Synthesis of amplitude and phase achromatized dielectric mirrors

Zdenek Knittl

SYNTHESIS OF AMPLITUDE AND PHASE ACHROMATIZED DIELECTRIC MIRRORS

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Résumé. — On déduit les conditions analytiques pour les indices de réfraction d'une couche multiple $\lambda/4$ générale, assurant la dispersion nulle du déphasage par réflexion. On indique les méthodes numériques pour chercher à l'aide du computeur automatique les solutions et, simultanément, les conditions d'achromatisme en amplitude, en conservant la réflexion nominale du miroir.

Abstract. — Conditions for an achromatic phase change upon reflection are derived in terms of the refractive indices of a quarter-wave dielectric multilayer. A numerical method of solution is indicated using a trial-and-error sequence on a digital computer. This method is extended to the simultaneous solution of achromatic amplitude, observing also the condition of a prescribed standard reflectivity value.

The well-known advantage of dielectric semi-reflectors of being non-dissipative is somewhat invalidated by the strong dispersion of reflected power and phase. Since the original idea that a quarter-wave stack of alternating refractive indices may simulate a metallic film of some thickness, design work has concentrated on improving the dispersion characteristics of such dielectric mirrors. There have been methods of varying the optical thicknesses and methods of selecting refractive indices. The first kind of procedure is of a rather empirical character, but well suited to the present day technologies with limited material resources. On the other hand, the variation of indices, although still ahead of technology, allows of a more compact theoretical treatment, since it is concerned with a " tuned ", i.e. strictly quarter-wave multilayer.

During the greater part of its history the quarter-wave stack has been regarded as an alternating sequence of two refractive indices, which in the domain of power-reflection implies the existence of a central maximum at

$$\varphi \equiv (2\pi/\lambda)\, nh = \pi/2$$

and a certain number of subsidiary maxima on both sides of it. The corresponding phase-change δ in dependence of φ is an oscillating function with a more or less negative slope at $\varphi = \pi/2$.

These typical characteristics of the orthodox quarter-wave system may be substantially changed by using a general sequence ot indices.

A drastic example how conditions may change is the theoretical case computed recently by R. J. Pegis [1] : by taking some five quarter-wave layers of various materials, with the refractive indices in the range $\Lambda \leqslant n \leqslant 12.00$, the central extreme becomes a subsidiary one, while two high maxima build up in the left and right hand part of the ρ-versus-φ diagram. The left one may then be used to cover a broad spectral range, which was the goal of the synthesis.

The aim of the present report is not to change the key position of the $\varphi = \pi/2$ region, but rather to influence the shape of the $\rho(\varphi)$ and $\delta(\varphi)$ functions in that region, observing simultaneously the condition of a prescribed light division $\rho : (1 - \rho)$, with ρ given as $\rho(\pi/2)$.

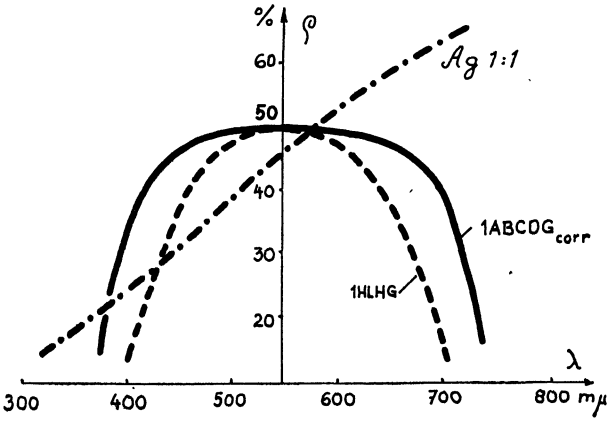

FIG. 1. — Power-dispersion of a metallic (Ag), non-corrected dielectric (1HLHG) and of a dielectric reflector (1ABCDG) corrected by proper choice of refractive indices.

The problem of shaping $\rho(\varphi)$ in the vicinity of $\varphi = \pi/2$ was theoretically solved in 1952 by Pohlack [2], [3].

His expansion of the inverse transmittance $1/\tau$ as a symmetrical polynimial in $\cos^2 \varphi$:

$$\frac{1}{\tau} = A_{k0} + A_{k2}\,\Lambda^2 + \ldots A_{k2\nu}\,\Lambda^{2\nu}\,; \quad \Lambda = \cos\varphi,$$

with the $A_{k\nu}$'s being functions of the refractive indices alone, has lead to a synthetic condition for

a flat (i.e. achromatic) extreme in terms of the refractive indices :

$$A_{k2} = 0.$$

Solving this equation proved to be a fairly laborious affair.

In a recent co-paper [4], the present author has contributed to the usefulness of this method by modifying it into a form suitable for automatic digital evaluation, observing at the same time the calibration condition

$$A_{k0} = 1/\tau = 1/(1 - \rho) \qquad (\varphi = \pi/2).$$

Another method of achromatizing a dielectric semireflector was worked out by Kard in [5], as an extension of his previous work on a similar problem in antireflection techniques [6].

To the author's knowledge no attempt has yet been made to flatten out the dispersion characteristics of the phase-change upon reflection, although the simple fact that it does exist and causes positive, negative or interesting effects in interference spectroscopy, metrology and filtering techniques gave rise to a number of papers : by Rank and Bennett, by Baumeister and Jenkins or Bruce and Ciddor [7, 8, 9], to mention at least some of them.

In the present paper a theoretical condition is derived ensuring a flat δ-versus-φ curve in the region of the central reflectivity extreme, and methods for its numerical solution are indicated. The problem is then extended to the simultaneous study of power and phase achromaticity and a procedure is found enabling a systematic search for achromatic power, with the phase-dispersion achromatized implicitly. By a simular procedure a higher-order achromatization of phase only may be investigated.

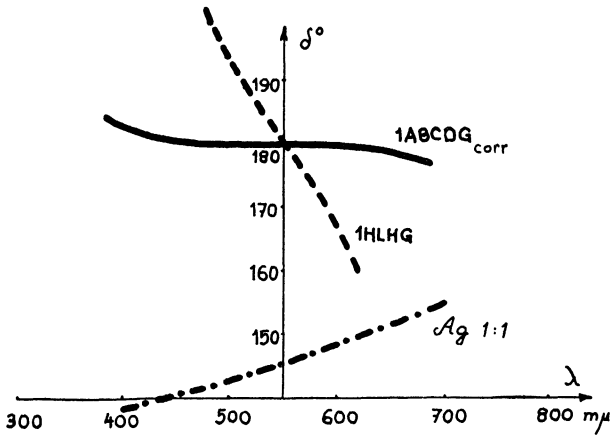

FIG. 2. — Phase-dispersion concerning the semireflectors in the figure 1. A comparison of the curves 1HLHG and 1ABCDG suggests what is to be expected from the theory to follow.

As a general introduction and for better understanding of the system of notation used, the reader is referred to the papers [2] and [4].

The total interference matrix of the quarter-wave system being $\|K_{\mu\nu}\|$, the reflected phase-change is given by the relations

$$\operatorname{tg} \delta = \frac{2n_0}{n_g} = \frac{y}{Z}$$

where

$$y = n_g(I_m\, K_{12})\, K_{22} - \frac{1}{n_g}(I_m\, K_{21})\, K_{11}$$

$$Z = \left(\frac{n_0}{n_g}\right)^2 K_{11}^2 - K_{22}^2 + n_0^2(I_m\, K_{12})^2 - \frac{1}{n_g^2}(I_m\, K_{21})^2. \quad (1)$$

Since for quarter-wave systems any central extreme at $\varphi = \pi/2$ entails either $\delta = 0$ or π, it is to be expected that the denominator Z is stationary at that point and the rate at which δ changes in the vicinity of $\varphi = \pi/2$ is essentially given by the numerator. In fact, it may be proved rigorously that a necessary and sufficient condition for δ to be stationary at the central extreme is

$$y = dy/d\varphi = 0,$$

so that the above given expression for y presents our main interest. We shall try to bring it into a form, where stationarity may easily be seen. The method consists in developing a polynomial in $\Lambda = \cos\varphi$ and $\Lambda^* = \sin\varphi$.

We shall make use of Pohlack's expressions defining the $K_{\mu\nu}$'s in terms of algebraic forms of the k-th degree (k, numbers of layers), with the coefficients given as special functions $N_{K\nu}^{(a)}$, $N_{K\nu}^{(b)}$ of the refractive indices.

After some tedious work y may be put into the form

$$y = \Lambda\Lambda^*(C_{K0} + C_{K2}\,\Lambda^2 + \ldots + C_{K/2(K-1)}\,\Lambda^{2K-1}) \quad (2)$$

where the $C_{K\nu}$ are functions of the $N_{K\nu}^{(a)}$, $N_{K\nu}^{(b)}$ and the outer indices n_0, n_g.

$dy/d\varphi$ at the central extreme $\varphi = \pi/2$ is easily seen to equal C_{k0} so that the condition for a stationary phase is

$$C_{k0} = 0. \quad (3)$$

Evaluating the second derivative $\ddot{y}$ for $\Lambda = 0$ this is found to vanish independently of C_{k0} and $\dot{y}$, so that there is always inflexion in the δ-versus-φ curve at $\varphi = \pi/2$. When in addition $\dot{y} = 0$ is satisfied, the inflexion point has a horizontal tangent. It may be assumed that, while power reflection may be achromatized in the way of a flat extreme, achromaticity of the phase-change corresponds to a horizontal inflexion.

The region of the horizontal tangent may, of course, be more or less extended. The flatness of the inflection will increase with the number of coefficients annulled. Thus the combined conditions

$$C_{k0} = 0, \qquad C_{k2} = 0 \quad (4)$$

should lead to a better phase-achromaticity than (3) alone.

By inspection of the way the expression y is built up, these two coefficients may be found to be, if k odd :

$$C_{K0} = n_g N^{(b)}_{KK} N^{(a)}_{KK^{-1}} - \frac{1}{n_g} N^{(a)}_{KK} N^{(b)}_{KK^{-2}}$$

$$C_{K2} = -(K-1) C_{K0} - \left[n_g N^{(b)}_{KK} N^{(a)}_{KK^{-2}} - \frac{1}{n_g} N^{(a)}_{KK} N^{(b)}_{KK^{-2}}\right] - \frac{n_g^2 - 1}{n_g} N^{(a)}_{KK^{-1}} N^{(b)}_{KK^{-}}$$

if k even,

$$C_{K0} = \frac{1}{n_g} N^{(b)}_{KK} N^{(a)}_{KK^{-1}} - n_g N^{(a)}_{KK} N^{(b)}_{KK^{-}}$$

$$C_{K2} = -(K-1) C_{K0} + \left[n_g N^{(a)}_{KK} N^{(b)}_{KK^{-2}} - \frac{1}{n_g} N^{(b)}_{KK} N^{(a)}_{KK^{-2}}\right] + \frac{n^2 - 1}{n_g^2} N^{(a)}_{KK^{-1}} N^{(a)}_{KK^{-}}.$$

The further coefficients are still more complicated and any conditions imposed on them would complicate matters enormously. First of all we shall deal only with C_{k_0}.

By the manner the $N^{(a)}_{K\nu}$ and $N^{(b)}_{K\nu}$ are built up of the refractive indices, already the simplified condition (3) represents a very intricate equation, which it is difficult to tackle for a solution. However, it is possible to apply the method developed in [4], where the solution of $A_{k2} = 0$ was reduced to a trial-and-error sequence on a digital computer. One of the indices is chosen as the independent variable, while the remaining ones are given by the calibration condition.

Since the $N^{(a)}_{K\nu}$, $N^{(b)}_{K\nu}$ computed for the Pohlack coefficient A_{k2} more than suffice to combine into the term C_{k_0}, the whole method of solving (3) is essentially identical with that described in [4], differing only in the way the $N^{(a)}_{K\nu}$ and $N^{(b)}_{K\nu}$ are combined into a final result.

Practical tests have shown, however, that a phase achromatized system is often extremely dispersive as regards power reflection. It is very difficult to keep both power and phase flat, because in the adopted trial-and-error system there are now two functions C_{k_0} and A_{k_2} to be annulled by one variable.

The computing procedure was therefore reshuffled in the following way : let there be three variable indices n_p, n_q, n_w, distributed in the complete system according to the pattern

$$n_1, n_2, \ldots n_q, \ldots n_{k-2}, n_q, n_w ; \qquad 1 \leqslant p \leqslant k-2$$

n_p again is the independent variable proper, n_q and n_w are to be determined implicitly.

Denote $k - 2 = k'$. If, during the recursion process according to [4], the first k' indices are consumed, the $N^{(a)}_{k'\nu}$, $N^{(b)}_{k'\nu}$ ($\nu = k', k'-1, k'-2$) so far computed already suffice to set up a quadratic equation for the last index $n_w = n_k$, whose solution, together with the first $k-2$ indices ensures $C_{k_0} = 0$ for the given calibration. This is secured in the last step by computing $n_k - 1$ so far skipped.

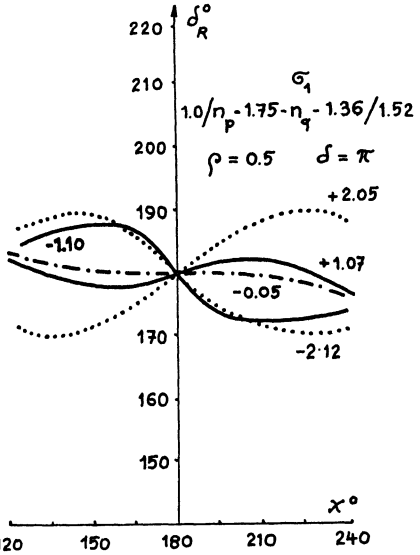

FIG. 3. — Describing five situations in a four-layer system, calibrated for $\rho = 50$ % and $\delta = \pi$. For various n_p the C_{k0} assumes the values $\pm$ 2.0, $\pm$ 1.0 and — 0.05. The last phase dispersion curve is indeed flat at the centre.

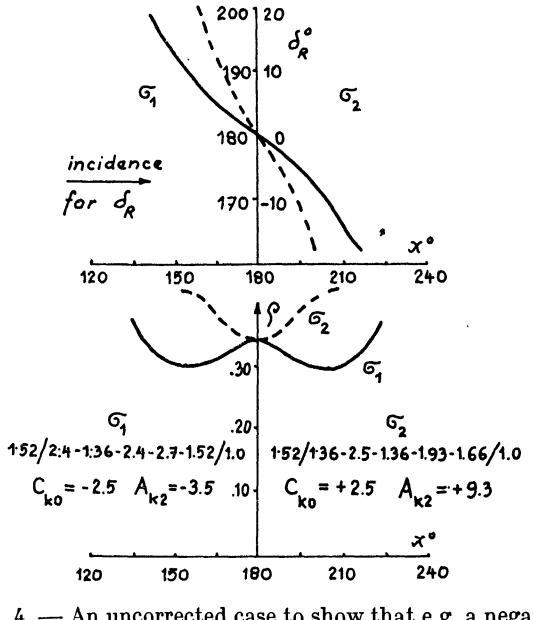

FIG. 4. — An uncorrected case to show that e.g. a negative slope of the δ-versus-φ function may correspond to a negative or positive C_{k_0}, according as the multilayer is of the $\sigma_1(\delta = \pi)$, or $\sigma_2(\delta = 0)$ class. This is in connection with the quantity z cooperating on tg δ and $dy/d\varphi$.

With the set of indices thus completed, the recursion may proceed to $N^{(a)}_{KK-1}\, N^{(a)}_{KK}$ and $N^{(b)}_{KK-11}\, N^{(b)}_{KK*}$ and the coefficient A_{k_2} is set up. Obviously, it is now only the function of n_p, with n_q, mnw implicitly involved by the phase and calibration condition. The operator now only follows this function $A_{k_2}(n_p)$ and tries to bring it to zero, as in the case of calibrated power-achromaticity.

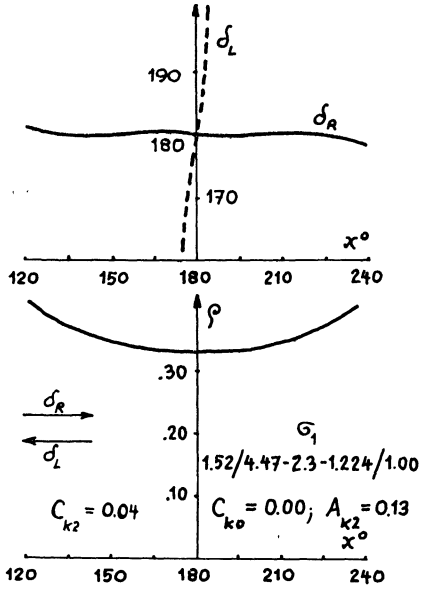

FIG. 5. — The simultaneous suppression of power and phase dispersion is an exception rather than a rule in the original system of computation designed for A_{k_2} alone. A further important feature of this case is that, in addition, C_{k_2} is small. The achromaticity has a broad range.

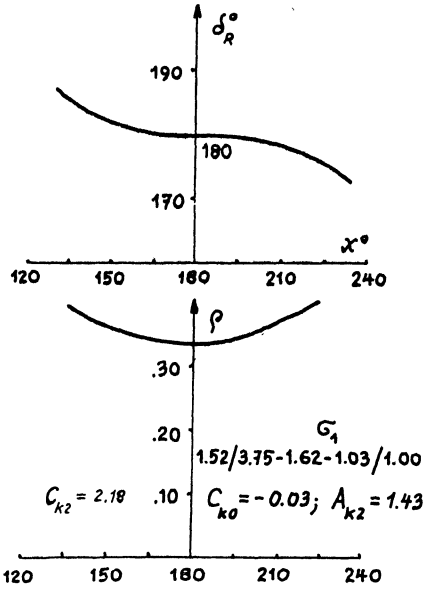

FIG. 6. — A less favourable compromise : $C_{k_0} \approx 0$ with $A_{k_2} = 1.43$. Moreover, $C_{k_2} = 2.185$, which is clearly seen in the shorter range of the horizontal tangent for δ.

The possibility of implicitly observing $C_{k_0} = 0$ and the calibration condition may also be applied to finding a solution to $C_{k_0} = 0$ instead of to $A_{k_2} = 0$. Thus a higher order phase-achromaticity may be achieved, losing of course the possibility of controlling A_{k_2}.

A difficult problem met in numerical work is the occurrence of non-technological indices of refrac-

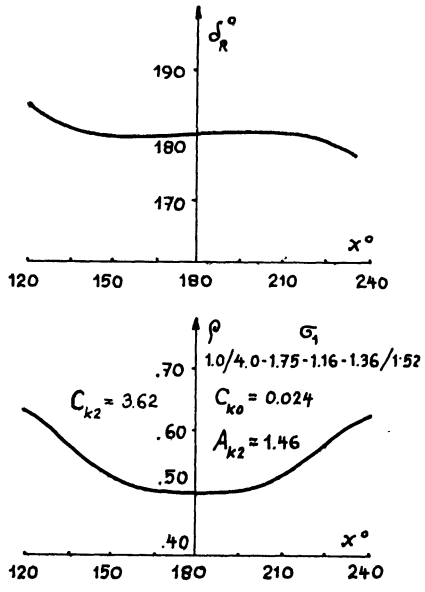

FIG. 7. — Another fair compromise, to be checked in the following figure for oblique incidence at 45°.

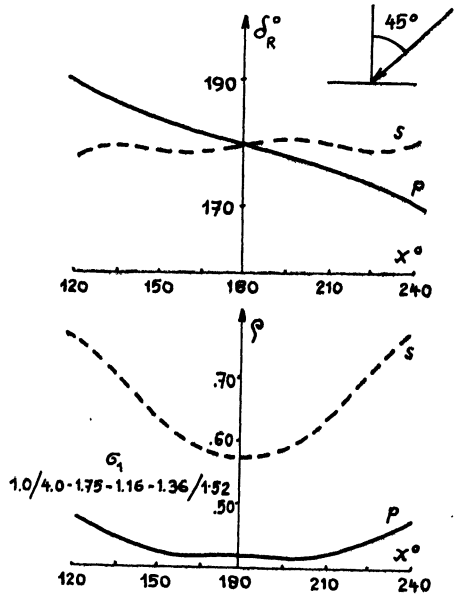

FIG. 8. — The normal design of the preceding figure proves to be stable as regards the angle of incidence. However, this is not a rule, and cases with complete breakdown may occur. A special design with optical admittances and observing polarization and de-tuning is needed in general.

TABLE

n_p	n_q	n_w	A_{k0}	A_{k2}	C_{k0}	C_{k2}
3.000	1.675	1.377	2.0	11.15	— 0.009	207.97
3.450	2.996	2.142	2.0	0.45	— 0.017	197.82
3.470	3.069	2.181	2.0	— 0.02	— 0.021	197.15
3.500	3.181	2.241	2.0	— 0.74	— 0.023	196.12

$$\rho\left(\frac{\pi}{2}\right) = .50 \qquad \delta_R\left(\frac{\pi}{2}\right) = \pi \qquad (\sigma_1)$$

$$1.00/n_p - 1.45 - 1.75 - n_q - n_w/1.52$$

The redesigned theory with three variable indices allows of an automatic correction for

$$A_{k0} = 1/(1 - \rho)$$

and C_{k0}, the operator checking A_{k2} and/or C_{k2}. Extract from a computer record. Compare A_{k2} and C_{k2}.

tion, at least in systems of up to five layers. The author was so far unable to carry out time-consuming procedures on systems with larger numbers of layers, where the probability of a realistic solution seems greater. Also, it might seem of interest to seek some kind of equivalence theorem, enabling to replace one layer of inconvenient refractive index by several normal layers.

Graphs of various achromatized systems will be shown.

Discussion

M. GABLER. — Are there possibilities to realise refractive indices as high as 3.5 ?

Réponse de M. WELFORD. — Some of the refractive indices which are found are not available in the visible but are perhaps available in other regions of the spectrum.

Alternatively it is possible that the theory of L. I. Epstein could be used, in which a layer of very high or low index is simulated by a three-layer symmetrical system in which known available indices are used.

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