A VELOCITY-BASED LPV MODELING AND CONTROL FRAMEWORK FOR AN AIRBREATHING HYPERSONIC VEHICLE

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ABSTRACT. This paper focuses on developing a linear parameter varying (LPV) controller for an airbreathing hypersonic vehicle using a velocity-based approach. The design of flight control systems for airbreathing hypersonic vehicles is a highly challenging task due to the unique characteristics of the vehicle dynamics. Motivated by recent results on a velocity-based linearization approach and LPV control theory, a velocity-based LPV modeling and control framework combined with a novel implementation method of nonlinear gain-scheduling controller has been developed, which provides a nonlinear tracking control structure. Within this framework, an accurate LPV modeling of nonlinear systems and advanced design of self-scheduled controllers are implemented, which relaxes the restriction to near equilibrium operation in traditional gain scheduling approaches. The framework is applied to a nonlinear longitudinal dynamic model of the airbreathing hypersonic vehicle. Simulation results demonstrate the effectiveness of the proposed method.

Keywords: Airbreathing hypersonic vehicle, Linear parameter varying (LPV) systems, Velocity-based approach, Nonlinear control, Gain scheduling control, Controller implementation

1. Introduction. Since there is a worldwide flurry of research and development activities toward building test vehicles and experimental facilities to fill the void of flight test data that characterizes the current state of the airbreathing hypersonic flight, the age of airbreathing hypersonic flight is upon us [1]. The researches on airbreathing hypersonic vehicles have become much more attractive in recent years mainly due to their promising prospects for reliable affordable access to space routine and global reach capabilities [2]. A key issue in making airbreathing hypersonic flight feasible and efficient is the control design, which is a highly challenging task due to the strong interactions among the elastic airframe, the propulsion system and the structural dynamics [3]. The requirements of flight stability and maneuvering performance for hypersonic flight control systems are higher than other aircrafts because of more significant dynamic characteristics due to facts such as various coupling effects, strong nonlinearities, high flight altitudes, large flight envelopes, extreme ranges of operating conditions and rapid changes of mass distribution. Therefore, seeking better control-oriented model and designing more appropriate controller of hypersonic vehicles are one of the major tasks in developing hypersonic vehicle technologies.
Recently, there have been several attempts to address the challenges of model development and control design of airbreathing hypersonic vehicles. In the cited references, due to the enormous complexity of the dynamics, only models of the longitudinal dynamics of airbreathing hypersonic vehicles have been developed and used for control design [4]. For the design of control systems of hypersonic vehicles based on linearized dynamic models, several results are available in the literature that consider representative control approaches based on feedback linearization techniques, such as stochastic robust control [5], dynamic inversion control [6] and adaptive sliding mode control [7]. Although the simulation results show the effectiveness of these approaches, the complexity of the model inevitably leads to very complicated expressions for the high-order Lie derivatives and tedious stability analysis and thus a robustness analysis cannot be performed. A few recent contributions have also attempted to design directly on nonlinear models [8-11]. However, each of the control designs presented above applies only to a certain flight condition. Therefore, in order to extend the applicability of the design to the entire flight envelopes, an advanced self-scheduling linear parameter varying (LPV) control approach is applied to the control design of airbreathing hypersonic vehicles [12-14].

In the past two decades, linear parameter varying control theory has been presented as a reliable alternative to classical gain scheduling for nonlinear systems [15,16]. LPV control methodology is an extension of $H_{\infty}$ control theory for LPV systems [17]. It explicitly takes into account the relationship between real-time parameter variations and performance. This enables controllers to be designed for whole ranges of operating conditions with theoretical guarantees of performance and robustness throughout the region [18,19]. LPV control synthesis techniques have already been widely used in various dynamical systems including high-performance aircraft, missiles, turbofan engines, robotic manipulators and industrial processes. A condition to apply LPV control synthesis is to transform the nonlinear model of the system into an LPV model; hence, LPV modeling becomes a key issue in the design of LPV controllers. There are several approaches used to obtain reliable LPV models, such as Jacobian linearization, state transformation, function substitution, linear fractional transformation and different types of linearization [15]. The velocity-based linearization techniques recently proposed in Leith and Leithead [20-23] provide very general and soundly-based methods for transforming systems into LPV/quasi-LPV form. In contrast to the conventional linearization method, the velocity-based analysis and design framework associates a linear system with every operating point of a nonlinear system, not just the equilibrium operating points.

In this paper, a LPV modeling and control framework for an airbreathing hypersonic flight using a recently developed velocity-based approach is presented. By employing velocity-based formulation, a set of nonlinear longitudinal equations of motion for the airbreathing hypersonic vehicle can be accurately transformed into quasi-LPV form in which the scheduling variables depend on the states of velocity and altitude. The resulting LPV model is suitable for a LPV control design framework in which a new and simple method for the implementation of self-scheduled control design is proposed where the idea is referred to Kaminer et al. [24]. The method can be applied to a fairly general class of control structures that are usually referred to as tracking controllers, which satisfies the design objective of the hypersonic flight control considered in the paper. The control objective is to design a controller that ensures asymptotic tracking of velocity and altitude reference trajectories. Simulation results are provided to validate the proposed approach.

This paper is organized as follows. In Section 2, the airbreathing hypersonic vehicle model and the formulation of tracking control problem are presented. Then, in Section 3, a framework employing velocity-based linearization for the analysis and design of LPV controllers combined with a novel implementation method is developed. Simulation results
on the nonlinear model of the hypersonic vehicle are presented and discussed in Section 4. Finally, we draw some conclusions in Section 5.

2. Problem Formulation. Consider the model for the longitudinal dynamics of an air-breathing hypersonic vehicle developed by NASA Langley Research Center [7]. All related parameters of the airbreathing hypersonic vehicle used in the modeling and simulation can be found in [9]. The equations for vehicle velocity \( V \), altitude \( h \), flight-path angle \( \gamma \), angle of attack \( \alpha \) and pitch rate \( q \) for the longitudinal dynamics of the hypersonic vehicle are given as follows [9]:

\[
\begin{align*}
\dot{V} &= \frac{T \cos \alpha - D}{m} - \frac{\mu \sin \gamma}{V^2} \\
\dot{h} &= V \sin \gamma \\
\dot{\gamma} &= \frac{L + T \sin \alpha}{mV} - \frac{(\mu - V^2 r) \cos \gamma}{V r^2} \\
\dot{\alpha} &= q - \dot{\gamma} \\
\dot{q} &= \frac{M_{yy}}{l_{yy}}
\end{align*}
\]

(1)

where \( L, D, T \) and \( M_{yy} \) denote the lift, drag, thrust and pitching moment, respectively, and they are defined as:

\[
L = \bar{q}S_{CL}, \quad D = \bar{q}S_{CD}, \quad T = \bar{q}S_{CT}, \quad M_{yy} = \bar{q}S\bar{c} [C_M (\alpha) + C_M (q) + C_M (\delta_e)] , \quad r = h + R_E
\]

where \( \bar{q} \) is the dynamic pressure defined as \( \bar{q} = \frac{1}{2} \rho V^2 \), and \( \rho \) denotes the air density computed as \( \rho = 0.00238 \exp \left( \frac{-h}{24000} \right) \). The force and moment coefficients are given by

\[
\begin{align*}
C_L &= \alpha \left( 0.493 + \frac{1.91}{M^2} \right) \\
C_D &= 0.0082 (171 \alpha^2 + 1.15 \alpha + 1) (0.0012M^2 - 0.054M + 1) \\
C_T &= \left\{ \begin{array}{ll}
C_T^* (1 + 0.15) \delta_t, & \text{if } \delta_t < 1 \\
C_T^* (1 + 0.15 \delta_t), & \text{if } \delta_t \geq 1
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
C_T^* &= 0.0105 \left[ 1 - 164 (\alpha - \alpha_0)^2 \right] (1 + \frac{17}{M}) \\
C_M (\alpha) &= 10^{-4} (0.06 - e^{-M/3}) (-6565 \alpha^2 + 6875 \alpha + 1) \\
C_M (q) &= \frac{q v}{2h} (-0.025M + 1.37) (-6.83 \alpha^2 + 0.303 \alpha - 0.23) \\
C_M (\delta_e) &= 0.0292 (\delta_e - \alpha)
\end{align*}
\]

The symbol \( M \) in the above equations is the Mach number defined as \( M = V/a \), with the sound speed \( a \) given by \( a = 8.99 \times 10^{-9} h^2 - 9.16 \times 10^{-4} h + 996 \). The symbols \( \delta_t \) and \( \delta_e \) are the throttle setting and the elevator deflection, respectively.

For the purpose of simplifying calculation, the aerodynamic coefficients are simplified around a trimmed cruise condition, which can be represented as

\[
p (t) = \{ V, h, \gamma, \alpha, q, \delta_t, \delta_e, M \}
\]

Then, the initial nominal cruising flight of the vehicle is chosen as [5]:

\[
p (0) = \{ 15060 \text{ft/s}, 110000 \text{ft}, 0 \text{rad}, 0.0315 \text{rad}, 0 \text{rad/s}, 0.183, -0.0066 \text{rad}, 15 \}\]

The state and control input/output vectors are chosen as follows:

\[
x = [V \ h \ \gamma \ \alpha \ q]^T, \quad u = [\delta_t \ \delta_e]^T, \quad y = [V \ h]^T.
\]

Then, the state equations of the hypersonic vehicle (1) can be rewritten in a compact form:

\[
\begin{align*}
\dot{x} &= f (x, w, u) \\
y &= g (x, w, u)
\end{align*}
\]

(2)
where \( w \) denotes the exogenous signals to the plant.

The commanded desired values of velocity and altitude are denoted by \( V_d(t) \) and \( h_d(t) \), respectively. Assume that the expected output vector is \( r(t) \) and consider that the controller design is a problem of output tracking control for nonlinear feedback systems. The tracking control problem can be described as follows.

**Problem 1.** Given system (2) and reference signal \( r(t) \), seek a proper control law \( u(t) \), such that the output of the closed-loop system tracks the reference signal asymptotically, that is

\[
\lim_{t \to \infty} [y(t) - r(t)] = 0
\]

In this paper, the output reference signal of the airbreathing hypersonic vehicle at the trimmed cruise condition is defined as \( r = [V_d \ h_d]^T \).

3. Velocity-based LPV Modeling and Control Framework. In this section, the velocity-based LPV modeling approach \[20-23\] for analysis of a nonlinear system whose dynamic characteristics are related to an associated family of linear systems, is briefly reviewed. Then, a LPV controller with guaranteed \( L_2 \)-gain performance is introduced. Finally, a novel method \[24\] that is proposed to implement gain-scheduled controllers for nonlinear plants is combined with the velocity-based LPV modeling and control framework, which has been developed to a tracking control structure.

3.1. Velocity-based LPV modeling. Consider a general nonlinear system

\[
\begin{align*}
\dot{x}(t) &= F(x(t), u(t)) \\
y(t) &= G(x(t), u(t))
\end{align*}
\]

where \( F(\cdot, \cdot) \) and \( G(\cdot, \cdot) \) are differentiable nonlinear functions with Lipschitz continuous first derivatives, and \( u(t) \in \mathbb{R}^m \) denotes the input to the plant, \( y(t) \in \mathbb{R}^p \) the output and \( x(t) \in \mathbb{R}^n \) the state.

To be able to apply the LPV control design framework, a transformation of the system (4) is necessary. The goal is to find a set of differential equation having the LPV form,

\[
\begin{align*}
\dot{x}(t) &= A(\varphi(t)) \dot{x}(t) + B(\varphi(t)) u(t) \\
\dot{y}(t) &= C(\varphi(t)) \dot{x}(t) + C(\varphi(t)) u(t)
\end{align*}
\]

where \( \varphi(t) \) is the parameter vector or scheduling variables of the system which is not known in advance but can be measured or estimated in real-time, such that the solution of (4) is as close to the solution of (5) as possible in a predefined region of \( \varphi(t) \in S \subseteq \mathbb{R}^S \).

Given an LPV system as defined in (5), if a scheduling variable \( \varphi(t) \) is also a state of the system, then this particular class of systems is called as quasi-LPV systems.

What has become a standard way of obtaining an LPV model of the form (5) is to linearize the nonlinear system (1) in different equilibrium operating points and map the linearization together to achieve an LPV system. There is, however, no direct connection between the nonlinear system and the obtained LPV system. This may result in a poor control design \[22\]. Other methods like analytic transformation are inadequate whenever the nonlinear system is of high order and/or contain look-up tables. The velocity-based linearization developed by Leith and Leithead \[20-23\] offers both a theoretical relation of the LPV system and the nonlinear one, and is applicable to more complicated system using numerical methods. The basic idea of velocity-based linearization is to make a differentiation of (4) in time, an alternative representation of the nonlinear system is

\[
\begin{align*}
\ddot{x}(t) &= A(\varphi(t)) \dot{x}(t) + B(\varphi(t)) \dot{u}(t) \\
\ddot{y}(t) &= C(\varphi(t)) \dot{x}(t) + C(\varphi(t)) \dot{u}(t)
\end{align*}
\]
where
\[
A(\rho(t)) = \frac{\partial F}{\partial x}(x(t), u(t)), \quad B(\rho(t)) = \frac{\partial F}{\partial u}(x(t), u(t)),
\]
\[
C(\rho(t)) = \frac{\partial G}{\partial x}(x(t), u(t)), \quad D(\rho(t)) = \frac{\partial G}{\partial u}(x(t), u(t)).
\]

It is noted that the velocity-based linearization system (6) is not an approximation for the original system (4), however, holds exactly. That is, the linearization system (6) is equivalent to the original nonlinear system (4). Moreover, the velocity-based linearization is hold at every operating point and not just the equilibrium operating points.

**Remark 3.1.** The basic assumption is that the original nonlinear system is differentiable nonlinear function with Lipschitz continuous first derivative. Once the assumption is satisfied, the velocity-based linearization is useful and practical to the controller design of general nonlinear system. It seems that the assumption is a bit strong for the original nonlinear system in application. Fortunately, many practical and physical systems with variously nonlinear models can satisfy the conditions of Lipschitz and differentiability above exactly or approximately, such as aircraft, missiles, turbofan engines, robotic manipulators and industrial processes.

### 3.2. LPV controller design.

This section introduces a practically valid LPV control design technique of output-feedback synthesis with guaranteed $L_2$-gain performance [25].

Consider LPV plant derived by the velocity-based linearization approach as follows:
\[
\begin{aligned}
\dot{x} &= A(\rho) x + B_1(\rho) w + B_2(\rho) u \\
z &= C_1(\rho) x + D_{11}(\rho) w + D_{12}(\rho) u \\
y &= C_2(\rho) x + D_2(\rho) w
\end{aligned}
\]

(7)

where the state-space entries have appropriate dimensions. The time-varying parameters \( \rho := (\rho_1, \cdots, \rho_L)^T \) as well as its rates of variation \( \dot{\rho} \) are assumed bounded as follows.

1) Each parameter \( \rho_i \) ranges between known extreme values \( \underline{\rho}_i \) and \( \overline{\rho}_i \), namely,
\[
\rho_i(t) \in [\underline{\rho}_i, \overline{\rho}_i]
\]

(8)

2) The rate of variation \( \dot{\rho}_i \) is assumed to be well-defined at all times and satisfies
\[
\dot{\rho}_i(t) \in [\underline{\dot{\rho}}_i, \overline{\dot{\rho}}_i]
\]

(9)

where \( \underline{\rho}_i \leq \overline{\rho}_i \) are known lower and upper bounds on \( \dot{\rho}_i \).

The gain-scheduled output-feedback control problem consists of finding a dynamic LPV controller with state space equations:
\[
\begin{aligned}
\dot{x}_K &= A_K(\rho, \dot{\rho}) x_K + B_K(\rho, \dot{\rho}) y \\
u &= C_K(\rho, \dot{\rho}) x_K + D_K(\rho, \dot{\rho}) y
\end{aligned}
\]

(10)

which ensures internal stability and a guaranteed $L_2$-gain bound \( \gamma \) for the closed-loop operator (7) – (10) from the disturbance signal \( w \) to the error signal \( z \), that is
\[
\int_0^T z^T z d\tau \leq \gamma^2 \int_0^T w^T w d\tau, \quad \forall T \geq 0
\]

Then, the LPV controller with guaranteed $L_2$-gain performance is presented in the following lemma where the dependence of data and variables on \( \rho \) and \( \dot{\rho} \) has been dropped for simplicity [25].
Lemma 3.1. Consider the LPV plant governed by (7), with parameter trajectories constrained by (8) and (9). There exists LPV output-feedback controller (10) enforcing internal stability and a bound $\gamma$ on the $L_2$ gain of the closed-loop system (7) and (10), whenever there exist parameter-dependent symmetric matrices $X(\varrho)$ and $Y(\varrho)$ and a parameter-dependent quadruple of state-space data $(\hat{A}_K, \hat{B}_K, \hat{C}_K, D_K)$ such that for all pairs of $(\varrho, \dot{\varrho})$ the following infinite-dimensional LMI holds

$$
\begin{bmatrix}
\dot{X} + XA + \hat{B}_K C_2 + (*) \\
\hat{A}_K^T + A + B_2 D_K C_2 \\
\left( XB_1 + \hat{B}_K D_{21} \right)^T \\
C_1 + D_{21} D_K C_2
\end{bmatrix}
\begin{bmatrix}
* & * & * \\
* & * & * \\
(B_1 + B_2 D_K D_{21})^T & -\gamma I & * \\
C_1 Y + D_{12} \hat{C}_K & D_{11} + D_{12} D_K D_{21} & -\gamma I
\end{bmatrix}
< 0 \ (11)
$$

where the terms denoted by $*$ will be induced by symmetry in large symmetric matrix expressions. For instance, with $S$ symmetric

$$
\begin{bmatrix}
S + M + N + (*) \\
Q & P
\end{bmatrix} \triangleq 
\begin{bmatrix}
S + M + M^T + N + N^T \\
Q & P
\end{bmatrix}
$$

In such case, a LPV controller of the form (10) is readily obtained with the following two-step scheme.

1) Solve $N$ and $M$ from the factorization problem

$$
I - XY = NM^T
$$

2) Compute $A_K$, $B_K$, and $C_K$ with

$$
A_K = N^{-1} \left( \dot{X} Y + N M^T + \hat{A}_K - X \left( A - B_2 D_K C_2 \right) Y - \hat{B}_K C_2 Y - XB_2 \hat{C}_K \right) M^{-T} \ (13)
$$

$$
B_K = N^{-1} \left( \hat{B}_K - XB_2 D_K \right) \ (14)
$$

$$
C_K = \left( \hat{C}_K - D_K C_2 Y \right) M^{-T} \ (15)
$$

Note that since all variables are involved linearly, the constraints (11) and (12) constitute a LMI system. Hence, the construction of LPV controller with guaranteed $L_2$-gain performance can be reduced to LMI problem and is numerically tractable.

Remark 3.2. Note that the LPV controller with guaranteed $L_2$-gain performance presented by Lemma 3.1 is only one of approaches of numerous LPV output feedback controllers. Thus, based on the status of each system and actual demand, any available LPV output feedback controller which is suited to the given system can be used to the proposed framework in this paper.

3.3. Implementation of the LPV tracking control. In this section, we focus on the implementation of LPV tracking control which is the generalization of the implementation of gain-scheduled control [24].

Consider the feedback system shown in Figure 1. The nonlinear system $G$ consists of a dynamical model of the physical plant to be controlled, together with the exogenous signals $w$, the control inputs $u$ and the generalized error signals $z$. The controller $K$ operates on the measured output variables to produce the control inputs.

In this paper, we consider general tracking control structures whereby some of the plant outputs are required to track reference commands. Therefore, the vector $w$ is decomposed
as \( w = [d^T \ w_m^T \ r]^T \), where \( d \) is the vector of exogenous signals that are not accessible for measurement (e.g., external disturbances and sensor noise), \( w_m \) is the vector of measurable exogenous signals (e.g., air speed in an airplane) and \( r \) is the vector of external reference signals to be tracked. Furthermore, \( y = [y_1^T \ y_2^T]^T \), where \( y_2 \) is the vector of output signals that must track the reference commands \( r \) and \( y_1 \) consists of an extra set of measurable output signals that will be used for feedback. To simplify the exposition, we allow \( y_1 \) including some or all of the components of \( y_2 \). With this notation, the generalized plant \( G \) can be described as:

\[
G \triangleq \begin{cases}
\dot{x} = f(x, w, u) \\
z = g(x, w, u) \\
y_1 = h_1(x, w) \\
y_2 = h_2(x, w)
\end{cases}
\tag{16}
\]

where \( f, g, h_1 \) and \( h_2 \) are differentiable nonlinear functions with Lipschitz continuous first derivatives, \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^m, w \in \mathbb{R}^l, z \in \mathbb{R}^q \) and \( y \in \mathbb{R}^p \).

Using the velocity-based approach, LPV model \( G(\varrho) \) associated with the generalized plant \( G \) is derived as follows:

\[
G(\varrho) \triangleq \begin{cases}
\dot{x} = A(\varrho) \dot{x} + B_1(\varrho) \dot{w} + B_2(\varrho) \dot{u} \\
\dot{z} = C_1(\varrho) \dot{x} + D_{11}(\varrho) \dot{w} + D_{12}(\varrho) \dot{u} \\
y_1 = C_{21}(\varrho) \dot{x} + D_{21}(\varrho) \dot{w} \\
y_2 = C_{22}(\varrho) \dot{x} + D_{22}(\varrho) \dot{w}
\end{cases}
\tag{17}
\]

where \( \varrho = \varrho(w, u, y) \) is a time-varying parameter vector, and state-space entries \( A(\varrho), B_i(\varrho), C_i(\varrho), D_i(\varrho) \) depend continuously on \( \varrho \):

\[
A(\varrho) = \frac{\partial f}{\partial x}(x, w, u), \quad B_1(\varrho) = \frac{\partial f}{\partial w}(x, w, u), \quad B_2(\varrho) = \frac{\partial f}{\partial u}(x, w, u), \\
C_1(\varrho) = \frac{\partial g}{\partial x}(x, w, u), \quad D_{11}(\varrho) = \frac{\partial g}{\partial w}(x, w, u), \quad D_{12}(\varrho) = \frac{\partial g}{\partial u}(x, w, u), \\
C_{21}(\varrho) = \frac{\partial h_1}{\partial x}(x, w), \quad D_{21}(\varrho) = \frac{\partial h_1}{\partial w}(x, w), \quad C_{22}(\varrho) = \frac{\partial h_2}{\partial x}(x, w), \quad D_{22}(\varrho) = \frac{\partial h_2}{\partial w}(x, w).
\]

To realize the tracking control of the reference commands \( r \), the LPV output-feedback controller \( K(\varrho) \) of the system (17) can be formulated by the foregoing LPV control design method. The closed-loop LPV control system structure can be represented in Figure 2.

Given the LPV model \( G(\varrho) \), a structure for LPV controller implementation of \( K(\varrho) \) referred to as the \( D \)-controller implementation methodology is developed as follows [24].
Figure 3. LPV controller implementation of $K(\varrho)$

(see Figure 3):

$$
K(\varrho) \triangleq \begin{cases} 
\dot{x}_c = \xi_c \\
\dot{\xi}_c = A_c(\varrho)\xi_c + B_c(\varrho)\dot{y}_1 + B_c(\varrho)\dot{e} \\
\dot{\xi}_2 = C_c(\varrho)\xi_c + D_c(\varrho)\dot{y}_1 + C_c(\varrho)\dot{e} \\
e = y_2 - r \\
\dot{u} = x_c + D_c(\varrho)e 
\end{cases}
$$

(18)

where the following assumptions hold:

(A1) $\dim (x_c) = \dim (u) = \dim (y_2)$;

(A2) the matrix $[sI A_c(\varrho) B_c(\varrho) C_c(\varrho) C_c(\varrho)]$ has full rank at $s = 0$ for each $\varrho$;

(A3) $w_2 \in C^1[0, \infty)$ or $y_1 = y_1(x)$; that is, $y_1$ is only a function of $x$.

Assumption (A1) implies that the number of integrators is equal to the number of control inputs. This is necessary if the controller is to provide independent control of the measured outputs $y_2$ using the control inputs $u$. Assumption (A2) implies that the realization $(A_c, B_c, C_c)$ has no transmission zeroes at the origin. Assumption (A3) is sufficient to ensure that $y_1$ exists and is continuous.

The main result of this section is as follows.

**Theorem 3.1.** Suppose Assumptions (A1) – (A3) hold. Consider a general tracking control structure of nonlinear system (16) with velocity-based LPV model (17). If there exist parameter-dependent symmetric matrices $X(\varrho)$ and $Y(\varrho)$ satisfying conditions (11) and (12), then the LPV controller (10) and the nonlinear gain-scheduled controller (18) solve Problem 1 and such that system (2) is asymptotically stable with the performance of guaranteed $L_2$-gain bound $\gamma$.

**Proof:** Let $C(G, K) : w \to z$ be the nonlinear closed loop system that consists of the nonlinear system (16) and a desired nonlinear controller, and let $C_l(G, K(\varrho))$ denote its LPV closed-loop control system that consists of velocity-based LPV model (17) and gain-scheduled controller (18). Furthermore, let $T(G, K)(s)$ and $T_l(G, K(\varrho))(s)$ denote the corresponding matrix transfer function of $C(G, K)$ and $C_l(G, K(\varrho))$ respectively.

Then, according to Theorem 4.1 in [24], we get following properties:

(i) The feedback system $C_l(G, K(\varrho))$ and $C(G, K(\varrho))$ have the same closed-loop eigenvalues;

(ii) The closed-loop transfer functions $T(G, K)(s)$ and $T_l(G, K(\varrho))(s)$ are equal.

Note that the velocity-based linearization system (17) is equivalent to the original nonlinear system (16) at every operating point, the foregoing two properties always hold.
Figure 4. Approximation implementation of $K(\varrho)$

From Lemma 3.1, we have that parameter-dependent symmetric matrices $X(\varrho)$ and $Y(\varrho)$ satisfying conditions (11) and (12), then the LPV controller (10) with the conditions (13) – (15) such that the tracking control problem (17) is asymptotically stable with guaranteed $L_2$-gain bound $\gamma$.

Since the nonlinear system (16) is equivalent to the system (17), then closed-loop control system that consists of the nonlinear system (16) and gain-scheduled controller (18) is also asymptotically stable with guaranteed performance.

Note that Problem 1 for system (2) is the special case of (16). Therefore, Problem 1 for system (2) is solvable and is asymptotically stable with guaranteed $L_2$-gain bound $\gamma$. This completes the proof of the theorem.

**Remark 3.3.** In contrast to the results of [26], the LPV controller implementation of this paper is suitable for every operating point of a nonlinear system, not just the equilibrium operating points. Also, this framework can guarantee that the whole tracking control system is global asymptotically stable with the robustness performance rather than so-called local linear equivalence property.

The $D$-method presented above requires differentiating some of the plant’s measured outputs. Except for the case where some of the derivatives have physical meaning and are available from dedicated sensors, this cannot be done in practice. For the implementation of gain-scheduled controllers, the differentiation operator can be replaced by a causal system with transfer function $s/(\epsilon s + 1)$ (see Figure 4), with $\epsilon > 0$, while having the linearization property recovered asymptotically as $\epsilon \to 0$. The result can be extended to the case where the differentiation operator is replaced by a strictly causal one. The result is given as follows. (The below corollary can be proven by combining the proof of Theorem 3.1 and Theorem 5.1 in [24].)

**Corollary 3.1.** Let the plant $G$ (16), velocity-based LPV model $G(\varrho)$ (17) and LPV controller (10) be as in Theorem 3.1. Consider the gain-scheduled controller

$$K_\epsilon(\varrho) \triangleq \begin{cases} 
\dot{x}_{c1} = \xi_{c1} \\
\dot{x}_{c2} = \xi_{c2} \\
\dot{\xi}_{c1} = A_{c1}(\varrho) \xi_{c1} + \tilde{B}_{c1}(\varrho) (y_1 - \tilde{z}) + B_{c2}(\varrho) \dot{e} \\
\dot{\xi}_{c2} = C_{c1}(\varrho) \xi_{c2} + \tilde{D}_{c1}(\varrho) (y_1 - \tilde{z}) + C_{c2}(\varrho) \dot{e} \\
\dot{\epsilon} = -\tilde{z} + y_1 \\
\dot{e} = y_2 - r \\
\dot{u} = x_{c2} + D_{c2}(\varrho)e
\end{cases}$$

where $\epsilon$ is a positive real parameter, $\tilde{B}_{c1}(\varrho) = B_{c1}(\varrho)/\epsilon$, $\tilde{D}_{c1}(\varrho) = D_{c1}(\varrho)/\epsilon$ and $\tilde{z} \in \mathbb{R}^r$ denote the fast states of the controller, with $\dim(\tilde{z}) = \dim(y_1) = m$.

Then, as $\epsilon \to 0$, the conclusion in Theorem 3.1 also holds asymptotically.
To summarize the results of the last three subsections, the following steps can be performed to realize the LPV modeling and control design for general nonlinear system under the certain conditions.

**Step 1:** Check the velocity-based linearization conditions of differentiability and Lipschitz continuous first derivative for the given nonlinear system. If satisfied, carry on with the next step. Otherwise, the framework is not available for this nonlinear system.

**Step 2:** Compute the LPV model (17) using the velocity-based linearization method.

**Step 3:** Find the LPV controller (10) by solving the LMI conditions (11) and (12) based on Lemma 3.1.

**Step 4:** Connect the LPV model (17) and LPV controller (10) with structure of gain-scheduled control (18) or (19).

**Remark 3.4.** Till then, a complete velocity-based LPV modeling and control framework has been developed. Although there are already many papers to the modeling, controller design and gain-scheduling implementation of LPV system, most of the available literature are only of interest for a certain aspect or two. To the best of our knowledge, however, intensive studies on the controller design for LPV systems not only with LPV modeling but also with an implementation method of LPV gain-scheduling controller have not been carried out thus far.

## 4. Control of the Airbreathing Hypersonic Vehicle

To validate the controller derived in the previous section, numerical simulations have been performed on the nonlinear model implemented in Matlab. As a representative case study, the vehicle is initially at the trim condition $p(0)$ discussed in Section 2. For simplicity, simulation studies are conducted for trimmed cruise conditions of 110,000 ft and Mach 15 to evaluate the responses of the vehicle to step changes of 100 ft/s in velocity and 2000 ft in altitude. The reference commands have been generated by filtering step increments in velocity and altitude by a second-order prefilter with appropriate natural frequency and damping factor.

In this section, we use the velocity-based approach to derive the LPV model of the airbreathing hypersonic vehicle. Furthermore, the LPV controller and implementation of LPV tracking control are performed. To derive the LPV model, the velocity $V$ and the altitude $h$ are selected as scheduling parameters, i.e., $\varrho = [V, h]^T$. It is obvious that the resulting LPV model is actually a quasi-LPV model. For lack of space, just some simulation results are given in Figures 5 – 10.

Figures 5 – 8 show that the tracking performance in closed-loop for the velocity and altitude, respectively. It can be seen that the tracking errors for the velocity and altitude converge to zero. The tracking performance is satisfactory, in spite of parameter
5. Conclusions. In this paper, a velocity-based LPV modeling and control framework combined with a novel implementation method of nonlinear gain-scheduling controller has been presented. The framework provides a general tracking control structure, which is applied to the flight control of nonlinear longitudinal dynamical systems of the airbreathing hypersonic vehicles. The effectiveness of the controller is demonstrated by simulation results of the responses of the step changes in altitude and velocity. These results are encouraging and motivate further investigation of potential applications of the proposed framework, such as a fairly general class of control structures that are usually referred to as tracking controllers. Furthermore, the proposed method in this paper makes it straightforward to implement antiwindup control schemes. This is necessary in applications where the input is constrained owing to actuator saturation or rate limit.

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