



**Paradoxes of Traffic Flow and
Economics of Congestion Pricing**

**交通流量悖论与
拥堵定价经济学分析**

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3. Solutions of Traffic Paradoxes - Congestion Pricing
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Introduction

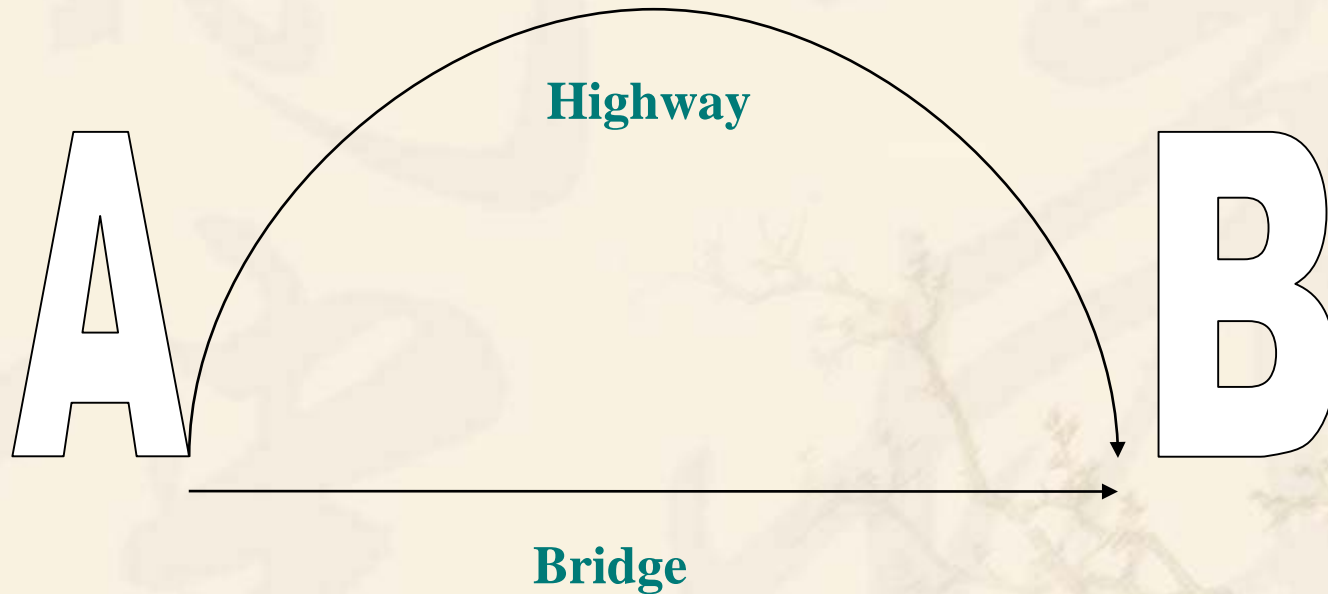
- ❖ **China saw dramatic growth of private vehicles in the past two decades. In 1990, China had 5.5 million private vehicles. This number swelled to 41.73 million in 2008 (Beijing 2.48 million). The same trend will continue, if not faster.**
- ❖ **China mainly depends on supply-side policies to mitigate urban congestion, such as through expanding network capacity. Unfortunately, supply-side policies are not effective to reduce urban traffic congestion because better roads induce more demand and urban commuting is subject to the theory of “triple convergences” (spatial, time, and modal convergences). They may also be subjected to some traffic paradoxes.**

Introduction

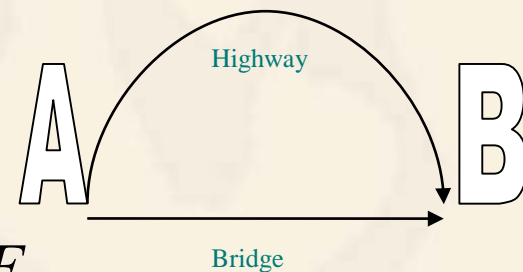
- ❖ **Some Chinese cities have also used regulatory instruments. In Beijing, for example, starting from April 11, 2009, private passenger vehicles are required to stay off roads one day every week, from 7AM to 8PM, with the date determined by the license numbers and being rotated quarterly. Beginning on April 12, 2010, Beijing staggers work schedule to spread peak hour trips in the morning and evening, with working hours from 9AM to 6PM for all employees in city's public sectors. Such regulations, however, leave fewer choices and cause a lot of inconvenience for commuters. Some families are forced to purchased more vehicles than they actually need.**
- ❖ **It is important to propose and implement demand-side policies to mitigate traffic congestion. Using price mechanism not only reduces congestion by changing commuting behavior but also generates toll revenues for governments to provide better transportation network.**

Traffic Paradoxes

A. Pigou-Knight-Downs Paradox



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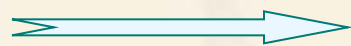


$$T_1 = a + b \left(\frac{F_1}{C_1} \right)$$

$$T_2 = d$$

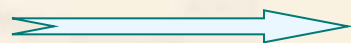
$$F_1 + F_2 = F$$

$$T_1 = T_2$$



$$C_1 = \frac{bF_1}{(d-a)}$$

$$F_1 = F$$



$$C_1(\text{boundary}) = \frac{bF}{(d-a)}$$

$$C_1 < \frac{bF_1}{(d-a)}$$



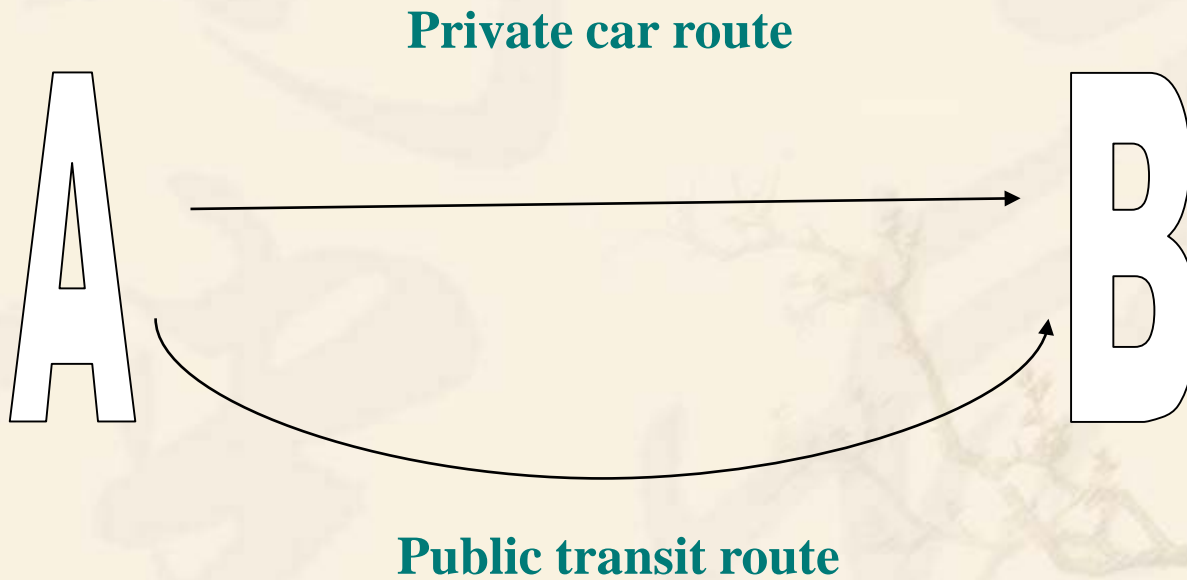
$$\frac{dF_1}{dC_1} = \frac{d-a}{b} > 0$$

$$T_1 = T_2 = d$$

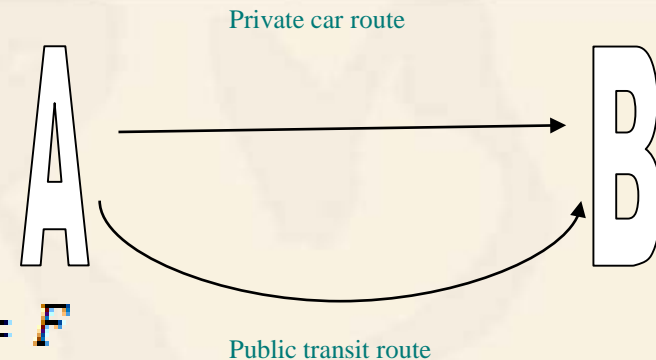
Pigou-Knight-Downs paradox shows that increasing bridge capacity may not reduce commuting time. It only attracts commuters from uncongested highway to congested bridge.

Traffic Paradoxes

B. Downs-Thomson Paradox



B. Downs-Thomson Paradox



$$T_1 = a + b \left(\frac{F_1}{C_1} \right) \quad T_2 = d - F_2/e \quad F_1 + F_2 = F$$

$$T_1 = T_2 \quad \Rightarrow \quad \begin{cases} F_1 = \frac{(de - ae - F)}{(be - C_1)} C_1 \\ T_1 = a + b \frac{(de - ae - F)}{(be - C_1)} \end{cases}$$

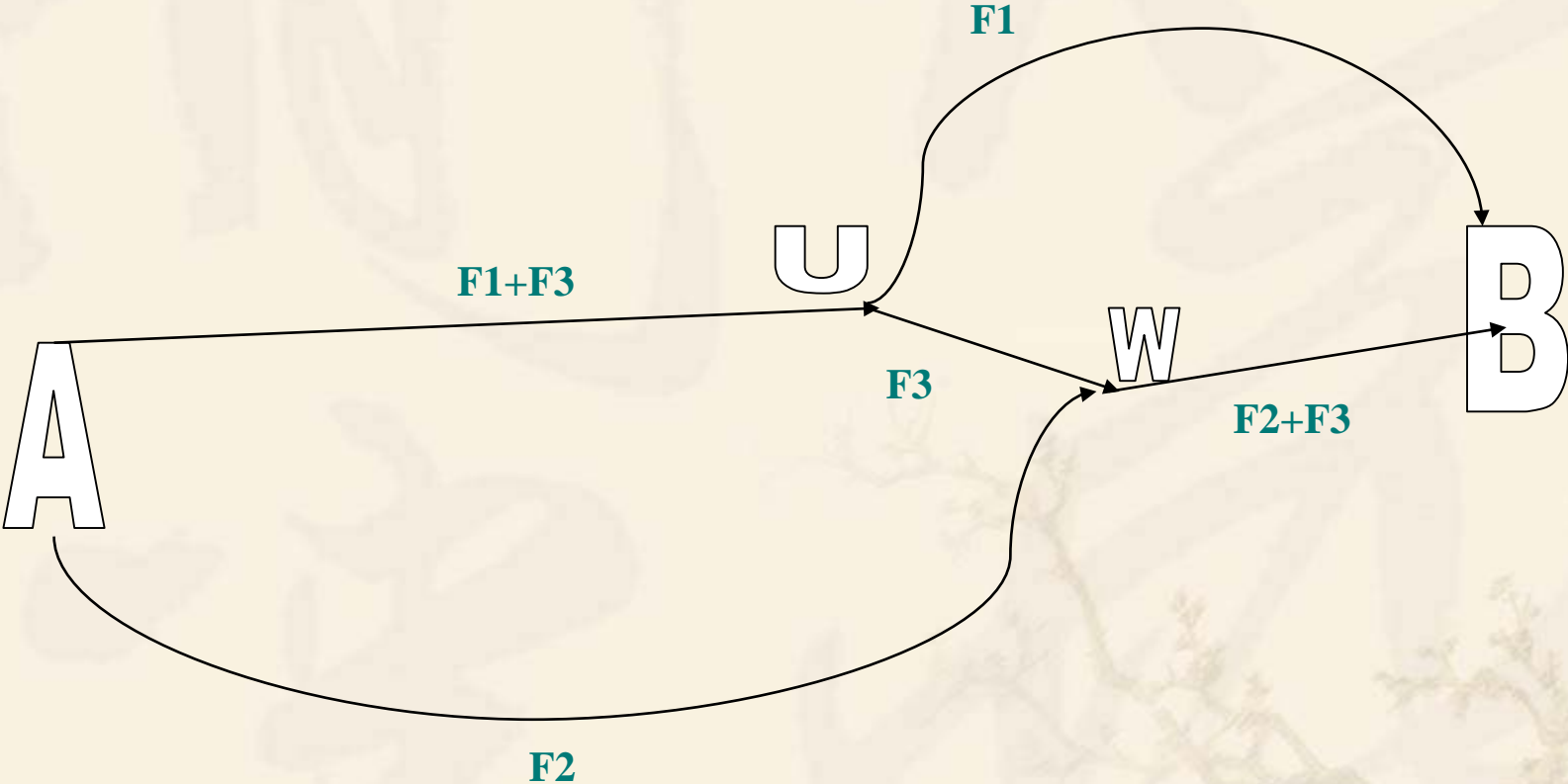
$$C_1(\text{boundary}) = bF / (d - a)$$

$$C_1 < bF / (d - a) \quad \Rightarrow \quad \frac{dT_1}{dC_1} = b(de - ae - F)(be - C_1)^{-2} > 0$$

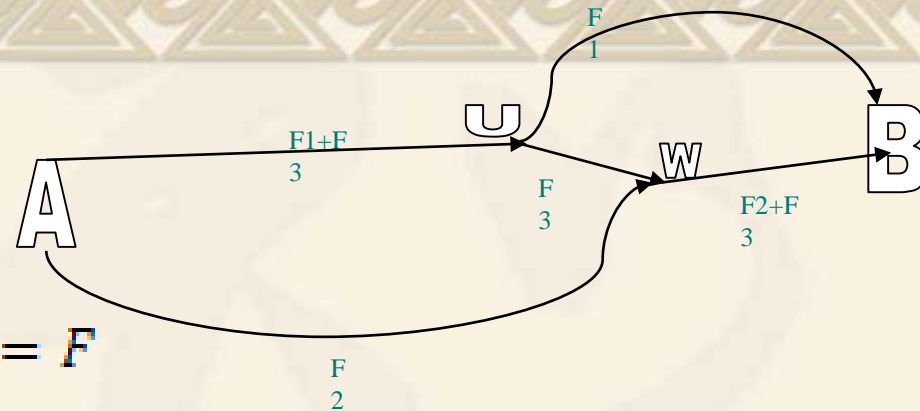
$$(F < (d - a)e)$$

Downs-Thomson shows, with economies of scale in public transit, expanding private car route could increase average commuting time.

C. Braess Paradox



C. Braess Paradox



$$T_1 = a + \left(\frac{F_1}{e}\right) \quad T_2 = a + \left(\frac{F_2}{e}\right) \quad F_1 + F_2 = F$$

$$T_1 = T_2 \implies T_{\text{without causeway}} = a + \frac{F}{2e}$$

$$T_1 = T_2 = T_3, \quad \left\{ \begin{array}{l} T_1 = a + \frac{F_1 + F_3}{e} \\ T_2 = a + \frac{F_2 + F_3}{e} \\ T_3 = k + \frac{F_1 + F_3}{e} + \frac{F_2 + F_3}{e} \\ F_1 + F_2 + F_3 = F. \end{array} \right. \implies T_{\text{with causeway}} = 2a - k.$$

$$\left. \begin{array}{l} F_1 = F_2 = ke + F - ae \\ F_3 = 2ae - 2ke - F \end{array} \right\} \implies T_{\text{without causeway}} - T_{\text{with causeway}} = \frac{F}{2e} - a + k.$$

$$F < 2(a - k)e. \implies \frac{F}{2e} - a + k < 0,$$

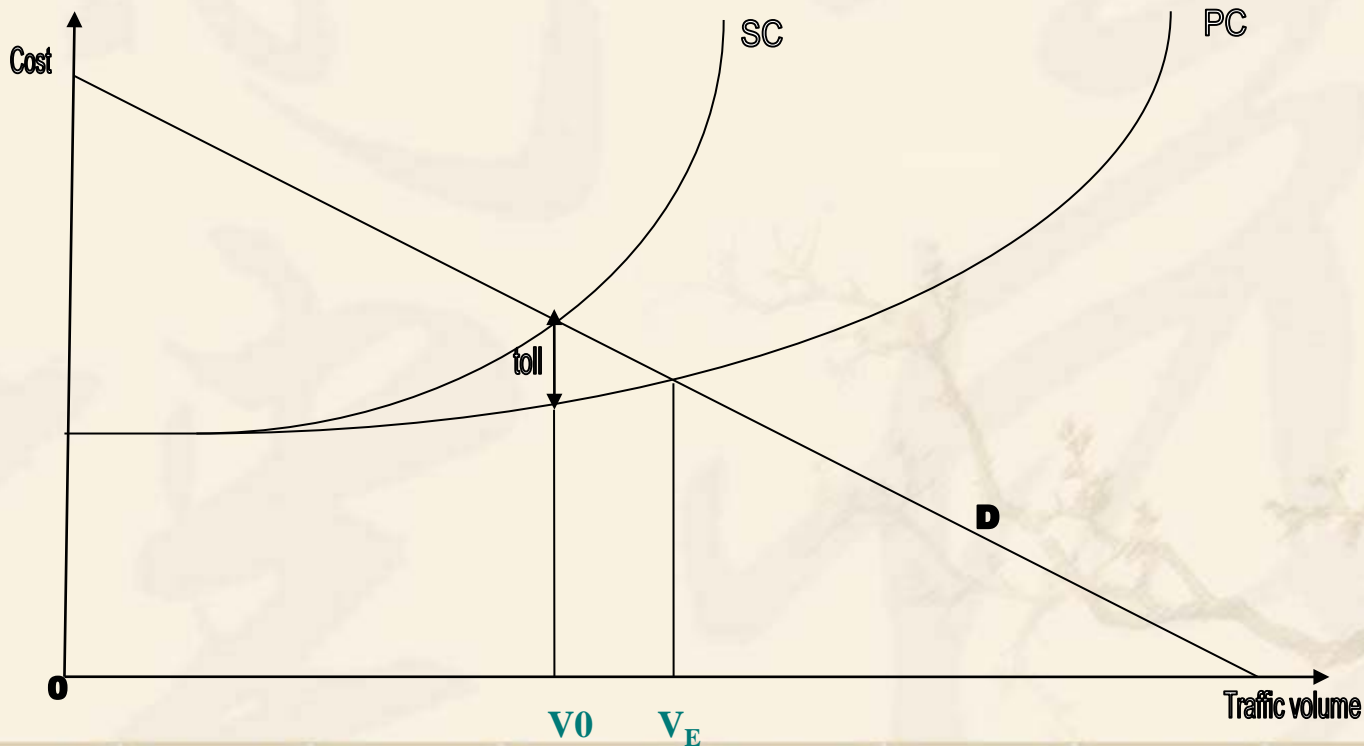
Braess shows that increasing capacity may actually increase commuting time.

Congestion Pricing – The Solution

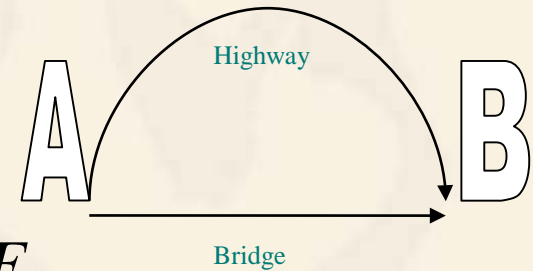
V —traffic volume

t —average commute time

$$SC = \frac{d(tV)}{dV} = t + V \frac{dt}{dV} = PC + EC$$



A. Solving Pigou-Knight-Downs Paradox



$$T_1 = a + b \left(\frac{F_1}{C_1} \right) \quad T_2 = d \quad F_1 + F_2 = F$$

$$TC = T_1 F_1 + T_2 F_2 = dF + (a - d)F_1 + \frac{b}{C_1} F_1^2$$

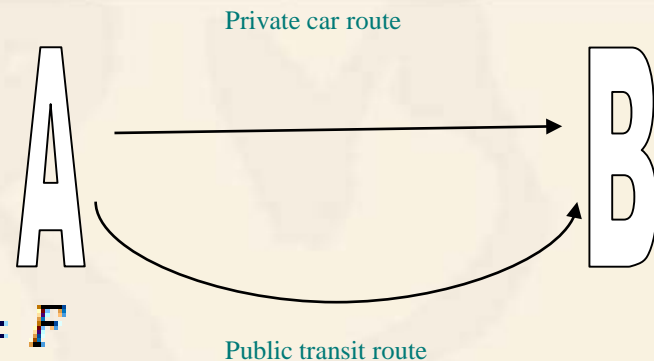
$$F_1 = \frac{d - a}{2b} C_1$$

$$TC = Fd - \frac{(d - a)^2}{4b} C_1$$

$$\frac{dTC}{dC_1} = -\frac{(d - a)^2}{4b} < 0$$

$$\left\{ \begin{array}{l} \text{Toll} = bF_1 / C_1 = \frac{d - a}{2} \\ \text{Revenue} = \frac{(d - a)^2 C_1}{2b} \end{array} \right.$$

B. Solving Downs-Thomson Paradox



$$T_1 = a + b \left(\frac{F_1}{C_1} \right) \quad T_2 = d - F_2/e \quad F_1 + F_2 = F$$

$$TC = T_1 F_1 + T_2 F_2 = dF - \frac{F^2}{e} + \left(a + \frac{2F}{e} - d \right) F_1 + \left(\frac{b}{C_1} - \frac{1}{e} \right) F_1^2$$

$$F_1 = \frac{de - ae - 2F}{2(be - C_1)} C_1$$

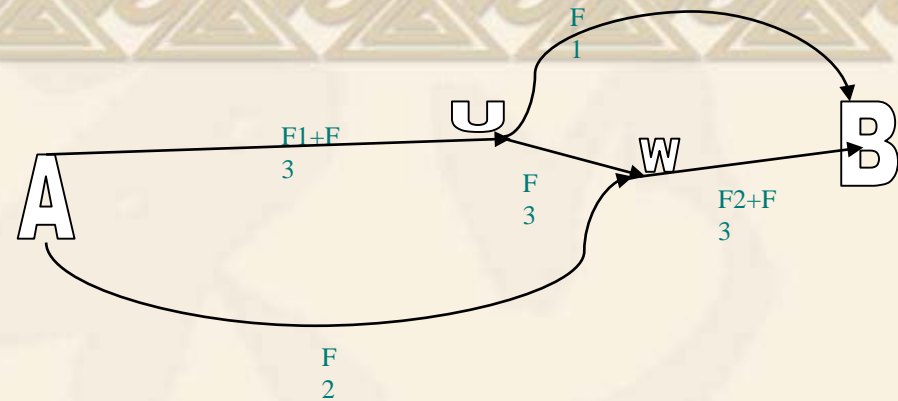
$$TC = Fd - \frac{F^2}{e} - \frac{(de - ae - 2F)^2}{4e(be - C_1)} C_1$$

$$\frac{dTC}{dC_1} = - \frac{b(de - ae - 2F)^2}{4(be - C_1)^2} < 0$$

$$\text{Toll} = \frac{bF_1}{C_1} = \frac{b(de - ae - 2F)}{2(be - C_1)}$$

$$\text{Revenue} = \frac{bF_1^2}{C_1} = \frac{bC_1(de - ae - 2F)^2}{4(be - C_1)^2}$$

C. Solving Braess Paradox



$$TC_{\text{without causeway}} = aF + \frac{F^2}{2e}$$

$$T_1 = a + \frac{F_1 + F_3}{e}$$

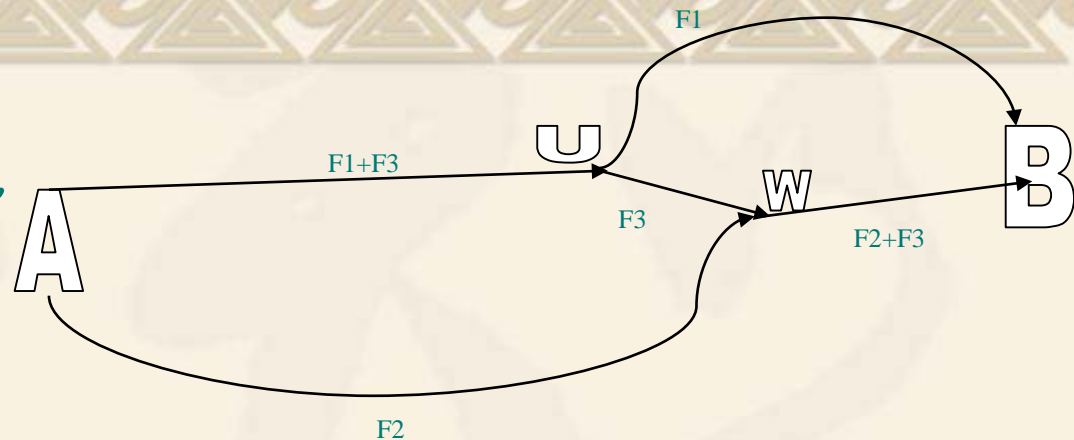
$$T_2 = a + \frac{F_2 + F_3}{e}$$

$$T_3 = k + \frac{F_1 + F_3}{e} + \frac{F_2 + F_3}{e}$$

$$F_1 + F_2 + F_3 = F.$$

$$TC = T_1 F_1 + T_2 F_2 + T_3 F_3 = \left(k + \frac{2F}{e}\right)F - \left(k + \frac{2F}{e} - a\right)F_1 - \left(k + \frac{2F}{e} - a\right)F_2 + \frac{F_1^2}{e} + \frac{F_2^2}{e}$$

C. 解决Braess悖论



$$\left. \begin{aligned} F_1 = F_2 &= \frac{ke + 2F - ae}{2} \\ F_3 &= ae - ke - F \end{aligned} \right\}$$

$$TC_{withcauseway} = \frac{2keF - (a - k)^2 e^2 + 4(a - k)eF}{2e}$$

$$TC_{withoutcauseway} - TC_{withcauseway} = \frac{(ae - ke - F)^2}{2e} > 0.$$

$$\left\{ \begin{aligned} Toll &= \frac{F_1 + F_3}{e} \text{ or } \frac{F_2 + F_3}{e} = \frac{a - k}{2} \\ Revenue &= \frac{(a - k)^2 e}{4} \end{aligned} \right.$$

Conclusions

- ❖ The three traffic paradoxes suggest that expanding transportation facilities may not always improve urban traffic. Under certain conditions, it may be counter productive.
- ❖ Internalizing travel externality by congestion pricing could solve the traffic paradoxes.
- ❖ To deal with traffic congestion in urban areas, it is necessary to implement both supply and demand side policies.