is the thigh mass center vector expressed in fixed frame; \(\mathbf{a}_c\) is the thigh mass center vector expressed in frame \{1\} (position vector from the origin of frame \{0\} to the mass center of thigh), and \(R_t\) is rotation matrix from fixed frame to ith frame of reference.

Similarly, the angular velocity and acceleration for the shank can be expressed by, \(\omega_s = \dot{q}_1 + \dot{q}_2, \omega_s = \dot{q}_1 + \dot{q}_2\), and linear acceleration of the mass center of shank is \(\mathbf{a}_s = \mathbf{a}_t + \mathbf{a}_s \times (\mathbf{a}_s \times \mathbf{a}_t)\), where \(\mathbf{a}_s = R_s \mathbf{a}_c\) is the link (shank) mass center vector expressed in fixed frame; \(\mathbf{a}_c\) is the shank mass center vector expressed in frame \{2\} (position vector from the origin of frame \{1\} to the mass center of shank), and linear acceleration \(\mathbf{a}_s\) can be found by substitution of \(\mathbf{a}_c\) instead of \(\mathbf{a}_t\) in equation of \(\mathbf{a}_c\).

Finally for the foot, the angular velocity and acceleration can be written as, \(\omega_f = \dot{q}_1 + \dot{q}_2 + \dot{q}_3, \omega_f = \dot{q}_1 + \dot{q}_2 + \dot{q}_3\), and linear acceleration is \(\mathbf{a}_f = \mathbf{a}_s + \mathbf{a}_f \times (\mathbf{a}_f \times \mathbf{a}_s)\), where \(\mathbf{a}_f = R_f \mathbf{a}_c\) is the foot mass center vector expressed in fixed frame, and \(\mathbf{a}_c\) is the foot mass center vector expressed in frame \{3\} (position vector from the origin of frame \{2\} to the mass center of the foot), and linear acceleration \(\mathbf{a}_c\) can be found by substitution of \(\mathbf{a}_c\) instead of \(\mathbf{a}_t\) in equation of \(\mathbf{a}_c\).

2.1 Cable attachments in ROPES

The combined human leg and orthosis is controlled in sagittal plane using four cables. More cables may unnec-
necessarily complicate the workspace design and analysis, and fewer than four cables makes it impossible for unilateral cable-tension constraints to be satisfied. The cables are connected to each link of orthosis using cuffs. The cuff for the thigh link is of radius \( d_{3y} \) and is positioned a distance \( d_{3x} \) from the origin of frame \{1\}. The cuff for the shank link is of radius \( d_{3y} \) and is placed a distance \( d_{3x} \) from the origin of frame \{1\}. Similarly, the cuffs for the foot link are of radius \( d_{3y} \) and \( d_{1y} \), respectively, and they are positioned a distance \( d_{3x} \) and \( d_{1x} \) from the origin of frame \{2\}. It is assumed each cable attachment point, \( C_i \), as shown in Fig. 2, is placed on a circle with center of \( C_0 \), of radius \( R \), and angle \( \theta_i, i = \{1, 2, 3, 4\} \) respect to the fixed coordinate frame.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Units</th>
<th>Parameter</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{1y} )</td>
<td>[45,85]</td>
<td>mm</td>
<td>( d_{2y} )</td>
<td>[50,85]</td>
<td>mm</td>
</tr>
<tr>
<td>( d_{3y} )</td>
<td>[55,95]</td>
<td>mm</td>
<td>( d_{3y} )</td>
<td>[75,125]</td>
<td>mm</td>
</tr>
<tr>
<td>( d_{1x} )</td>
<td>[70,220]</td>
<td>mm</td>
<td>( d_{2x} )</td>
<td>[70,220]</td>
<td>mm</td>
</tr>
<tr>
<td>( d_{3x} )</td>
<td>[110,440]</td>
<td>mm</td>
<td>( d_{4x} )</td>
<td>[150,450]</td>
<td>mm</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>[170,195]</td>
<td>deg</td>
<td>( \theta_2 )</td>
<td>[260,330]</td>
<td>deg</td>
</tr>
<tr>
<td>( \theta_3 )</td>
<td>[-10,80]</td>
<td>deg</td>
<td>( \theta_4 )</td>
<td>[110,165]</td>
<td>deg</td>
</tr>
</tbody>
</table>

The cable configuration parameters, as described in Fig. 2, are allowed to vary within the ranges given in Table 1. For example, the radius of cable cuff attached to the thigh, \( d_{3y} \), is allowed to vary from the 75 mm to 125 mm in increments of 5 mm, and its position, \( d_{3x} \), is allowed to range from 150 mm to 450 mm in increments of 5 mm.

### 3 DYNAMIC EQUATIONS OF MOTION OF LOWER LIMBS DRIVEN WITH CABLES

Guaranteeing positive cable tensions for the cable-driven articulated orthosis (within an adequately large work-region) remains a critical issue. Hence the dynamic equations of human lower limb (orthosis + human leg) were developed to facilitate model-based control of the tension during the rehabilitation exercises. The exercises include normal walking in the sagittal plane and lateral leg-lifting in frontal plane. For this purpose, two dimensional equations of motion for each segment of the human leg was formulated separately in the sagittal and frontal planes. In sagittal plane, the hip and knee flexion/extension motion and the dorsiflexion/plantar flexion motion of the ankle are considered. Similarly, the equation of motion for lateral leg-lifting exercise is derived in the frontal plane while considering additional degrees-of-freedom at the hip (abduction/adduction) and ankle (inversion/eversion) joints.

As shown in [25], adding springs between ground and multibody or between links can improve wrench feasible workspace of cable-driven system. Exploiting this idea within the cable-articulated orthosis, one linear tensional spring is attached between the fixed ground and the thigh, \( K_{f} \), and another tensional spring is connected the thigh to the shank, \( K_{s} \). Such springs help in keeping cables taut, and improve the wrench feasible workspace without adding redundant cables to cover whole operational workspace of the predefined ankle trajectory. Without loss of generality, the springs are modeled as linear springs with stiffness constant \( K \) and zero free-length. The generated force will then become \( K_{f}l_{f} \) which \( l_{f} \) is the vector of zero-free length springs.

Many designers of orthoses and prostheses have sized their devices based on the average kinetic and kinematic data of human [26, 27]. The compliance of lower extremity joints during locomotion can be investigated by the concept of quasi-stiffness. This term can be distinguished from the passive and active stiffness of a joint typically used to describe the local tangent to the moment-angle curve shown for given joint at a specific angle [28]. As illustrated in Fig. 3, the quasi-stiffness of a joint is defined more globally, as the slope of the best linear fit on the moment-angle curve of a joint over a whole stride or specific phase of a stride [29]. A series of empirical studies [29–31] have been conducted to characterize the quasi-stiffness and linear behavior of lower extremity joints during walking for adult humans as function of body size (height and weight). This forms the basis of a normative anthropometric statistical model which we will employ to predict the hip, knee and ankle quasi-stiffness for adults walking on level ground. The mean value of quasi-stiffness of each joint is enumerated in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip quasi-stiffness in extension phase ( K_{He} )</td>
<td>320</td>
</tr>
<tr>
<td>Hip quasi-stiffness in flexion phase ( K_{Hf} )</td>
<td>335</td>
</tr>
<tr>
<td>Knee quasi-stiffness in extension phase ( K_{Ke} )</td>
<td>263</td>
</tr>
<tr>
<td>Knee quasi-stiffness in flexion phase ( K_{Kf} )</td>
<td>304</td>
</tr>
<tr>
<td>Ankle quasi-stiffness in plantar-flexion phase ( K_{Ap} )</td>
<td>202</td>
</tr>
<tr>
<td>Ankle quasi-stiffness in dorsiflexion phase ( K_{Ad} )</td>
<td>246</td>
</tr>
<tr>
<td>Hip stiffness on the orthesis ( K_{OH} )</td>
<td>327</td>
</tr>
<tr>
<td>Knee stiffness on the orthesis ( K_{OK} )</td>
<td>283</td>
</tr>
<tr>
<td>Ankle stiffness on the orthesis ( K_{OA} )</td>
<td>224</td>
</tr>
</tbody>
</table>

Moreover, additional torsional springs are placed on the orthesis at hip \( K_{OH} \), knee \( K_{OK} \), and ankle \( K_{OA} \) joints, and their values are defined in Table 2.

#### 3.1 Newton-Euler dynamic formulation in sagittal plane

In this section, a Newton-Euler formulation is used to derive dynamics equations of the cable-driven articulated orthosis (Fig. 2) in the sagittal plane. Such a formulation is
very useful in a design/simulation setting in helping analyze force-profiles (both internal forces/moments within as well as the external forces needed to drive) the cable-articulated orthosis. Each segment’s dynamic equations, subjected to cable-and-constraint-forces, is derived separately. Note that each segment is subjected to cable forces as well as constraint forces and moments. Since the weight and moment of inertia of the orthosis segments and cuffs are much smaller than lower limbs segments, their weight and moment of inertia are ignored in this simulation. The recursive Newton-Euler formulation in the sagittal plane is written for the foot segment in matrix form as follows (notice that boldface symbols have been used for vectors and matrices),

\[
\begin{bmatrix}
  \mathbf{t}_1 \\
  \mathbf{t}_2 \\
  \mathbf{t}_{sf} \\
  \mathbf{u}_{f1} \times \mathbf{t}_1 \\
  \mathbf{u}_{f2} \times \mathbf{t}_2 \\
  \mathbf{u}_{f3} \times \mathbf{t}_{sf} \\
  \mathbf{m}_{f1} \\
  \mathbf{m}_{f2} \\
  \mathbf{m}_{f3}
\end{bmatrix}
\begin{bmatrix}
  \mathbf{T}_1 \\
  \mathbf{T}_2 \\
  \mathbf{T}_{sf} \\
  \mathbf{F}_{f1} \\
  \mathbf{F}_{f2} \\
  \mathbf{F}_{f3} \\
  \mathbf{M}_{f1} \\
  \mathbf{M}_{f2} \\
  \mathbf{M}_{f3}
\end{bmatrix} =
\begin{bmatrix}
  \mathbf{t}_3 \\
  -\mathbf{t}_{sf} \\
  \mathbf{t}_{ts} \\
  \mathbf{u}_{f3} \times \mathbf{t}_3 \\
  -\mathbf{u}_{f2} \times \mathbf{t}_{sf} \\
  \mathbf{u}_{f1} \times \mathbf{t}_{ts} \\
  -\mathbf{m}_{f} \\
  \mathbf{m}_{s} \\
  \mathbf{m}_{m}
\end{bmatrix}
\]

where \( \mathbf{J}_k = (K_{Ke/KF} + K_{0K})q_2 - \mathbf{u}_{s2} \times \mathbf{K}_{2i} \mathbf{l}_2 \), \( \mathbf{I}_k = \mathbf{R}_k^T \mathbf{I}_k \mathbf{R}_k \); \( \mathbf{t}_3 \) is the unit vector along the cable with magnitude \( T_3 \); \( \mathbf{t}_{ts} \) and \( \mathbf{m}_{s} \) are the unit vectors of force \( F_{ts} \) and moment \( M_{ts} \) exerted from thigh to the shank; \( m_{s} \) is the mass of the shank; \( \mathbf{u}_{si} \), \( i = \{1, 2, 3\} \) are the position vectors from the mass center of the shank to the point of application of the applied force expressed in the fixed frame; \( K_{Ke/KF} \) is quasi-stiffness of knee which depends on the stance phase during walking, for example, \( K_{Ke/KF} = K_{Ke} \) in flexion phase, \( K_{Ke/KF} = K_{Ke} \) in extension phase; \( \mathbf{J}_h \) is the position vector from the mass center of the thigh to the point of application of the applied force expressed in the fixed frame; \( K_{He/Hi} \) is quasi-stiffness of hip which depends on the stance phase during walking, for example, \( K_{He/Hi} = K_{He} \) in flexion phase, \( K_{He/Hi} = K_{He} \) in extension phase; \( \mathbf{Q}_i \) is the initial angular position of the shank to the spring attachment point on the orthosis; the generated force by the linear axial springs is equal to \( K_{2i} \mathbf{l}_2 \) which \( \mathbf{l}_2 \) is the vector of zero-free length spring stiffness and vector along the spring, respectively; \( \mathbf{s}_{2i} \) is the position vector from the mass center of the shank to the spring attachment point on the orthosis; the generated force by the linear axial springs is equal to \( K_{2i} \mathbf{l}_2 \) which \( \mathbf{l}_2 \) is the vector of zero-free length spring. This spring provides a tensile force proportional to its length.

Equation (6) can be written compactly as \( \mathbf{J}_f \mathbf{T}_f = \mathbf{F}_f \), Newton-Euler formulation for thigh can also be expressed by \( \mathbf{J}_f \mathbf{T}_f = \mathbf{F}_f \), where

\[
\begin{bmatrix}
  \mathbf{t}_4 \\
  \mathbf{u}_{f4} \times \mathbf{t}_4 \\
  -\mathbf{u}_{f2} \times \mathbf{t}_{si} \\
  \mathbf{u}_{f1} \times \mathbf{t}_{si} \\
  -\mathbf{m}_{f4} \\
  \mathbf{m}_{s4} \\
  \mathbf{m}_{m4}
\end{bmatrix}
\begin{bmatrix}
  \mathbf{T}_4 \\
  \mathbf{T}_{sf} \\
  \mathbf{F}_{f4} \\
  \mathbf{M}_{f4} \\
  \mathbf{M}_{s4} \\
  \mathbf{M}_{m4}
\end{bmatrix} =
\begin{bmatrix}
  \mathbf{t}_3 \\
  -\mathbf{t}_{sf} \\
  \mathbf{t}_{ts} \\
  \mathbf{u}_{f3} \times \mathbf{t}_3 \\
  -\mathbf{u}_{f2} \times \mathbf{t}_{sf} \\
  \mathbf{u}_{f1} \times \mathbf{t}_{ts} \\
  -\mathbf{m}_{f} \\
  \mathbf{m}_{s} \\
  \mathbf{m}_{m}
\end{bmatrix}
\]

\[
\begin{bmatrix}
  \mathbf{t}_4 \\
  \mathbf{u}_{f4} \times \mathbf{t}_4 \\
  -\mathbf{u}_{f2} \times \mathbf{t}_{si} \\
  \mathbf{u}_{f1} \times \mathbf{t}_{si} \\
  -\mathbf{m}_{f4} \\
  \mathbf{m}_{s4} \\
  \mathbf{m}_{m4}
\end{bmatrix}
\begin{bmatrix}
  \mathbf{T}_4 \\
  \mathbf{T}_{sf} \\
  \mathbf{F}_{f4} \\
  \mathbf{M}_{f4} \\
  \mathbf{M}_{s4} \\
  \mathbf{M}_{m4}
\end{bmatrix} =
\begin{bmatrix}
  \mathbf{t}_3 \\
  -\mathbf{t}_{sf} \\
  \mathbf{t}_{ts} \\
  \mathbf{u}_{f3} \times \mathbf{t}_3 \\
  -\mathbf{u}_{f2} \times \mathbf{t}_{sf} \\
  \mathbf{u}_{f1} \times \mathbf{t}_{ts} \\
  -\mathbf{m}_{f} \\
  \mathbf{m}_{s} \\
  \mathbf{m}_{m}
\end{bmatrix}
\]
the spring attachment point on the orthosis, and \(K_s\) and \(l_1\) are zero-free length tensile spring stiffness constant and vector along the spring, respectively.

By assembling Eqns. (5), (6) and (7) the number of independent action-reaction forces and moments at the joint can be reduced, and the compact form dynamic equations of lower limb motion with cable-driven system in sagittal plane can be obtained as

\[ J_{9 \times 10} T_{10 \times 1} = F_{9 \times 1} \]  

where

\[
J = \begin{bmatrix}
J_f & J_s & J_l & 0 & 0 & 0 & J_f & J_s & J_l & 0 \\
0 & 0 & 0 & J_l & J_f & J_s & 0 & 0 & J_l & J_f \\
0 & 0 & 0 & 0 & J_s & J_l & J_f & 0 & 0 & J_s & J_l & J_f \\
\end{bmatrix}
\]

\[
T = [T_1 \ T_2 \ T_3 \ T_4 \ F_{f1} \ F_{b1} \ M_{f1} \ M_{b1}]^T
\]

\[
F = [F_f^T \ F_s^T \ F_l^T]^T
\]

where \(J_f\), \(J_s\) and \(J_l\) are the \(i\)th column of the \(J_f\), \(J_s\) and \(J_l\), respectively, defined in Eqns. (5), (6) and (7). Similarly, dynamic equations of each segment of human leg can be derived separately for the orthosis in the frontal plane (but omitted here for brevity [32]).

### 3.2 Closed-form dynamic formulation in joint-space

The Lagrangian formulation is developed to reduce the size of structure matrix \(J\) in Eqn. (8). The general form of dynamic equations for the cable-driven system in joint-space can be written as

\[ M(q) \ddot{q} + V(\dot{q}, q) + G(q) = \dot{Q} \]  

where vector \(q = [q_1, q_2, q_3]^T\) are the generalized coordinates; \(M\) is the inertial matrix which is positive-definite, symmetric and hence invertible; \(V = [V_1, V_2, V_3]^T\) is the velocity coupling vector which includes velocity-squared terms (centrifugal forces) and velocity product terms (Coriolis forces), and \(G = [G_1, G_2, G_3]^T\) is combination of gravitational forces and spring forces.

Except for gravitational and inertial forces, the generalized forces \(Q\) account for all other forces acting on lower limb. So, the contribution of cable forces to the dynamics of multi-body system is modeled as joint forces applied to the links. By the principle of virtual work

\[ Q^T \delta q = F_{c}^T \delta P + \sum_{i=1}^{4} T_{i}^T (\delta X_{Ti}) \]  

where \(J_c\) is the conventional Jacobian matrix which maps end-effector output force into n-dimensional joint torques and \(J_T\) is a Jacobian which maps cable tensions into the joint torques (Eqn. (4)); \(\delta P\) and \(\delta X_{Ti}\) denote the virtual displacement vector of the end-effector and cable attachment point on each segment of lower limb, then they are substituted by the relations \(\delta P = J_c \delta q\) and \(\delta X_{Ti} = \frac{\partial u_i}{\partial q} \delta q\), respectively; the vector \(u_i\) denotes the position vector of cable attachment points with respect to fixed frame of reference.

One can find the generalized coordinates as

\[ Q = J_T^T \dot{T} + J_f^T F_c \]  

where

\[
J_T = \begin{bmatrix}
t_1^T \frac{\partial u_1}{\partial q_1} & t_2^T \frac{\partial u_2}{\partial q_1} & t_3^T \frac{\partial u_3}{\partial q_1} & t_4^T \frac{\partial u_4}{\partial q_1} \\
t_1^T \frac{\partial u_1}{\partial q_2} & t_2^T \frac{\partial u_2}{\partial q_2} & t_3^T \frac{\partial u_3}{\partial q_2} & t_4^T \frac{\partial u_4}{\partial q_2} \\
t_1^T \frac{\partial u_1}{\partial q_3} & t_2^T \frac{\partial u_2}{\partial q_3} & t_3^T \frac{\partial u_3}{\partial q_3} & t_4^T \frac{\partial u_4}{\partial q_3} \\
\end{bmatrix}
\]

where \(J_T^T\) is the part of generalized force related to cable forces and \(J_f^T F_c\) includes generalized external forces and moments \((F_c = [F_{ext}, M_{ext}]^T)\).

This form is well suited for trajectory tracking control applications where the desired end-effector trajectory is presented in terms of joint angles, velocities, and accelerations for which a feedback linearization controller is developed.

### 3.3 Closed-form dynamic formulation in task-space

However, in other circumstances (e.g. development of an impedance controller) we may wish to move the ankle on a trajectory in Cartesian space without converting task-space variables. In such cases, the joint-space dynamics can be projected into the task-space to realize the (often lower-order) task-space dynamics equations as follows

\[ \dot{M} \ddot{P} + \dot{V} + \dot{G} = \dot{Q} \]  

where \(M = J_c^T M J_c^{-1}\), \(\dot{V} = J_c^T (V - MJ_c^{-1} J_c \dot{q})\), \(\dot{G} = J_c^T G\), and \(\dot{Q} = J_c^T \dot{Q}\).

It is noteworthy that although the equations are expressed in task-space, still some terms such as \(V\), \(G\), and \(Q\) are written as function of joint variables \(q\). Due to nonlinearity of inverse kinematics, it is impossible to write everything in terms of task-space variables, \(P\).

### 4 PRELIMINARY CONFIGURATION ANALYSIS IN ROPES VIA MONTE CARLO APPROACH

In order to account for the positive tension condition present in cable-driven systems, more useful workspaces have been proposed. Two of the more commonly utilized ones are introduced here. The wrench-closure workspace (WCW) [33] (also called the controllable workspace [34], or force-closure workspace [35, 36]), is defined as the set of end effector poses for which any arbitrary wrench can be resisted/exerted by the platform while maintaining positive...
cable tensions. This workspace depends only on the geometric parameters of the system i.e., the locations of the cable attachment points on the base and platform and the pose of the platform. Several algorithms have been proposed for efficiently computing the boundary of the wrench-closure workspace.

If \( J_T^T \) is full rank, then Eqn. (11) is under-determined and has many tension solutions for a given pose, \( q \), and desired set of \( Q - J_T^T F_r \). In general, these tension solutions may not be strictly positive, but only positive cable tensions are valid for controlling ROPES. Solving Eqn. (11) for the tension vector, \( T \), yields the following Eqn. (13):

\[
T = (J_T^T)^\dagger (Q - J_T^T F_r) + \eta_{T_{null}} \lambda
\]

where \((J_T^T)^\dagger\) is the Moore-Penrose pseudo-inverse of matrix \( J_T^T \), \( \lambda \) is an arbitrary constant, and \( \eta_{T_{null}} \) is a basis for the null space of \( J_T^T \) such that \( J_T^T \eta_{T_{null}} = 0 \).

For this case, since \( J_T^T \) is \( 3 \times 4 \), then \( \eta_{T_{null}} \) is \( 4 \times 1 \) vector. The \( \lambda = 0 \) case of Eqn. (13) is the minimum norm solution but the tension vector components are not guaranteed to be non-negative.

If all elements of the null vector, \( \eta_{T_{null}} \), have the same sign, then it can be seen that regardless of minimum norm solution values a \( \lambda \) can be chosen such that the \( \eta_{T_{null}} \) term be more than particular solution and gives a tension solution which all components are positive. Here, we calculate the wrench-closure workspace by exploiting this characteristic (i.e., if all elements of the null vector are the same sign, then the point belongs to the wrench-closure workspace). Since the conditions of the wrench-closure workspace are quite strict, and set of wrenches that the end-effector will have to resist/exert are known, a more useful workspace called wrench-feasible workspace (WFW) [24] can be used such that cable tensions are greater than some prescribed minimum and less than some prescribed maximum values. This workspace depends not only on geometric parameters, but also on the allowable tension ranges, gravitational effects, and the required wrench set.

The choice of cable attachment both on fixed frame and cuffs have significant effects on the operational workspace of the ROPES. Inappropriate selections in cable attachments can mitigate or restrain the ability of ROPES to perform the prescribed task, while a proper configuration might provide additional capabilities and enlarge the wrench-feasible workspace.

4.1 Optimization of configuration parameters

Table 3 shows parameter values for subject’s thigh, shank, and foot (length, weight, moment of inertia, and mass-center location) taken from an anthropometric database [37]. From the a typical walking gait in task-space, the desired task in joint-space is generated and shown Fig. 4. The desired workspace in joint-space is also defined as \( \theta_b = -10^\circ \) to \( 50^\circ \) (or \( q_1 = 220^\circ \) to \( 280^\circ \)), \( \theta_k = q_2 = 0^\circ \) to \( 60^\circ \), and \( \theta_d = -10^\circ \) to \( 25^\circ \) (or \( q_3 = 245^\circ \) to \( 280^\circ \)).

The total number of independent parameters is 12 (eight cuff parameters \( d_{ix} \) and \( d_{iy} \) which \( i = \{1, 2, 3, 4\} \), and four angles for the position of cable attachment points to the fixed frame, \( \theta_i \)). Systematically determining each configuration in the range defined in Table 1 would be extremely time consuming. Thus, alternatively, a Monte Carlo approach is employed to rapidly explore configurations within quite large independent configurations.

To accomplish this analysis, a random value for each of these 12 independent parameters in a given range, shown in Table 1, is selected. Then, the subsequent randomly selected configuration is checked for wrench feasible workspace. Configurations which satisfy the wrench feasibility constraints are accumulated in database of desired configurations.

To visualize the trends of these high-dimensional data-set, each possible parameter value is shown in separate x-axis on the plot, in a range defined for parameter values in Table 1, and the number of wrench feasible configurations including each possible parameter value is drawn on y-axis (see Fig. 5, 6, and 7).

Figures 5, 6, and 7 show configuration analysis plot which all configuration parameters were randomly selected with uniform distribution. These plots can provide insight into the parameter value trends which result in high-performance (higher WFW). If higher WFW be achieved in a range defined for parameter values shown in Table 1, ROPES would be more robust to errors in setup and assembly. As shown in Fig. 5, the trend for \( d_{ix} \) suggests that parameter should be as close as possible to ankle joint, and trend for \( d_{2x} \) suggests that the parameter should be as far as possible from the ankle joint. The trends for \( d_{3x} \) and \( d_{4x} \) exhibit that it is preferable to select a value in range of 220°.
Table 3. Anthropometry and mass distribution for human body [37]

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5 TENSION DISTRIBUTION IN ROPES

A variety of tension distribution algorithms have been proposed for resolving the actuation redundancy present in most cable-driven robots. Each of these approaches have different characteristics and varying computational cost [39].

Perhaps the most commonly implemented method in the literature relies upon the minimization (or maximization) of some function of the cable tensions. In [34], it was shown that while the $L_\infty$-norm provides optimal solutions, they may be discontinuous along a given trajectory. The optimal solution can, however, be approximated using a $p$-norm ($1 < p < \infty$), and the resulting minimum-norm solution is proven to be unique and continuous except at singular configurations.

In [40], a discussion on the potential limitations of the more traditional $L_1$ and $L_2$ norms is provided in the context of tension distribution. Use of the $L_1$-norm results in a linear programming problem which seeks to minimize (or maximize) the sum of the tensions. The primary drawback of this approach is that it is susceptible to discontinuities, as the optimal operating point may jump from one vertex of the feasible polyhedron to another between successive computations. This can potentially excite high-frequency modes and degrade the stability of the system. The $L_2$-norm, which results in a quadratic programming problem and seeks to minimize (or maximize) the sum of the squared tensions, improves upon this limitation by providing a smooth objective function.

Both methods, however, result in tensions that frequently lie on the lower or upper tension limits. Operation at the lower tension limits increases the risk of a cable
becoming slack, and can result in low stiffness properties. Operation at the upper tension limits results in excessively high torque requirements. Thus, the alternative methods presented by these authors allow for the cable tensions to be steered towards a desired region of operation. Despite the potential limitations of utilizing the $L_1$ and $L_2$ norms for the purposes of cable tension distribution, efficient algorithms for these problems are readily found in general optimization packages.

The tensionability of the multibody system, driven by cables, can be evaluated by the analysis of rank and null space of the structure matrix $J_f^T$ of the multibody system. This equation at each instant of time is a set of linear equations in terms of $T$. The size of Jacobian matrix $J_f^T$ is $3 \times 4$ which indicates that the number of unknowns is one more than the number of equations. If the number of unknowns was equal to the number of equations, there would be one and only one solution for each cable.

While a variety of redundancy resolution techniques have been proposed, one approach commonly used is based on minimizing the norm of the cable tensions. This has the beneficial effect of minimizing the energy and torque requirements of the system. The optimization problem can be formulated as:

$$\min \|T\|_p \quad (14)$$

subject to: $J_f^T T = Q - J_f^T F_e$ and $T_{\min} \leq T_i \leq T_{\max}, i = \{1, 2, 3, 4\}$. Thus, the optimized cable tensions must satisfy the dynamic equilibrium equations and remain within some specified upper and lower bounds. In general, the lower limit corresponds to the amount of tension required to keep the cables taut, while the upper bound depends on the torque capacity of the motors and/or the failure point of the cables.

6 TRAJECTORY TRACKING CONTROLLER DESIGN FOR THE MULTIBODY CABLE-DRIVEN SYSTEM

Appropriate controller design is a critical aspect of development of rehabilitative robots and motor therapy. In [41], control strategies have been categorized in four groups. The first group is assisting controllers which move the patients injured limbs in a predefined trajectory to stretch the limb muscles and rebuild the human motor control system. Effort is thought to be essential for provoking motor plasticity [42], and stretching can help prevent stiffening of soft tissue and reduce spasticity, at least temporarily [43].

Impedance control is the main approach in assisting control paradigm which helps the patient to follow the desired trajectory with some deviation, depending on the impedance gains [44]. More recent controllers have used more sophisticated forms of mechanical impedance than stiffness, including for example viscous force fields [45], and creating virtual objects that assist in achieving the desired movement [46].

The second group is challenge-based controllers which try to strengthen the muscles by providing resistance against the movement, hence it increases the error [47, 48]. Simulating everyday normal activities by using haptic interfaces is the third group of controllers [49]. Finally, the last group is robot for encouraging patients for performing exercises [50]. All these controllers are always implemented using position, force or impedance control.

Here, the trajectory tracking controller tracks the desired normal cycle using a feedback linearized PD controller. The desired trajectory was obtained from recorded data of healthy subject during walking with attached markers to human leg as shown in Fig. 4 [38].

In this controller, the desired trajectory in terms of generalized coordinates is defined as a function of time $q_d = q_d(t)$. The block diagram of the proposed control law for trajectory tracking system is shown in Fig. 8. Hence, from closed-form dynamic equations (Eqn. 9) one can write the virtual control law as

$$\tau_1 = M(q) (\ddot{q}_d + K_p q_e + K_v q_v) + V(q, q) + G(q) \quad (15)$$

This law linearizes the equations to an exponentially stable system, $\ddot{q}_e + K_p q_e + K_v q_v = 0$, where $q_e = q_d - q$, $K_p = \text{diag}\{K_{p1}, K_{p2}, K_{p3}\}$, and $K_v = \text{diag}\{K_{v1}, K_{v2}, K_{v3}\}$ are positive matrices. In this simulation the controller gain coefficients are selected to be $K_{pi} = 125$ and $K_{vi} = 15$, and user-determined allowable minimum and maximum cable tensions are chosen to be 2 and 80N, respectively.

As previously noted, there are more cables than the DOF of orthosis, therefore there are many solutions for cable tensions. As we discussed in cable tension distribution section, it is desirable to find the set of cable tensions with smaller positive values. This can be solved by quadratic programming approach. Then, the cable force distribution can be obtained from Eqn. (14) and $J_f^T T = \tau_1 - J_f^T F_e = \tau_2$.

With this controller and quadratic programming, the ROPES is able to follow the desired trajectory as shown in Fig. 9. The hip, knee and ankle angles corresponding to the closed-loop ankle trajectory, and cable length variations are shown in Fig. 9, and tensile cable forces are illustrated in Fig. 10. Moreover, the Fig. 11 shows the internal forces/moments at the lower extremity joints obtained from Eqns. (5), (6) and (7). These results exhibit and ensure that the internal stresses at these joints never exceed the corresponding forces/moments during normal walking.
implemented before for rehabilitation purposes, and have shown their capabilities for providing compliant interaction with the human limbs. Here, we present the development of impedance and force-field control for human lower limbs.

7 Impedance control

The goal of the position-based impedance controller is to create a virtual force to the leg while it is moving along a target path. Let \( \mathbf{P} \) be the current position of ankle in a Cartesian reference frame attached to subject’s trunk. The more distance between the current position and desired position, the more forces will be applied to thigh and shank to bring back ankle point toward the target path in sagittal plane. The impedance controller tries to control the lower limb such that against an external force it acts as a mass-spring-damper system. So,

\[
\mathbf{F}_d = K_a (\mathbf{P}_d - \mathbf{P}) + K_f (\mathbf{P}_d - \mathbf{P}) + K_x (\mathbf{P}_d - \mathbf{P})
\]

where \( K_a = \text{diag}\{K_{a1}, K_{a2}\} \), \( K_f = \text{diag}\{K_{f1}, K_{f2}\} \), and \( K_x = \text{diag}\{K_{x1}, K_{x2}\} \) are impedance gains matrices, \( \mathbf{P} \) is the current position of the ankle, and \( \mathbf{P}_d \) represents the desired position.

Similarly, if foot angle deviates from the desired foot angle trajectory \( q_{3d} \), a torque \( \tau_d \) at the ankle joint will bring the foot towards the desired trajectory, so one can define \( \tau_d \) as

\[
\tau_d = K_{fa} (q_{3d} - q_3) + K_{f\theta} (q_{3d} - q_3) + K_f (q_{3d} - q_3)
\]

where \( K_{fa}, K_{f\theta}, K_f \) are impedance gains, \( q_3 \) is the current foot angle, and \( q_{3d} \) represents the desired foot angle. In this simulation, the parameters are defined in Table 4.

As illustrated in the block diagram Fig. 12, the haptic interface in impedance controller design consists of two major components, hardware and software. The hardware consists of motors, load cells, orthosis and IMU sensors, and is simulated for purposes of the current study. The software component consists of forward kinematics, gravity compensation, and impedance controller blocks which are implemented in Matlab/Simulink. The ankle point position (or foot angle) sent from Simulink to virtual reality will be compared to desired target \( \mathbf{P}_d \) (or desired angle \( q_{3d} \)), and then the desired force \( \mathbf{F}_d \) (or torque \( \tau_d \)) will be generated. Impedance controller based on the current ankle position/foot angle and desired target reflects forces to cable-driven system. Cable-based impedance controller utilizes the principle of virtual work to create the forces in cable system.

7.2 Force-field control

The goal of force-field controller is to assist the individual to move ankle point along target path, also help the individual in plantar- and dorsiflexion motion of the foot during normal walking, for those who suffer from a significant weakness of ankle muscles [53]. Force-field control constructs a virtual tunnel like force-field around the ankle
point target path, and along the desired foot angle trajectory. If the ankle point (or foot angle) deviates from the target path (or foot desired angle trajectory), the controller acts as spring and brings them back to target.

The high level force-field controller generates: (i) a force \( F_{n} \) at the ankle point that has a normal (\( F_{n} \)) and tangential (\( F_{t} \)) components, i.e., \( F_{d} = F_{n} + F_{t} \), (ii) a torque vector \( \tau_{d} \) which similarly has a normal (\( \tau_{n} \)) and tangential (\( \tau_{t} \)) components. The normal components i.e. \( F_{n} \) and \( \tau_{n} \) are responsible for pulling ankle point (or foot angle) towards the target path (or desired joint angle trajectory), and tangential components i.e. \( F_{t} \) and \( \tau_{t} \) are tangential to target path/desired joint angle trajectory, and assist in tracking the target path (or desired joint angle trajectory) [17]. So,

\[
\|F_{n}\| = K_{n}\left(1 - e^{-\frac{2q_{min}}{q_{n}}}\right)^{2}, \quad \|F_{t}\| = K_{t}e^{-\frac{2q_{min}}{q_{t}}} \tag{18}
\]

\[
\|\tau_{n}\| = \zeta_{n}\left(1 - e^{-\frac{2q_{min}}{q_{n}}}\right)^{2}, \quad \|\tau_{t}\| = \zeta_{t}e^{-\frac{2q_{min}}{q_{t}}} \tag{19}
\]

where \( K_{n} \) and \( \zeta_{n} \) are gain vectors for normal force-field, and \( K_{t} \) and \( \zeta_{t} \) are gain vectors for tangential force-field, respectively; \( q_{min} \) is minimum distance from the ankle point to a point on the target path, and \( q_{min} \) is the distance between the current foot angle and desired joint angle.

Two virtual force-field tunnels with diameters \( r_{n} \) and \( r_{t} \) are created around the target path in task-space, and similarly two force-field tunnels with diameters \( \delta_{n} \) and \( \delta_{t} \) are created around the desired joint angle trajectory in the joint angle space. And also, two upper and lower bounds are created as joint angle limits, \( q_{up} \) and \( q_{low} \) for the safety of foot to avoid increasing the absolute value of the foot angle (see Fig. 14). To understand the force field tunnels concept, a force field around the ankle target path (thick red line) is illustrated in Fig. 13. For the normal force \( F_{n} \), outside the tunnel \( r_{n} \), the magnitude almost equals to \( K_{n} \) and apply force along the stream line (blue lines) and normal to target path, and the magnitude close to the target path equals to zero. Likewise, for the tangential force \( F_{t} \), outside the tunnel \( r_{t} \), the magnitude roughly equals to zero and gradually increase to \( K_{t} \) inside the tunnel.

### Table 4. Parameters/gains of impedance and force-field controller

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8 Discussion

In this paper we examined various aspects of the modeling, analysis and simulation of an articulated-multibody cable-driven system for rehabilitative exercises on lower extremity. The sagittal-plane dynamic model of lower limbs with 3-DOF was formulated based on the both Newton-Euler and Lagrangian formulations to support the design/control.
efforts. The Newton-Euler approach allows for monitoring of the internal forces (both within the orthosis as well as the human) which is critical from a design perspective. The Lagrangian formulation aids development of controllers for simulation- or hardware-testing after elimination of internal constraint forces. We highlighted how the wrench feasible workspace of ROPES depends on the selection of cable placement both on fixed frame and mobile frames (attached to the orthosis). We identified and analyzed the design-space for cable configuration including cuff sizes, distance of cuffs from local frames, and cable attachment points to the ground frame. This analysis helped to identify not only the most-sensitive configurations of ROPES but also the most-robust (with respect to assembly- and set-up errors). We highlight the opportunity and intend to pursue this as one aspect of future work.

For the subsequent control analysis, we down-selected one particular configuration (from among the numerous high-performance configurations). Two types of controllers were implemented for Lagrangian model, trajectory tracking PD controller in joint-space, and force-control strategies in task-space. Simulation results for both controllers were presented to show how model-based controller can apply forces through cable-driven system. Finally, using the cable tension results, and splitting the Jacobian matrix derived in Newton-Euler formulation, we were able to calculate the internal forces and moments at the hip, knee and ankle joints due to forces through cable-driven systems to avoid increasing of internal stresses at lower extremity joints.

For future work, there is an opportunity to include mobility in fixed bases and add springs in series with cables to (i) generate appropriate pretension in cables without becoming slack, (ii) be able to change the ROPES stiffness, and (iii) increase the safety of mechanism as well. Last but not least is the need for experimental validation of this overall framework.

Acknowledgment

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