# Precise Orbit Determination for CHAMP using GPS Data from BlackJack Receiver

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## ABSTRACT

The CHAllenging Minisatellite Payload (CHAMP) was launched into a 450-km low-Earth orbit in July, 2000, to support geoscientific and atmospheric research. The mission is being managed by GeoForschungsZentrum Potsdam (GFZ) in Germany, and NASA is one of three international partners. Among the scientific instruments on board is an advanced codeless, dual-frequency GPS receiver developed by the Jet Propulsion Laboratory (JPL). This "Blackjack" receiver supports several important operational and scientific functions. In addition to disseminating precise timing and navigation information to the spacecraft, the Blackjack receiver collects GPS tracking information to support precise orbit determination (POD) activities on the ground. The receiver also supports the collection of atmospheric limb sounding measurements and GPS specular reflection signals through side- and down-looking antennae respectively.

In this paper we present results for CHAMP POD using the precise GPS measurements collected by the BlackJack receiver through the up-looking antenna. We will describe the quality of the tracking data, the tuning of the reduced-dynamic model for the 400-km orbit, and the various methods of evaluating the orbit accuracy. Comparisons of overlapping orbit solutions suggest that the GPS-based CHAMP orbits are accurate to better than 10 cm in all three components. This is further supported by test of independent measurements with precise satellite laser ranging (SLR) systems. We will also describe orbit solutions obtained with different strategies and dynamic models, and discuss the possible remaining error sources and ways to further improve the orbit solutions.

## INTRODUCTION

CHAMP (CHAllenging Minisatellite Payload) is a German small satellite mission managed by GFZ (GeoForschungsZentrum Potsdam). The CHAMP satellite was launched into a near polar, low-Earth orbit in July, 2000. This geoscientific and atmospheric research mission has several scientific objectives, including precise measurement of the Earth's gravity field and magnetic field, high resolution profile of temperature and water vapor content of the Earth's atmosphere, and mapping of electron density of the Earth's ionosphere. NASA is one of the international partners of CHAMP mission. NASA's Jet Propulsion Laboratory developed and provided a new generation flight GPS receiver, the "BlackJack" receiver. The BlackJack receiver collects GPS measurements through three different antennas. It collects direct GPS measurements through the up-looking antenna for precise orbit determination (POD), collects atmospheric limb sounding measurements through the rear-looking antenna for atmospheric profile, and collects GPS specular reflection signals from the ocean surface through the nadir-looking antenna for GPS-altimetry experiment. In addition to the GPS tracking for POD, CHAMP also has a laser retro-reflector Satellite Laser Ranging (SLR) measurement to support the POD activity.

Shortly after the CHAMP was deployed into the orbit, it's onboard Blackjack GPS receiver started collecting precise dual-frequency measurements. At JPL we analyzed the GPS tracking to determine the precise orbit. Our primary goal for this analysis is to obtain the precise position and velocity of the CHAMP orbit, without adjusting the Earth gravity field model. This precise orbit information is a product essential for fulfilling the mission's scientific goal on geomagnetic and atmospheric study. The surface force perturbation on CHAMP learned through this orbit determination process will also be helpful to the precise Earth gravity field mapping.

## STRATEGY FOR CHAMP ORBIT DETERMINATION

Before flown on CHAMP, BlackJack receivers had flown on NASA's space shuttle for the SRTM, and successfully met the mission' requirement of 60cm orbit determination [Bertiger et al., 2000]. After that, the performance of BlackJack receiver has been improved significantly through several software upgrading. Ground test shows 3-4 cm kinematic positioning accuracy with about 98% of the time.

CHAMP flies in a low Earth orbit satellite at altitude of 450 km. This is the first time for a high-accuracy scientific satellite orbit to be determined with GPS tracking at such low altitude. To determine the CHAMP orbit, we use the reduced dynamic technique that has proved successful for other low Earth orbit missions [Yunck et al., 1990, Wu et al., 1991, Bertiger et al., 1994].

In our processing of the CHAMP GPS data, GPS satellite orbit and transmitter clock were held fixed to the precise values determined from an independent process that analyzes data from a globally distributed ground network [Jefferson, 1998]. Using the reduced dynamic technique for orbit determination, we first develop a dynamic model for the CHAMP orbit motion that is best to our knowledge, and estimate the orbit initial position and velocity and a few dynamic parameters. Upon this dynamic model, we estimate a series of stochastic accelerations to compensate the perturbation that is missing in the dynamic model and whose physical nature is unknown. Finding an optimum combination of the dynamic model and the stochastic series is usually referred to as "tuning" the model.

*Dynamic Models*

The dynamic model for CHAMP orbit includes JGM-3 70x70 Earth gravity field [Tapley, et al., 1995], atmospheric drag, solar radiation and Earth radiation pressure force, and relativity acceleration. Six initial state parameters, one drag coefficient and one radiation pressure coefficient were estimated. For satellite shape, we tested sphere body model, and a model of six surfaces plus a boom. Tracking data were processed by daily orbit arc, each arc contains 30 hours of data, with 24 hours in current day, 3 hours in previous day and 3 hours in next day. In this way, 6 hours of orbit overlap can be formed between each two orbital arcs, which is useful in orbit precision evaluation and model tuning processes. Correction to receiver clock is estimated as a white noise series.

*Tuning Process*

The stochastic accelerations were treated as a first order Markov process [Bierman, 1977]. To estimate/update the time series, a correlation time and a process noise level need to be pre-selected. The purpose of the tuning process is to find the best values of these parameters in combination with the dynamic model. The performance of the combination is judged by examining a set of quantities that were set as our goal. In our process, the orbit overlap difference is the primary quantity been examined. An optimally tuned model should have:

--minimum orbit overlap differences, the orbit overlap should be close to the corresponding formal sigma of orbit position;

--minimum postfit data residual RMS;

We developed the following procedure to search for the optimum parameters:

1. Use the orbit overlap for the dynamic orbit solution to evaluate the process noise level. For an orbit component, the approximate process noise level is

$$\sigma_p = \sqrt{3n}\,\sigma_n / T^2 \qquad (1)$$

   where T is half of the orbit arclength, n = T/$\Delta$t is the total number of data time points over T, and $\sigma_n$ is the orbit overlap difference RMS.

2. Use the ratio of formal sigma for kinematic solution over the process noise level to approximate the correlation time. We choose the correlation time as

$$\tau \approx \frac{1}{2}\sqrt{\frac{\sigma_k}{\sigma_p}} \qquad (2)$$

   where $\sigma_k$ is the formal sigma for kinematic orbit position solution, and $\sigma_p$ is the process noise level for the same component (see Appendix).

3. Fine-tune the parameters around the values estimated in step 1 and 2.

In our tuning process for CHAMP orbit, we chose four day's data, and used the three orbit overlap sessions to decide the best acceleration parameters for radial, cross-track and along-track components. Once the orbit model is tuned, our automated system processes the tracking data and generates orbit solution everyday. The accuracy of these orbit products is similar for each day, until the satellite status or tracking data quality changes, when we need to re-tune the model.

## CHAMP ORBIT ACCURACY AND PRECISION

Similar to the orbit evaluation in the tuning process, the quality of our CHAMP orbit product was primarily evaluated and routinely monitored through orbit overlap comparison. Theoretically the orbit overlap difference measures the orbit precision, not the orbit accuracy. Certain types of systematic error may not be revealed in this self-consistency test. However, past experience [e.g., Bertiger et al., 1994] has suggested that orbit overlap difference can be a good approximation of orbit accuracy if we use it with caution and use other test to detect possible systematic errors.

*Orbit overlap Comparison*

Figure 1 shows the orbit overlap difference of one overlapping session (between two daily solutions). To compute the RMS of the orbit overlap difference, we excluded half hour from each end of the 6 hour overlapping session. Since the orbit solution at the edge of an orbit arc is not well smoothed in the reduced dynamic orbit determination process, those orbit points typically has quality below the normal solution. We compute the mean orbit overlap RMS (RMS of 3 component RMSs) of each overlap session, and use it as the measure of one component orbit precision for that day. Figure 2 shows the computed mean orbit overlap RMS over 100 days.

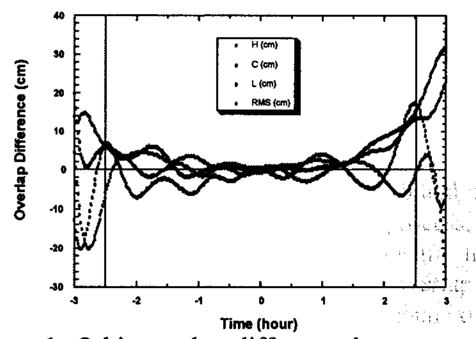


Figure 1. Orbit overlap difference between solution on July 29 and July 30, 2000.

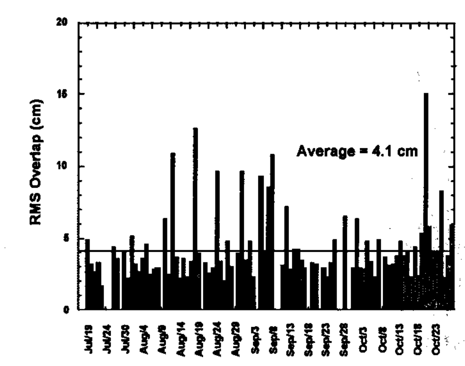


Figure 2. Mean overlap RMS for daily orbit solution from July 18 through October 25, 2000.

*SLR Residual Test*

To evaluate the orbit accuracy independently, we computed the SLR (Satellite Laser Ranging) data residual, using the GPS data determined the CHAMP orbit position. Figure 3 shows the SLR data residual statistics for 6 SLR tracking sites over 48 days. This is the residual RMS after a range bias and a time bias are removed from each pass of data. The residuals were significantly higher on the early days, before the satellite status stabilized. The average of the standard deviation in Figure 3 is lower than that in Figure 2, this number may be a little optimistic because many of the passes are short ones, and in a short pass some orbital error can be absorbed into the residual time bias.

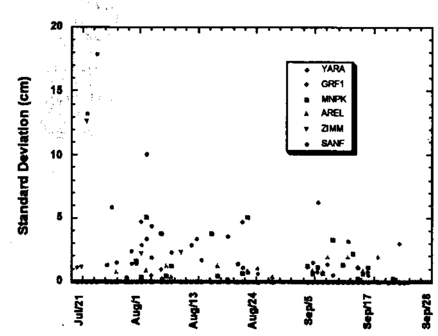


Figure 3. SLR residual standard deviation from July 18 through September 23, 2000 for six sites.

*Comparison with Kinematic Solution*

We also made a case of kinematic solution and compare the orbit with the normal reduced dynamic solution. In the kinematic solution, the process noise level was set to extremely large value, so that dynamic model was practically excluded. By comparing with the orbit of kinematic solution, we can detect possible gross error in our dynamic modeling. Orbital position points with formal error less than 0.5 meter from the kinematic solution were chosen to make the comparison, that counts about 80% of the total number of solution points. Table 1 shows the statistics of the orbit difference between the two solutions on August 7, 2000.

Table 1. Statistics of orbital position differences.

| | Radial (m) | Cross-track (m) | Along-track (m) |
|---|---|---|---|
| Mean | -0.05 | -0.03 | 0.03 |
| Std | 0.40 | 0.16 | 0.20 |

The standard deviation shown in the table agrees with the average formal sigma value for corresponding component. This table shows that there is no significant bias between our reduced dynamic solution and the kinematic solution, and excludes the possibility of gross error in our dynamic modeling.

## SUMMARY

The BlackJack flight GPS receiver onboard CHAMP demonstrated good data quality. With the dual-frequency tracking data, and with a fine-tuned reduced-dynamic model, the 450-km high CHAMP orbit is determined to sub-decimeter accuracy routinely. Orbit overlap comparison and Satellite

Laser Ranging residual test suggest the orbit precision of 5 cm for each component. An automated process is generating the precise orbit products every day to support the scientific task of the mission.

## ACKNOWLEDGMENTS

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## APPENDIX

### 1. Evaluation of Process Noise Level

First examine the effect of stochastic acceleration over a time interval. Let the dynamic equation be

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a \tag{a1}$$

where x and v are the position and velocity, the state variable, a is the stochastic acceleration, the process noise term. For a small enough time interval $[t_k, t_{k+1}]$, a can be regarded as a constant $a_k$. Integration of equation (a1) over this time interval is

$$\begin{bmatrix} x \\ v \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}_k + \begin{bmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix} a \tag{a2}$$

where $\Delta t = t_{k+1} - t_k$ is the span over the time interval. The increments of the state variables over $\Delta t$ are:

$$x_{k+1} - x_k = \Delta t\, v_k + \frac{1}{2}\Delta t^2\, a_k$$

$$v_{k+1} - v_k = \Delta t\, a_k \tag{a3}$$

where subscript k marks the time epoch of the state variable and process noise. Summing up the increments from $t_0$ to $t_{k+1}$, we have

$$x_{k+1} - x_0 = \Delta t \sum_0^k v_i + \frac{1}{2}\Delta t^2 \sum_0^k a_i$$

$$v_k - v_0 = \Delta t \sum_0^{k-1} a_i \tag{a4}$$

Let's consider the case where $x_0 = 0$, $v_0 = 0$, and focus on the process noise, then position error at $t_{k+1}$ is

$$x_{k+1} = \Delta t^2 \sum_0^k \sum_0^{i-1} a_j + \frac{1}{2}\Delta t^2 \sum_0^k a_i$$

$$= \Delta t^2 \sum_{0}^{k}(\sum_{0}^{i-1} a_j + a_i - \frac{1}{2}a_i)$$

$$= \Delta t^2 \sum_{0}^{k}(\sum_{0}^{i} a_j - \frac{1}{2}a_i)$$

$$= \Delta t^2 \left[\sum_{0}^{k}(k+1-i)a_i - \sum_{0}^{k}\frac{1}{2}a_i\right]$$

$$= \Delta t^2 \sum_{0}^{k}(k - i + \frac{1}{2})a_i \tag{a5}$$

If $a_i$ is a white Gaussian process

$$E[a_i] = 0 \;; \qquad E[a_i\, a_j] = \sigma^2\, \delta(i-j)\,; \tag{a6}$$

then the mean and variance of $x_{k+1}$ are

$$E[x_{k+1}] = 0$$

$$E[x_{k+1}\, x_{k+1}] = \Delta t^4\, \sigma^2 \sum_{0}^{k}(k + \frac{1}{2} - i)^2 \tag{a7}$$

Denote n = k + 1, then we have

$$\sigma^2_{x_{k+1}} = E[x_{k+1}\, x_{k+1}] = \Delta t^4\, \sigma^2 \sum_{1}^{n}(j - \frac{1}{2})^2$$

$$= \Delta t^4\, \sigma^2\, (\frac{1}{3}n^3 - \frac{1}{12}n) \tag{a8}$$

When n is a big number, the cubic term grows much faster than the square term and linear term, we can use the first term as an approximation:

$$\sigma_{x_{k+1}} \approx \Delta t^2\, \sigma n \sqrt{n/3} = T^2\, \sigma \,/ \sqrt{3n} \tag{a9}$$

where $T = n\Delta t$ is the total time from $t_0$ to $t_n$. The equation approximately shows the position error accumulated over time T due to n steps of stochastic acceleration. We can use this formula to inversely evaluate the process noise if we know the position error.

Orbit overlap difference of dynamic orbit solution can be regarded as orbit position error due to unmodeled dynamic error (process noise), since the measurement error effect is much smaller over relatively long arc. Given the overlap error and arc length, we can evaluate the process noise level (stead state sigma) as

$$\sigma_p = \sqrt{3n}\, \sigma_n \,/\, T^2 \tag{a10}$$

where T is half of the orbit arclength, and $n = T/\Delta t$, sn is the overlap error RMS.

## 2. Choosing the Correlation Time

The geometric strength of measurement at one epoch can be judged by the formal error of kinematic position solution. Adding of dynamic information should improve this strength. Now imagine we estimate the perturbation acceleration as piecewise constant and update it over every n measurement time ($n\Delta t$), then the effect of measurement error will decrease as data strength improves by

$$\sigma_m \approx \sigma_k \,/ \sqrt{n} \tag{a11}$$

where $\sigma_k$ is the formal sigma of kinematic positioning and sm is formal sigma of measurement error effect over n epoch. On the other hand, effect of dynamic error will increase as the dynamic noise that is different from the constant accumulates, according to equation (a9)

$$\sigma_d \approx \Delta t^2\, \sigma_p\, n \sqrt{n/3} \tag{a12}$$

The optimum time span for updating the piecewise constant would be such that the combination of the effect of the two is minimum over the time span, i.e.,

$$\frac{\partial}{\partial n}(\sigma_m^2 + \sigma_d^2) = 2(\sigma_m \frac{\partial \sigma_m}{\partial n} + \sigma_d \frac{\partial \sigma_d}{\partial n}) = 0$$

or

$$(-\frac{1}{2})\frac{\sigma_k^2}{n^2} + (\frac{1}{2})\Delta t^4 n^2 \sigma_p^2 = 0$$

This leads to

$$T^2 \approx n^2 \Delta t^2 = \frac{\sigma_k}{\sigma_p} \tag{a13}$$

This is an approximate optimum time for updating piecewise constant acceleration. For updating correlated acceleration we can use $2\tau = T = n\Delta t$. The process noise estimated from equation (a10), and correlation estimated from equation (a13) can serve as the nominal values for the tuning process.